Status of the dispersive analysis of K^L_{l3} form factors based on KTeV data

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Paper: in preparation

Outline

1. Introduction and Motivation : Callan-Treiman theorem in $K^L_{\mu 3}$ decays.

2. Dispersive representation of the scalar and vector form factors.

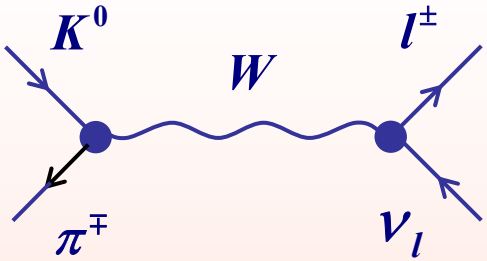
3. Measurement of lnC and Λ_+ with KTeV data: fit results.

4. Applications :
   - Branchings Ratios $R_{\mu/e}$.
   - Extraction of $|f_+(0)V_{us}|$.
   - Test of the SM and the lattice results on F_K/F_π, and $f_+(0)$.

5. Conclusion and outlook.

1. Introduction and Motivation

1.1 Definition



- K_{l3} decays $K^0 \rightarrow \pi^{\mp} l^{\pm} \nu_l$

- The hadronic element :

$$\langle \pi^-(p_\pi) | \bar{s}\gamma_\mu u | K^0(p_K) \rangle = f_+(t)(p_K + p_\pi)_\mu + f_-(t)(p_K - p_\pi)_\mu$$

→ $f_+(t)$, $f_-(t)$: form factors

→ $t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2$

- We consider : - the vector form factor $f_+(t)$.
  - the scalar form factor $f_s(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$

→ Normalization : $\bar{f}_+(t) = \frac{f_+(t)}{f_+(0)}$, $\bar{f}_+(0) = 1$ and $\bar{f}_0(t) = \frac{f_s(t)}{f_+(0)}$, $\bar{f}_0(0) = 1$

- $\bar{f}_+(t)$ accessible in K_{e3} and $K_{\mu 3}$ decays, $\bar{f}_0(t)$ only accessible in $K_{\mu 3}$ (suppressed by m_l^2/M_K^2) + correlations ⇨ difficult to measure.

1.2 Interests

- Decay rate formula: $l = (e, \mu)$

$$\Gamma_{K^{+/0}l3} = \frac{Br_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW} \left(1 + 2\Delta_{K^{+/0}l}^{EM}\right) \left| f_+^{K^{+/0}}(0) \mathcal{V}_{us} \right|^2 I_{K^{+/0}}^l$$

Kaon life time

Radiative Corrections

$$I_{K^{+/0}}^l = \int dt \frac{1}{m_{K^{+/0}}^8} \lambda^{3/2} F\left(t, \overline{f}_+(t), \overline{f}_0(t)\right)$$

Form factor shapes from Dalitz plot ⇨ phase space integrals.

- Extraction of $f_+(0)|\mathcal{V}_{us}|$
- Prediction of $R_{\mu/e} = \Gamma_{K\mu3}/\Gamma_{Ke3}$

- Test of the Standard Model :
  - Callan Treiman relation

1.3 Theoretical knowledge for the scalar FF: CT relation

- Callan-Treiman Theorem: SU(2) x SU(2) theorem

$$C = \overline{f_0}(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+^{K^0}(0)} + \Delta_{CT}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

- Corrections of order m_u, m_d
  - → No chiral logarithms : $\Delta_{CT} \sim \mathcal{O}\left(m_{u,d} / 4\pi F_\pi\right)$
  - → Isospin limit m_d=m_u : $\Delta_{CT}^{NLO} \sim -3.5.10^{-3}$ **[Gasser & Leutwyler]**
  - → K^0 decay : no small denominators due to π^0 - η mixing $(\mathcal{O}((m_d - m_u)/m_s))$.
  - → K^+ decay case : enhancement by π^0 - η mixing in the final state

    ⇨ $\Delta_{CT}^{K^+} \sim few\ 10^{-2}$ (K^0 ideal decay)

- Estimations of the higher order terms: corrections in $\mathcal{O}(m_{u,d})$ and $\mathcal{O}(m_s)$

