The Final Measurement of $Re(\epsilon'/\epsilon)$
by KTeV Collaboration

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for KTeV collaboration
Introduction to $\epsilon'/\epsilon$

- Weak eigenstates contain admixture of “wrong” CP state
  
  \[ |K_S\rangle \sim |K_1\rangle + \epsilon|K_2\rangle \]
  
  \[ |K_L\rangle \sim |K_2\rangle + \epsilon|K_1\rangle \]
  
  \[ K_L = K_2 + \epsilon K_1 \]

  “Direct” in decay process
  
  “Indirect” from asymmetric $K^0 - \overline{K^0}$ mixing

- Useful to define the following measurable quantities
  
  \[ \eta_{+-} \equiv \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} = \epsilon + \epsilon' \]
  
  \[ \eta_{00} \equiv \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)} = \epsilon - 2\epsilon' \]

- \[ \left| \frac{\eta_{+-}}{\eta_{00}} \right|^2 \simeq 1 + 6 \text{Re}(\epsilon'/\epsilon) \]
Define amplitudes to $\pi\pi$ states of a definite isospin:

\[
< I|T|K^0 > = (A_I + B_I) e^{i\delta_I} \\
< I|T|\bar{K}^0 > = (A_I^* - B_I^*) e^{i\delta_I}
\]

$Im(A_I)$ — CP violation
$Re(B_I)$ — CP & CPT violation
$\delta_I$ — final state interaction phase shifts, $\delta_2 - \delta_0 = (-42 \pm 4)^\circ$.

\[
\epsilon'_\text{CP} \approx \frac{i}{\sqrt{2}} \frac{Re(A_2)}{Re(A_0)} \left[ \frac{Im(A_2)}{Re(A_2)} - \frac{Im(A_0)}{Re(A_0)} \right] e^{i(\delta_2-\delta_0)}
\]

\[
\epsilon'_\text{CPT} \approx \frac{1}{\sqrt{2}} \frac{Re(A_2)}{Re(A_0)} \left[ \frac{Re(B_2)}{Re(A_2)} - \frac{Re(B_0)}{Re(A_0)} \right] e^{i(\delta_2-\delta_0)}
\]

As numerically $\epsilon$ is almost parallel to $\epsilon'_\text{CP}$,

- $Re(\epsilon'/\epsilon)$ — Measure of direct CP violation.
- $Im(\epsilon'/\epsilon)$ — Measure of CPT violation.
Kaon Sector Parameters Measurements

Kaon parameters:

- $\Delta m = m_{KL} - m_{KS}$
- $\tau_S$, $\phi_\epsilon$
- $\phi_{+-} \approx \phi_\epsilon + \text{Im}(\epsilon'/\epsilon)$,
  $\phi_{00} \approx \phi_\epsilon - 2\text{Im}(\epsilon'/\epsilon)$
- $\Delta \phi = \phi_{00} - \phi_{+-}$
- $\text{Im}(\epsilon'/\epsilon) \approx -\frac{1}{3} \Delta \phi$

Using interference in the regenerator beam, KTeV can measure not only decay rates but also phases as well as other kaon parameters.

CPT requires:

$$\phi_\epsilon = \phi_{SW} \equiv \arctan \frac{2\Delta m}{1/\tau_S - 1/\tau_L}$$
History of $Re(\epsilon'/\epsilon)$ measurements

KTeV Data yields of $K_L \rightarrow \pi^+\pi^-$ and $K_L \rightarrow \pi^0\pi^0$ events:

<table>
<thead>
<tr>
<th>Year</th>
<th>$K \rightarrow \pi^+\pi^-$</th>
<th>$K \rightarrow \pi^0\pi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>96</td>
<td>−</td>
<td>$0.8 \times 10^6$</td>
</tr>
<tr>
<td>97</td>
<td>$8.6 \times 10^6$</td>
<td>$2.1 \times 10^6$</td>
</tr>
<tr>
<td>99</td>
<td>$14.9 \times 10^6$</td>
<td>$3.1 \times 10^6$</td>
</tr>
</tbody>
</table>

Today: analysis of the complete KTeV data set.
The KTeV Detector

- Two almost parallel neutral beams, $K_L/n \sim 1/1$.
- Movable Regenerator to create $K_S$ beam.
- Large Vacuum decay volume.
- Low material drift chamber spectrometer, high precision CsI calorimeter
Charged Mode Reconstruction

