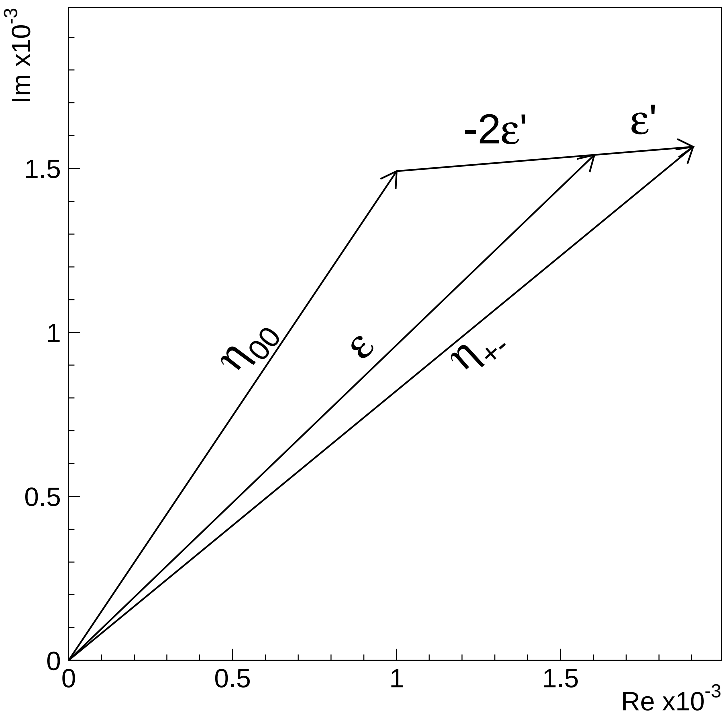
Outline

- New strategy of the kaon parameters fits.
- Re-evaluation of the screening corrections uncertainty.
- Kaon parameters results.

Fit for the Kaon Parameters



← KTeV data measures $Re(\epsilon'/\epsilon)$, $Im(\epsilon'/\epsilon)$, $\phi_\epsilon \equiv \arg(\epsilon)$ as well as δ_M and τ_S. $|\epsilon|$ has been measured by KTeV via $Br(K \to \pi\pi)/Br(K \to \pi e\nu)$ and other braniching ratios. The figure shows KTeV central values with ϵ'/ϵ scaled by $\times 50$.

The kaon parameters can be determined from the fit to the Z vertex distribution in the regenerator beam. CPT symmetry requires

$$\begin{aligned} \Delta\phi &\equiv \arg(\eta_{+-}) - \arg(\eta_{00}) \approx -3Im(\epsilon'/\epsilon) = 0 \\ \phi_\epsilon &= \phi_{SW} \equiv \tan^{-1}(2\Delta m/\Delta\Gamma) \end{aligned} \quad (1)$$

Standard Z-binned fits

Four separate fits:

- ΔM, τ_S fit in charged and neutral mode, assuming CPT invariance. The results are averaged taking into account systematic uncertainties.
- ΔM, τ_S, ϕ_{+-} fit in charged mode, without CPT assumption.
- $Re(\epsilon'/\epsilon)$, $Im(\epsilon'/\epsilon)$, fit without CPT assumption.

The systematic uncertainties are evaluated separatelly for different Z binned fits, taking into account correlation between them. Finally total errors errors with their correlations are reported.

Problems:

- Many fits with various systematic variations have to be repeated many times. For statistics limited systematics (i.e. $< 2\sigma$ change) rather arbitrary decision which is different for different set of variables.
- As the result systematic test are not exactly equivalent. CPT constraint applied *a posteriori* to ΔM, τ_S, ϕ_{+-} fit may lead to different results vs ΔM, τ_S fit in which CPT is required *a priory*.

New procedure

- Perform *one* CPT assumption free fit to neutral and charged mode data simultaneously in which float $\Delta M, \tau_S, \phi_{+-}, Re(\epsilon'/\epsilon)$ and $Im(\epsilon'/\epsilon)$. Report the values obtained without CPT assumption.
- Evaluate systematic uncertainties for this fit, determine total errors covariance matrix.
- Apply CPT constraint for $\Delta\phi$ and ϕ_ϵ to obtain ΔM and τ_S, with the CPT constraint.

Cross checks:

- For statistical errors only the two procedure should yield identical results — yes, for example $\tau_{S_{old}} = 89.634 \pm 0.0177$ and $\tau_{S_{new}} = 89.634 \pm 0.0175$
- For the total errors a rough agreement of the old and new procedure are expected — yes, all agree within $\sim 10\%$.

Systematic Uncertainties – Charged mode

	τ_S	Δm	ϕ_ϵ	$\Re(\epsilon'/\epsilon)$	$\Im(\epsilon'/\epsilon)$
Trigger	-0.004	+2.4	+0.08	+0.13	+1.16
Track reconstruction					
maps	+0.000	0.0	0.00	0.04	0.48
resolution	+0.001	-2.6	-0.08	0.10	-1.20
p_t kick	+0.009	-0.7	0.00	-0.14	1.75
Z DC	+0.002	-0.1	0.00	-0.28	0.39
Selection efficiency					
pt cut	+0.008	-3.6	-0.10	-0.16	+0.96
accidental	0.000	0.1	0.02	+0.05	-0.73
scattering	0.001	-0.3	-0.10	-0.15	+0.17
Apertures					
Cell separation	+0.036	-10.0	-0.31	+0.42	+2.57
Background	0.001	0.0	0.01	0.1	0.6
Acceptance					
Z slope	-0.007	-1.4	-0.04	-0.13	-3.05

Larger uncertainties from the charged mode for $\tau_S, \Delta m, \phi_\epsilon$ – these measurements are dominated by the charged mode.