  ⇨ $\Delta_{CT} = (-3.5 \pm 8).10^{-3}$

1.4 Test of the Standard Model.

$$C_{SM} = \overline{f}_0(\Delta_{K\pi}) = \underbrace{\frac{F_K |\mathcal{V}^{us}|}{F_\pi |\mathcal{V}^{ud}|} \frac{1}{f_+(0) |\mathcal{V}^{us}|} |\mathcal{V}^{ud}|}_{B_{exp}} + \Delta_{CT}$$

- C is predicted in the Standard Model using the measured Br: $Br(K_{l2}/\pi_{l2})$, $\Gamma(K_{e3})$ and $|\mathcal{V}_{ud}|$. ($\mathcal{V}_{us}$ not needed in this prediction.)

$$\Rightarrow \quad C_{SM} = 1.2439 \pm 0.0042 + \Delta_{CT}$$

- Relation which tests the Standard Model very accurately.
  If physics beyond the SM: ~1% difference between C and B_{exp}. Uncertainties from Δ_{CT} and B_{exp} on the permile level ⇨ opportunity to see a possible effect.

- Possible test of the lattice results for F_K/F_π, $f_+(0)$.

How to measure C ?

1.5 What do we know experimentally ?

- Data available from KTeV, NA48 and KLOE.

- Necessity to parametrize the 2 form factors $\overline{f}_+$ and $\overline{f}_0$ to fit the measured distributions.

- Different parametrizations available : usually use of a quadratic parametrization for $\overline{f}_+(t)$ and a linear one for $\overline{f}_0(t)$ (λ_0'' not accessible) to fit the measured distributions.

$$\overline{f}_+^{quad}(t) = 1 + \lambda_+' \frac{t}{m_\pi^2} + \frac{1}{2}\lambda_+'' \frac{t}{m_\pi^2}$$

and

$$\overline{f}_0^{lin}(t) = 1 + \lambda_0 \frac{t}{m_\pi^2}$$

NB: A pole parametrization is also often used for the vector form factor

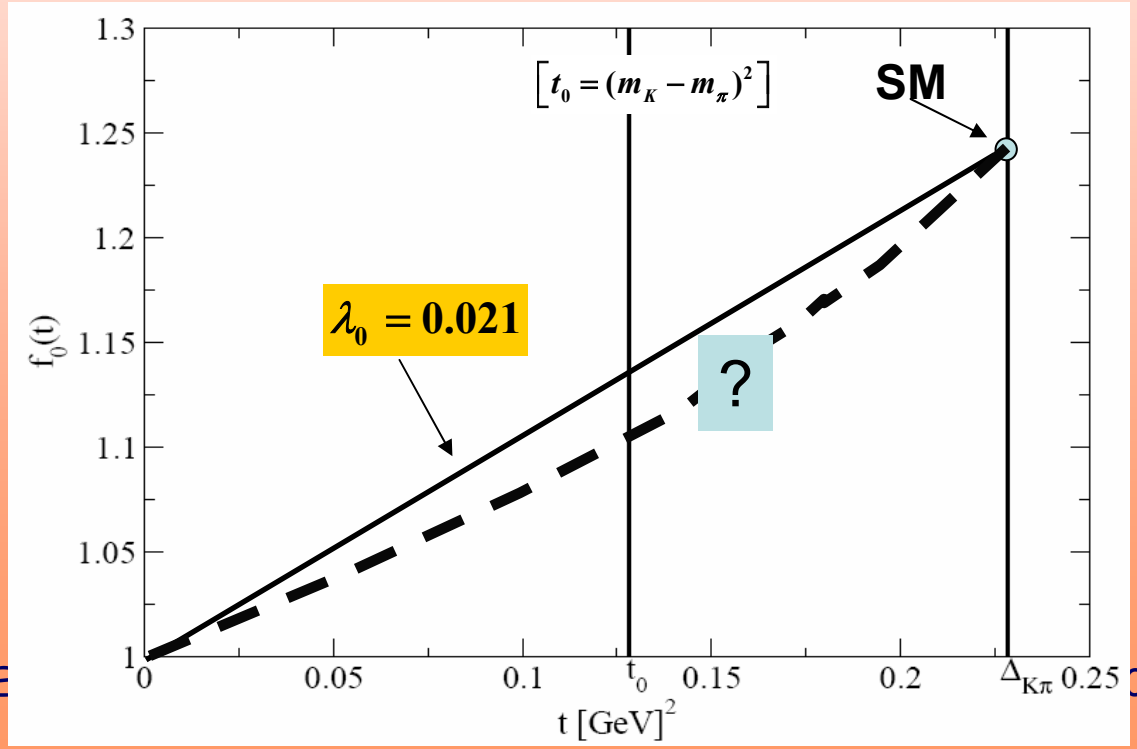
$$f_{pol}(t) = \frac{m_V^2}{m_V^2 - t}$$