- Magnetic spectrometer to reconstruct kinematics.
- Regenerator/Vacuum beam identification using X-vertex position.
- Clearance cuts to define detector volume.
Neutral Mode Reconstruction

- CsI calorimeter to reconstruct photons energies and positions
- $Z_v$ determined as average of $Z_{\pi^0} = \sqrt{E_1 E_2 R_{12}}/m_{\pi^0}$
- Regenerator/Vacuum beam identification using X-center of energy
- Detector volume defined by veto detectors and $Z_v$
Charged Mode Improvements

Improvements in reconstruction and simulation of backgrounds, veto detectors thresholds. Mass resolution is $1.4 \text{ MeV}/c^2$, 15% better vs KTeV03.

Better description of detector material, scattering and $\delta$-rays. Good description of tails in mass and $p_t^2$ distributions.
Neutral Mode Improvements

- Major improvements in CsI calorimeter simulation.
- Better description of the calorimeter nonlinearities.

Leading systematics for $Re(\epsilon'/\epsilon)$ from energy scale uncertainty reduced by factor of $\sim 2$. 
Acceptance Check

The measurement of $Re(\epsilon'/\epsilon)$ depends on how well MC describes decay vertex distribution for $K_L$ decays. Difference of an average vertex between Reg. and Vac. beam is about 6 m.

$$\delta Re(\epsilon'/\epsilon) \approx \text{slope}$$
Fitting Kaon Parameters

\[ K \rightarrow \pi\pi \] decay rate in Regenerator beam:

\[ N(p, z) \sim |\rho|^2 e^{-\Gamma_{st}} + |\eta|^2 e^{-\Gamma_{Lt}} + |\rho||\eta|\cos(\Delta m t + \phi_{\rho} - \phi_{\eta}) e^{-\Gamma_{t}} \]

Clear interference effect.

- Regeneration amplitude \( \rho \) cancels out for \( \epsilon'/\epsilon \).
- For \( \Re(\epsilon'/\epsilon) \) fit integrated yield in Regenerator beam. Assume CPT, fix \( \Delta m, \tau_S, \Im(\epsilon'/\epsilon) = 0 \).
- For \( \Im(\epsilon'/\epsilon) \) fit shape in Regenerator beam. Float \( \Delta m, \tau_S, \epsilon'/\epsilon, \phi_{\epsilon} \).
Regenerator Properties

![Graph showing the raw regenerator beam transmission as a function of kaon momentum.](attachment:image.png)

- Raw regenerator beam transmission as a function of kaon momentum.

- Measurements of $Re(\epsilon'/\epsilon)$ and $Im(\epsilon'/\epsilon)$ are weakly sensitive to regenerator parameters $\rho$ and transmission.

- For $\Delta m$, $\tau_S$ and $\phi_\epsilon$ regenerator beam transmission and screening corrections are dominant systematic sources.

- Measure transmission directly from data ($K_L \to \pi^+ \pi^- \pi^0$ decays — nine fold increase, special trigger in 99).

- Compare screening corrections directly with $K_L \to \pi\pi$ data.

$\rightarrow$ Total $\phi_\epsilon$ error reduced by factor 2.
Fit is performed without CPT assumption. Band indicate CPT constraint $\phi \epsilon = \phi_{SW}$ applied a posteriori to obtain $\Delta m|_{CPT}$ and $\tau_S|_{CPT}$.

<table>
<thead>
<tr>
<th></th>
<th>KTeV03</th>
<th>PDG08</th>
<th>KTeV08</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m</td>
<td>_{CPT} \times 10^6 \text{fs}^{-1}$</td>
<td>$5261 \pm 15$</td>
<td>$5292 \pm 9$</td>
</tr>
<tr>
<td>$\tau_S</td>
<td>_{CPT} \times 10^{-12} \text{s}$</td>
<td>$89.65 \pm 0.07$</td>
<td>$89.53 \pm 0.05$</td>
</tr>
<tr>
<td>$\phi_{+-}$, degrees</td>
<td>$44.12 \pm 1.40$</td>
<td>$43.4 \pm 0.7$</td>
<td>$43.76 \pm 0.64$</td>
</tr>
</tbody>
</table>