Systematic uncertainty – Neutral and Total

	τ_S	Δm	ϕ_ϵ	$\Re(\epsilon'/\epsilon)$	$\Im(\epsilon'/\epsilon)$
Trigger	+0.002	+0.9	+0.02	-0.08	-1.71
CsI Reconstruction					
Energy linearity	+0.003	-0.8	-0.01	+2.30	-2.43
Energy scale	-0.008	+0.8	-0.01	-1.72	+12.29
Selection Efficiency					
Ring	-0.002	-0.3	+0.01	-0.18	-2.19
Pairing χ^2	-0.012	+2.2	+0.07	-0.02	+2.19
Shape χ^2	0.0	-0.2	-0.02	-0.06	+0.90
Apertures					
CsI size	+0.006	+0.2	+0.04	+0.64	-8.35
MA	0.	+0.1	0.00	+0.27	-0.21
CA	0.	-0.2	-0.01	+0.47	+0.32
Background	-0.008	-0.3	+0.04	+0.43	-6.69
Acceptance	-0.002	-0.1	-0.01	+0.13	+2.81
Fitting					
Attenuation Norm	-0.003	0.3	-0.01	0.01	0.01
Attenuation Slope	0.003	-2.1	0.05	0.05	0.00
Target K_S	-0.026	+4.7	+0.11	0.00	0.00
Screening	-0.018	+5.6	-0.02	-0.57	+1.35
Analytisity	0.0	0.0	0.25	0.0	0.0
MC statistics	0.016	4.9	0.15	0.36	2.78
Total Syst	0.056	14.7	0.49	3.17	18.06
Stat Error	0.042	12.8	0.40	1.31	9.04
Total Error	0.070	19.5	0.63	3.43	20.20

For $\Re(\epsilon'/\epsilon)$ and $\Im(\epsilon'/\epsilon)$ the neutral mode errors dominate. Screening corrections errors are from new evaluation (was 0.75° for ϕ_ϵ).

Part II: Regenerator Screening and Analyticity

- Standard treatment
- Second Power law fit
- Fit in P_K bins + derivative analyticity relation (DAR).
- How to assign systematic uncertainty

Standard treatment

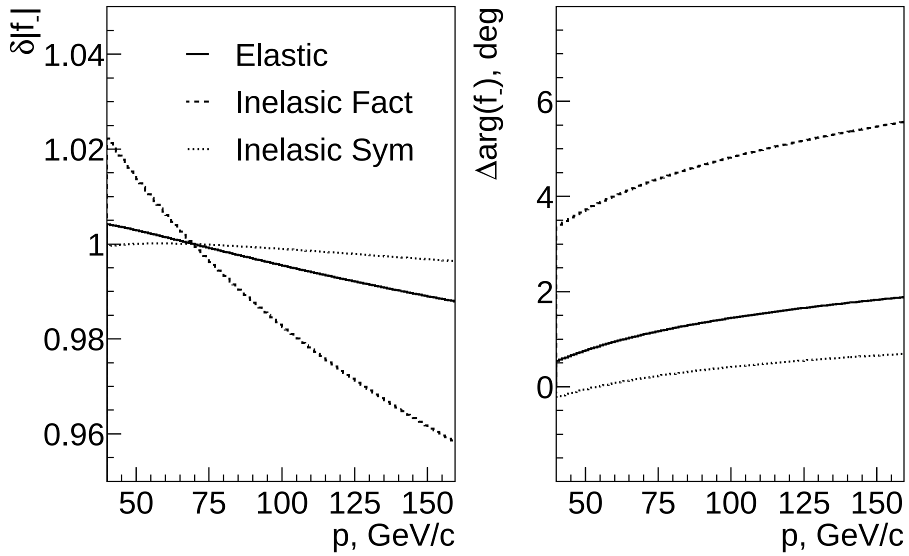
- Kaon propagation through regenerator uses complete matix formalism, using complete material information as a function of Z:

$$\begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}' = T \begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}$$

- Forward scattering regeneration amplitude, $f_- = \hbar \frac{f(0) - \bar{f}(0)}{p}$, is assumed to follow a fixed power law in lead and fixed double power law in hydrogen.
- f_- in plastic is a density weighted sum of f_- in **C** and **H**.
- Forward scattering amplitude in **C** is considered to be a single power law, $f_- = A_{70} \left(\frac{p}{70\,\mathrm{GeV/c}}\right)^{\alpha}$ (ω-trajectory) with screening corrections. The normalization A_{70} and slope α are free parameters of the fit to $K \to \pi\pi$ data. The phase of f_- is given by analyticity relation:

$$\arg(f_-) = -\pi(1 + \alpha/2)$$

Screening Corrections

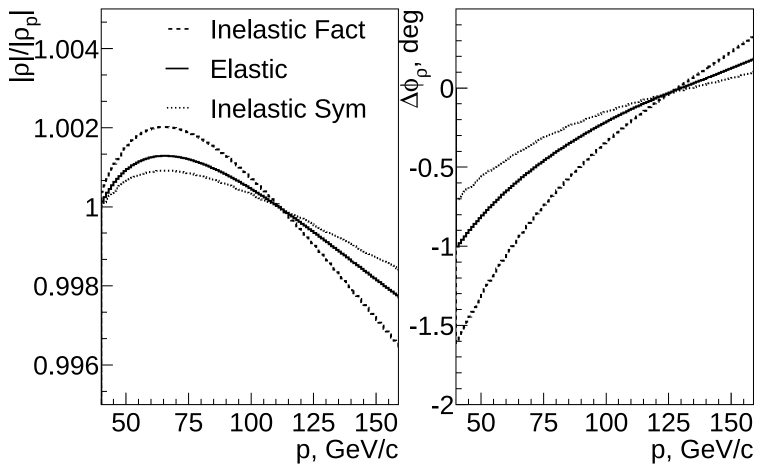


- Screening corrections are calculated for carbon:

$$A_{70}^{full} = A_{70} \times \delta|f_-|(p)$$
$$\arg(f_-^{full}) = \arg(f_-) + \Delta \arg(f_-)$$

- Precise calculation for elastic screening, less certain for inelastic. Use "Inelastic Fact" as default, "Inelastic Sym" to estimate systematics.

Screening Corrections 2



← Ratio of the reg. amplitude and difference in phase for different screening models vs fit with no screening.

- Screening corrections modify the slope of the power law + add extra p dependent correction.
- Results of z-binned, CPT free fit to $K \to \pi\pi$ data:

	$1+\alpha$	ϕ_ϵ	χ^2/dof
No screening	0.4187(5)	42.69(43)	471/(432-33)
Elastic screening	0.4285(5)	43.43(43)	431/(432-33)
Elastic+Inelastic Fact.	0.4624(5)	43.86(42)	425/(432-33)
Elastic+Inelastic Sym.	0.4197(5)	43.15(43)	438/(432-33)

ω intersept is 0.43(1) from scattering, 0.436 from Chew-Frautschi plot 0.437(7) from Roy and Bruce and 0.424(5) from CPLEAR fit.

Fit with second power law

- Perturbation of a power law dependence can be approximated by an additional power law trajectory.
- Total f_- is calculated as a sum of the two amplitudes, phase of each amplitude is given by its slope.

	$1+\alpha$	$A'_{70}/A_{70},\%$	α'	ϕ_ϵ	χ^2
No screeing	0.4248(11)	0.95(15)	2.26(1)	43.96(48)	421/397
Elastic screeing	0.4302(11)	0.48(15)	2.32(2)	44.15(47)	420/397
Inelastic Fact.	0.4624(5)	0.00(5)	—	43.86(41)	425/397
Inelastic Sym.	0.4225(11)	0.62(15)	2.29(2)	44.05(47)	420/397

- Much smaller spread in ϕ_ϵ
- All χ^2 are consistent with each other.

Data is sensitive enough to measure the screening corrections (?)

Amp12/Phase12 fit

Perform a fit with individual A_{70} at each p bin. This allows to study *amplitude* dependence vs p. For the *phase*, there are 2 options:

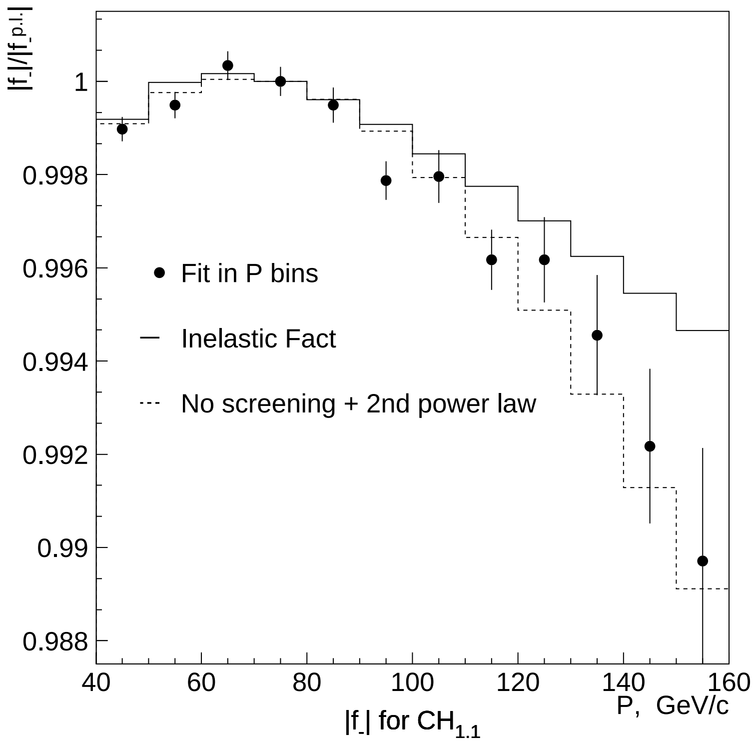
- Float $\arg(f_-)$ for each p bin individually. There is a large common offset correlation vs p, via common (floated) τ_S and Δm, fix $\arg(f_-)$ for $p = 70$ GeV/c.
- Deremine $\arg(f_-)$ using derivative analyticity relation (DAR) numerically:

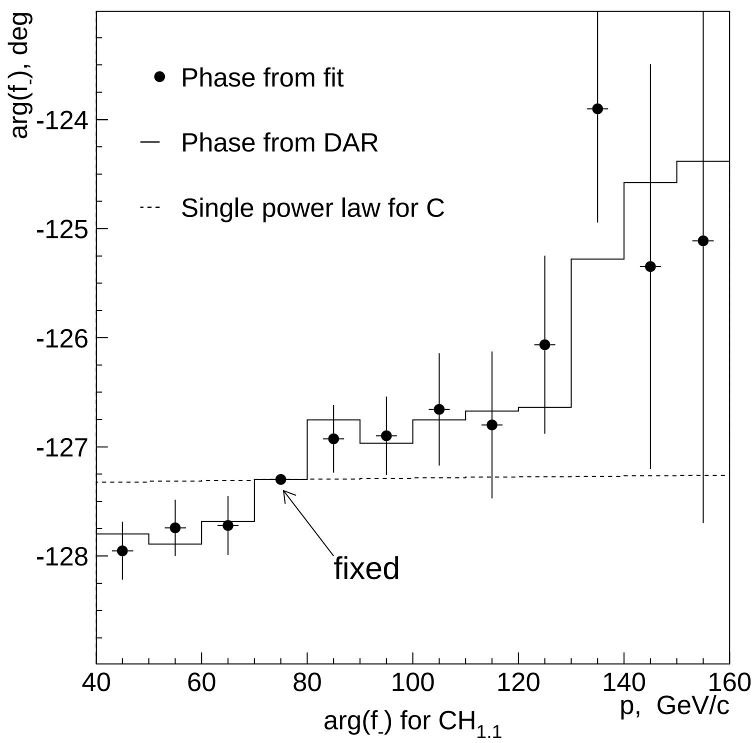
$$\arg(f_-) = -\pi - \tan\left(\frac{\pi}{2} \frac{\mathrm{d}}{\mathrm{d} \ln p}\right) \ln |f_-|.$$

For exact power law DAR leads to a constant phase, i.e. we need to parameterize $|f_-|$ locally with a power law. For 2 points "parameterization" is unique, for example phase for $p = 50$ GeV/c can be determined using A_{70}^{45} and A_{70}^{55}:

$$\delta \arg(\epsilon) = -\frac{\pi}{2} \frac{\ln A_{70}^{55} - \ln A_{70}^{45}}{\ln 55 - \ln 45}$$

Results of Amp12/Phase12 fit





- Charged mode only, no screening corrections.
- Data prefers larger corrections to the $|f_-|$ than predicted by inelastic-fact screening model for higher kaon momentum.
- Phase determined from a fit agrees very well with an estimate using DAR.

Summary of Amp12/Phase12 fit results

The phase measurements in the fit correlate with each other and with measurements of A_{70}. To quantify agreement between the DAR prediction and measured phase, fit slope $s = \frac{d\arg(f_-)}{dp}$.

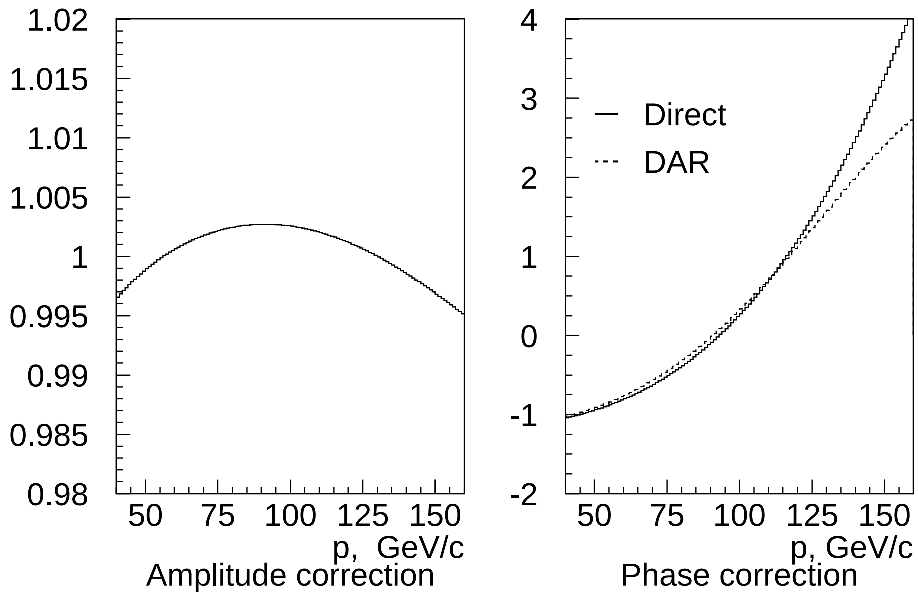
- For a fit with no DAR correction: $s = (0.026 \pm 0.005)\ c/GeV$
- for a fit with DAR correction to the phase: $s = (0.002 \pm 0.005)\ c/GeV$.

In other words deviation of the reg. amplitude from a single power law predicts a phase walk of $\sim 3°$ for kaon momentum changing from 40 to 160 GeV/c which is confirmed by data to $0.6°$.

DAR fit result for the z-binned fit:

	ϕ_ϵ	χ^2/dof
Amp12 fit, with DAR	43.84(44)	412/388

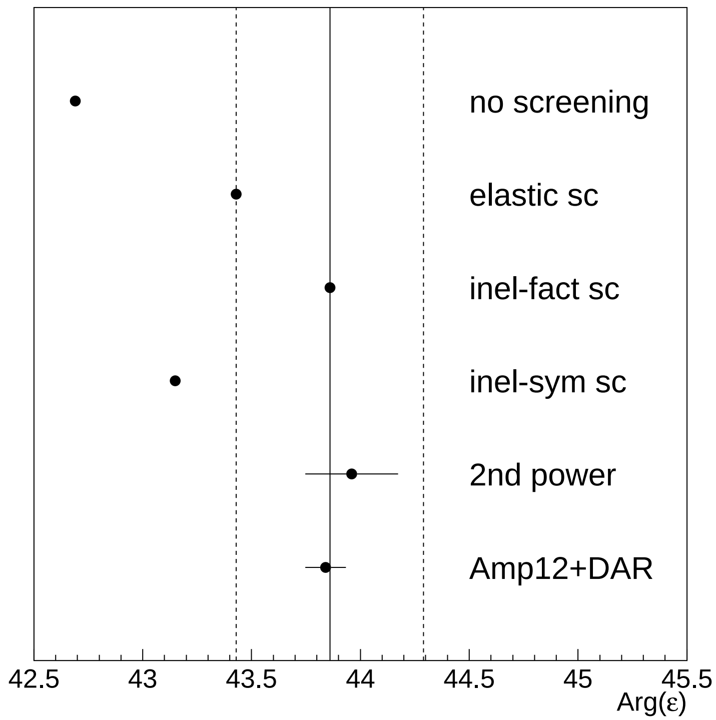
DAR vs 2nd power law

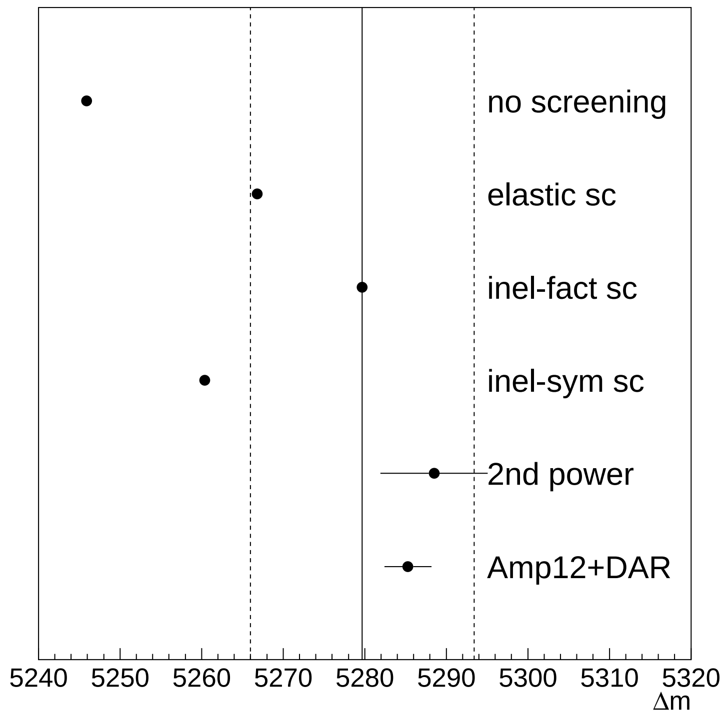


For a 2-power law fit, the phase determined using DAR can be compared to direct calculation. Agrees well for low p, but diverges for high. I'm not sure about DAR validity for positive α (pole at $p = \infty$). Use DAR instead of direct phase (apply as a screening correction), see the effect:

	ϕ_ϵ	δm	τ_S	χ^2/dof
Direct	43.96(48)	5288.4(152)	89.573(48)	421/397
DAR	43.96(42)	5287.6(136)	89.569(45)	425/399

ϕ_ϵ with different corrections





For ϕ_ϵ, good agreement for inelastic factorized vs fits to data (2nd power law and Amp12 fit). About 2σ difference for Δm between inelastic factorized and Amp12 fit.

Systematics: screening, analyticity, transmission

- 0.25° for *analyticity* based on low energy data and effect of electromagnetic regeneration (based on Bruce and Roy PRL)
- 0.05° for *transmission* (based on $K \to \pi^+\pi^-\pi^0$ measurement).
- 0.02° from the difference between binned-DAR vs inelastic factorized screening calculation for *screening* and subleading trajectories.
- A possible additional regeneration at higher momentum due to Odderon exchange, up to 0.6°, (based on Bruce and Roy PRL) — assume no effect, keep as a comment

Z-binned fit results

$$
\begin{aligned}
\tau_S &= [89.589 \pm 0.042_{\text{stat}} \pm 0.056_{\text{syst}}] \times 10^{-12}\ \text{s} \\
&= [89.589 \pm 0.070] \times 10^{-12}\ \text{s} \\
\Delta m &= [5279.7 \pm 12.8_{\text{stat}} \pm 14.7_{\text{syst}}] \times 10^{6}\ \hbar/\text{s} \\
&= [5279.7 \pm 19.5] \times 10^{6}\ \hbar/\text{s} \\
\phi_\epsilon &= [43.863 \pm 0.40_{\text{stat}} \pm 0.49_{\text{syst}}]^\circ \\
&= [43.863 \pm 0.63]^\circ \\
Re(\epsilon'/\epsilon) &= [21.10 \pm 1.3_{\text{stat}} \pm 3.17_{\text{syst}}] \times 10^{-4} \\
&= [21.10 \pm 3.43] \times 10^{-4} \\
Im(\epsilon'/\epsilon) &= [-17.20 \pm 9.0_{\text{stat}} \pm 18.06_{\text{syst}}] \times 10^{-4}, \\
&= [-17.20 \pm 20.20] \times 10^{-4},
\end{aligned}
\tag{2}
$$

$\chi^2/\nu = 425.4/(432 - 33)$, no CPT consevation assumption. For $\tau_S, \Delta m$ and ϕ_ϵ comparable stat. and systematic uncertainty.