CPT test:

$$\delta \phi = \phi \epsilon - \phi_{SW} = [0.40 \pm 0.56]^{\circ}$$
New result for $\text{Im}(\epsilon'/\epsilon)$:

$$\text{Im}(\epsilon'/\epsilon) = [-17.2 \pm 9.0_{\text{stat}} \pm 18.1_{\text{syst}}] \times 10^{-4} = [-17.2 \pm 20.2] \times 10^{-4}.$$  

or $\Delta \phi = [0.30 \pm 0.35]^{\circ}$, improved vs $[0.39 \pm 0.50]^{\circ}$ (KTeV03) mostly due to energy scale improvement.
\[ \text{Re}(\epsilon'/\epsilon) \text{ results} \]

\[ \text{Re}(\epsilon'/\epsilon) = [19.2 \pm 1.1_{\text{stat}} \pm 1.8_{\text{syst}}] \times 10^{-4} = [19.2 \pm 2.1] \times 10^{-4} \]

\( \chi^2 / \text{dof} = 22.8 / 21 \)

\begin{center}
\begin{tabular}{ccc}
\text{E731 93} & & \text{7.4} \pm \text{5.9} \\
\text{NA31 93} & & \text{23.0} \pm \text{6.5} \\
\text{NA48 02} & & \text{14.7} \pm \text{2.2} \\
\text{KTEV 08} & & \text{19.2} \pm \text{2.1} \\
\text{New World Ave.} & & \text{16.8} \pm \text{1.4} \\
\end{tabular}
\end{center}

KTeV03: \[ \text{Re}(\epsilon'/\epsilon) = [20.7 \pm 1.5_{\text{stat}} \pm 2.4_{\text{syst}}] \times 10^{-4} \]
Conclusions

- Final Result from KTeV collaboration on $Re(\epsilon'/\epsilon)$, based on entire data set:
  \[ Re(\epsilon'/\epsilon) = [19.2 \pm 2.1] \times 10^{-4} \]

- NA48 and KTeV results are consistent with each other, precise value of Direct CP violation parameter $Re(\epsilon'/\epsilon) = [16.8 \pm 1.4] \times 10^{-4}$ is established.

- KTeV data is consistent with no CPT violation:
  - $\phi_\epsilon - \phi_{SW} = [0.40 \pm 0.56]^\circ$
  - $Im(\epsilon'/\epsilon) = [-17.2 \pm 20.2] \times 10^{-4}$
Extras
Screening Corrections

- Determine regeneration amplitude in 10 GeV kaon momentum bins. Agrees with screening corrections calculations for low $P$.

- Calculate phase at a given $P$ using Derivative Analyticity Relation, using the 12 amplitudes.

- Compare variation of the phase vs $P$ from DAR to direct fit to the data — good agreement.
<table>
<thead>
<tr>
<th>Source</th>
<th>$K \to \pi^+\pi^-$</th>
<th>$K \to \pi^0\pi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trigger</td>
<td>0.23</td>
<td>0.20</td>
</tr>
<tr>
<td>CsI reconstruction</td>
<td>—</td>
<td>0.75</td>
</tr>
<tr>
<td>Track reconstruction</td>
<td>0.22</td>
<td>—</td>
</tr>
<tr>
<td>Selection efficiency</td>
<td>0.23</td>
<td>0.34</td>
</tr>
<tr>
<td>Apertures</td>
<td>0.30</td>
<td>0.48</td>
</tr>
<tr>
<td>Acceptance</td>
<td>0.57</td>
<td>0.48</td>
</tr>
<tr>
<td>Background</td>
<td>0.20</td>
<td>1.07</td>
</tr>
<tr>
<td>MC statistics</td>
<td>0.20</td>
<td>0.25</td>
</tr>
<tr>
<td>Total</td>
<td>0.81</td>
<td>1.55</td>
</tr>
<tr>
<td>Fitting</td>
<td></td>
<td>0.31</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>1.78</td>
</tr>
</tbody>
</table>

CsI reconstruction error is reduced from $1.47 \times 10^{-4}$. 
Stability of $\text{Re}(\epsilon'/\epsilon)$ is studied for various data sub-samples as a function on run period, and as a function of kaon momentum. No systematic trends are observed.