CPT tests

$$\begin{aligned}\Delta\phi &= -3Im(\epsilon'/\epsilon) \\ &= [0.30 \pm 0.15_{\rm stat} \pm 0.31_{\rm syst}]^\circ \\ &= [0.30 \pm 0.35]^\circ,\end{aligned} \tag{3}$$

The superweak phase calculated using
$\tau_L = 5.099 \pm 0.021 \times 10^{-8}$ (PDG) is

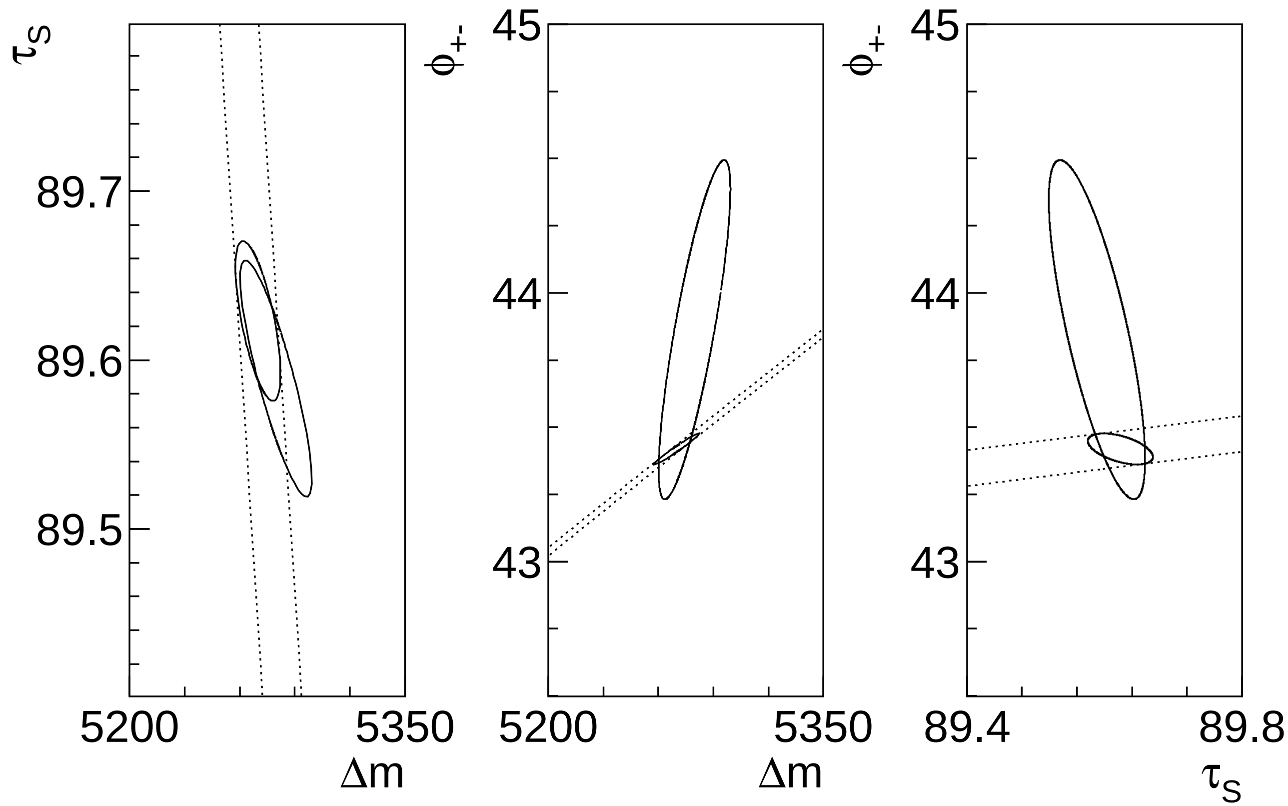
$$\begin{aligned}\phi_{SW} &= [43.461 \pm 0.069_{\rm stat} \pm 0.070_{\rm syst}]^\circ \\ &= [43.461 \pm 0.098]^\circ\end{aligned} \tag{4}$$

The difference of ϕ_ϵ and ϕ_{SW}

$$\begin{aligned}\delta\phi &= \phi_\epsilon - \phi_{SW} \\ &= [0.40 \pm 0.37_{\rm stat} \pm 0.42_{\rm syst}]^\circ \\ &= [0.40 \pm 0.56]^\circ\end{aligned} \tag{5}$$

is consistent with zero as expected from CPT invariance.

CPT constraint



Bigger ellipse: no CPT constraint, smaller: CPT imposed, lines: $\phi_\epsilon = \phi_{SW}$ condition.

$$\phi_{SW}\,|_{\rm cpt} = [43.419 \pm 0.058]^\circ. \tag{6}$$

Δm and τ_S with CPT constraint

$$\begin{aligned}
\Delta m\,|_{\rm cpt} &= [5269.9 \pm 3.8_{\rm stat} \pm 11.7_{\rm syst}] \times 10^{-12}\ {\rm s} \\
&= [5269.9 \pm 12.3] \times 10^{-12}\ {\rm s} \\
\tau_S\,|_{\rm cpt} &= [89.623 \pm 0.018_{\rm stat} \pm 0.044_{\rm syst}] \times 10^{6} \hbar/{\rm s} \\
&= [89.623 \pm 0.047] \times 10^{6} \hbar/{\rm s}
\end{aligned} \quad (7)$$

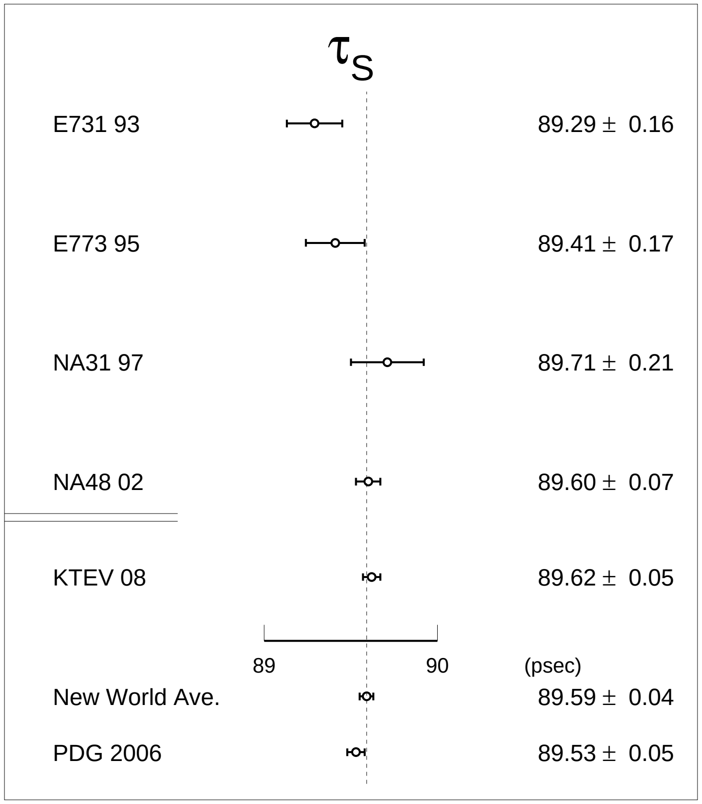
Cross check — Δm and τ_S in charged and neutral mode separately:

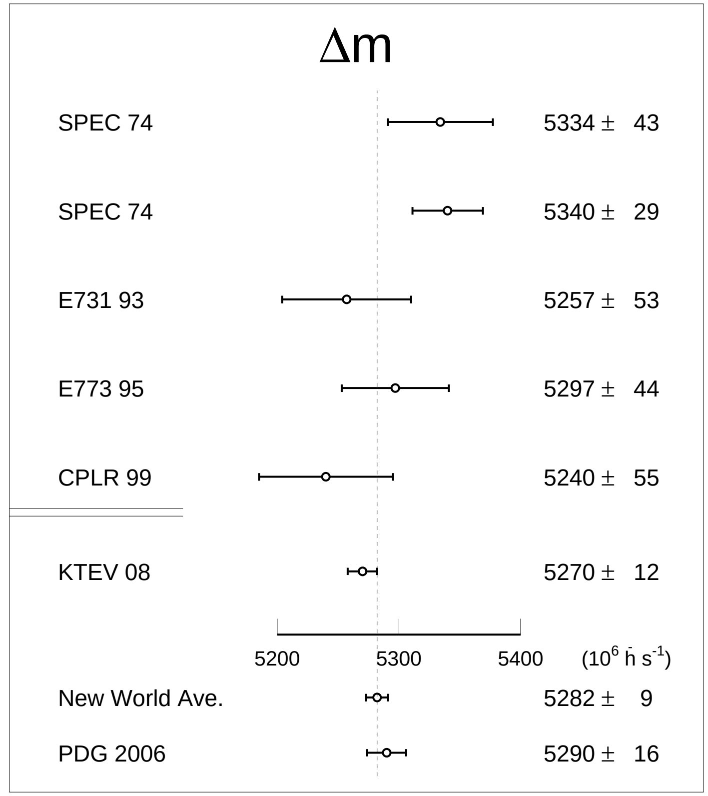
$$\begin{aligned}
\Delta m^{00} &= (5257.6 \pm 8.3_{\rm stat}) \times 10^{6} \hbar/{\rm s} \\
\Delta m^{+-} &= (5269.0 \pm 4.2_{\rm stat}) \times 10^{6} \hbar/{\rm s} \\
{\tau_S}^{00} &= (89.667 \pm 0.039_{\rm stat}) \times 10^{-12} {\rm s} \\
{\tau_S}^{+-} &= (89.620 \pm 0.020_{\rm stat}) \times 10^{-12} {\rm s}
\end{aligned} \quad (8)$$

PRD03 vs now

	PRD03	PRD08
$\Delta m^{00} \times 10^6 \hbar/\mathrm{s}$	$5237.3 \pm 10.6_{\mathrm{stat}}$	$5257.6 \pm 8.3_{\mathrm{stat}}$
$\Delta m^{+-} \times 10^6 \hbar/\mathrm{s}$	$5266.7 \pm 5.9_{\mathrm{stat}}$	$5269.0 \pm 4.2_{\mathrm{stat}}$
${\tau_S}^{00} \times 10^{-12}\mathrm{s}$	$89.637 \pm 0.050_{\mathrm{stat}}$	$89.667 \pm 0.039_{\mathrm{stat}}$
${\tau_S}^{+-} \times 10^{-12}\mathrm{s}$	$89.650 \pm 0.028_{\mathrm{stat}}$	$89.620 \pm 0.020_{\mathrm{stat}}$
$\Delta m \rvert_{cpt} \times 10^6 \hbar/\mathrm{s}$	5261 ± 15	5269.9 ± 12.3
$\tau_S \rvert_{cpt}$	89.65 ± 0.07	89.623 ± 0.047
$\Delta m \times 10^6 \hbar/\mathrm{s}$	5288 ± 42	5279.7 ± 19.5
$\tau_S \times 10^{-12}\mathrm{s}$	89.58 ± 0.13	89.589 ± 0.070
$\phi_{+-}, ^\circ$	44.12 ± 1.40	
$\phi_{+-} - \phi_{SW}, ^\circ$	0.61 ± 1.19	
$\phi_{\epsilon}, ^\circ$		43.863 ± 0.63
$\phi_{\epsilon} - \phi_{SW}, ^\circ$		0.40 ± 0.56
$\Delta\phi, ^\circ$	0.35 ± 0.50	0.30 ± 0.35

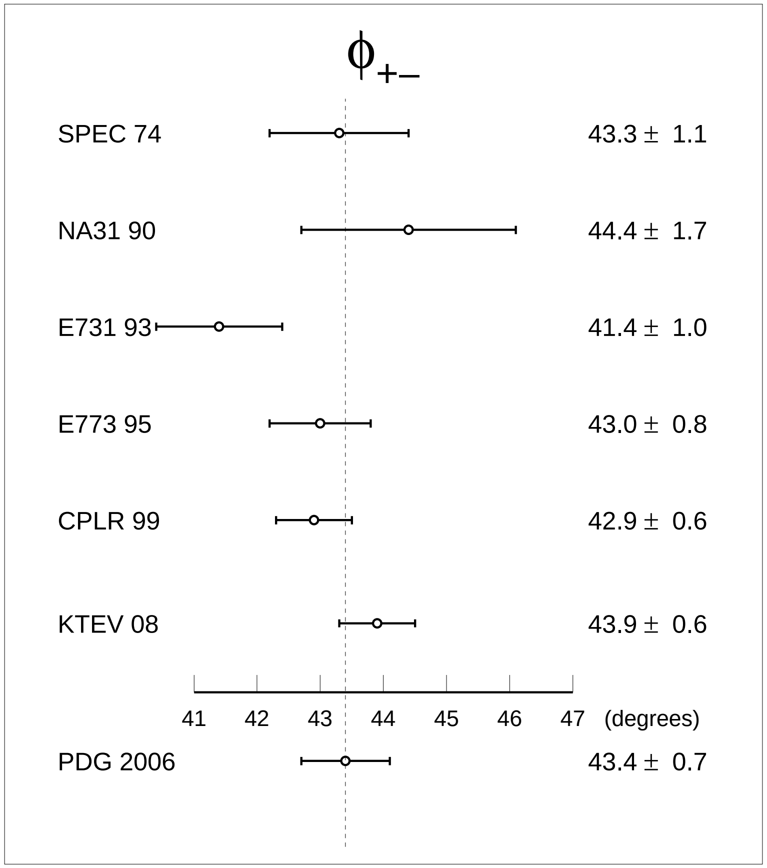
Comparisons with other results

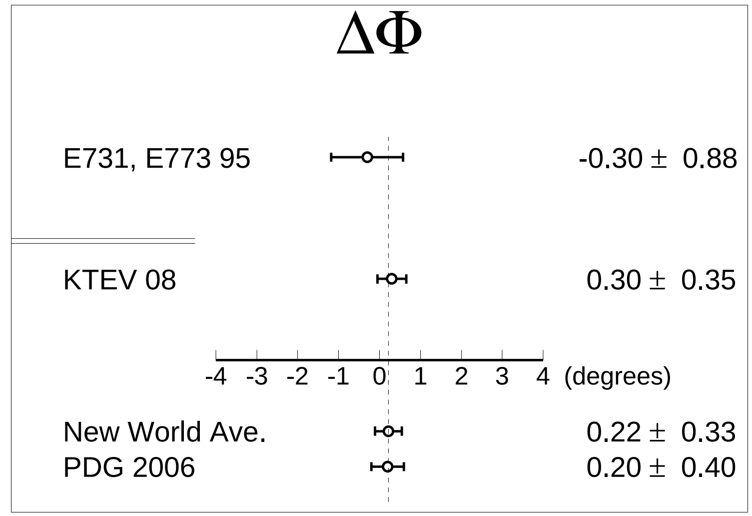




Good agreement with NA48 for τ_S. For Δm, KTeV results moves slightly to PDG average.

Comparisons with other results





Summary

- The strategy of determination of the kaon parameters is simplified and unified for CPT assumption free and CPT concerving case.
- Screening corrections are reexamined. The data is sensitive enough to measure modification of the momentum depence of $|f_-|$ from a simple power law. The variation of $\arg(f_-)$ vs p_K, about 3° for $40 - 160$ GeV momentum range, calculated using derivative analyticity relation (DAR) agrees well with the screening calculation. The phase variation can be also determined directly from the data, it agrees very well with the estimaion using DAR:
  $\Delta \arg(f_-)|_{DAR} - \Delta \arg(f_-)|_{data} = (0.2 \pm 0.6)^\circ$ for the same 120 GeV momentum range.
- The systematic uncertainty due to screening corrections is evaluated using difference between the calculation and DAR fit to the data, the uncertainty for ϕ_ϵ is reduced from 0.75° to 0.02°
- Reduced uncertainty for screening allows to improve sensitivity of the CPT test $\phi_\epsilon - \phi_{SW}$ from $\sim 0.9^\circ$ to $\sim 0.6^\circ$.