- Problem: higher order terms (curvature…) neglected :

$$\Rightarrow \quad \lambda_0^{exp} > \lambda_0 = m_\pi^2 f'(0)$$

- λ_0^{exp} may depend on the fitted distribution: different weight given to different areas of the Dalitz plot. λ_0^{exp} is likely method dependent. It could explain different results between experiments.

- How to extrapolate up to the CT point to test the SM ?



How to improve the parametrization to measure $C=\overline{f}_0(\Delta_{K\pi})$?

2. Dispersive Representation of the $K\pi$ form factors

[Bernard, Oertel, E.P. and Stern'06]

2.1 Why a dispersive parametrization ?

- Use of the physical knowledge :
  - $K\pi$ scattering phase.
  - Presence of resonances....
- 2 subtractions ⇨ insensitive to the unknown high energy phase. This allows for a good precision in the region of interest (at low energy).
- Only one free parameter to be fitted.

2.2 Dispersive parametrization for the scalar FF.

- Problem : How to construct a very precise representation of $\bar{f}_0(t)$ between 0 and $\Delta_{K\pi}$?
- Knowledge :
  - → $\bar{f}_0(0) = 1$
  - → $\bar{f}_0(\Delta_{K\pi})$ = C, Callan-Treiman point
  - → $K\pi$ scattering phase
  - → Asymptotic behaviour of the form factor : $\bar{f}_0(s) \underset{s\to-\infty}{=} \mathcal{O}(1/s)$

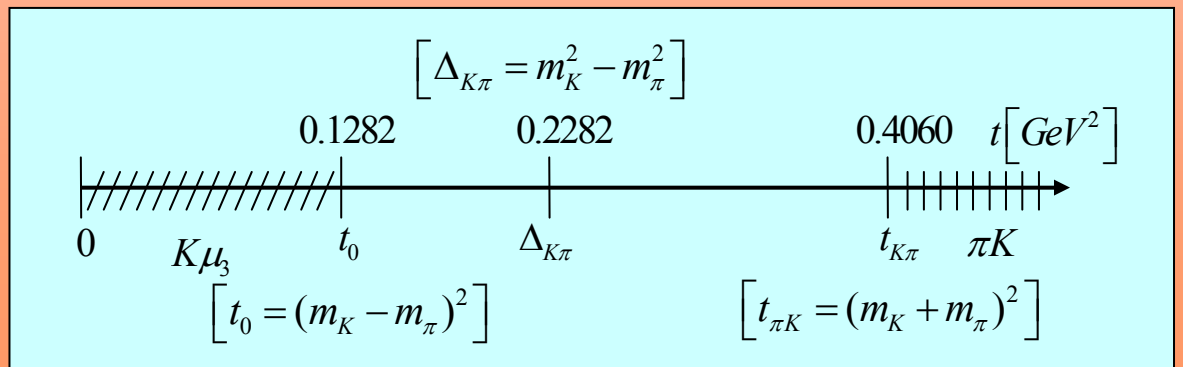
- A dispersion relation with two substractions at 0 and $\Delta_{K\pi}$ for $\ln(\bar{f}_0(t))$:

$$\bar{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right]$$

with

$$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s}\frac{\phi(s)}{(s-\Delta_{K\pi})(s-t)}$$

  - → $\phi(t)$ phase of form factor : $\bar{f}_0(t) = \left|\bar{f}_0(t)\right| e^{i\phi(t)}$



2.3 Description.

$$\overline{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\,(\ln C - G(t))\right]$$ with $$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s}\frac{\phi(s)}{(s-\Delta_{K\pi})(s-t)}$$

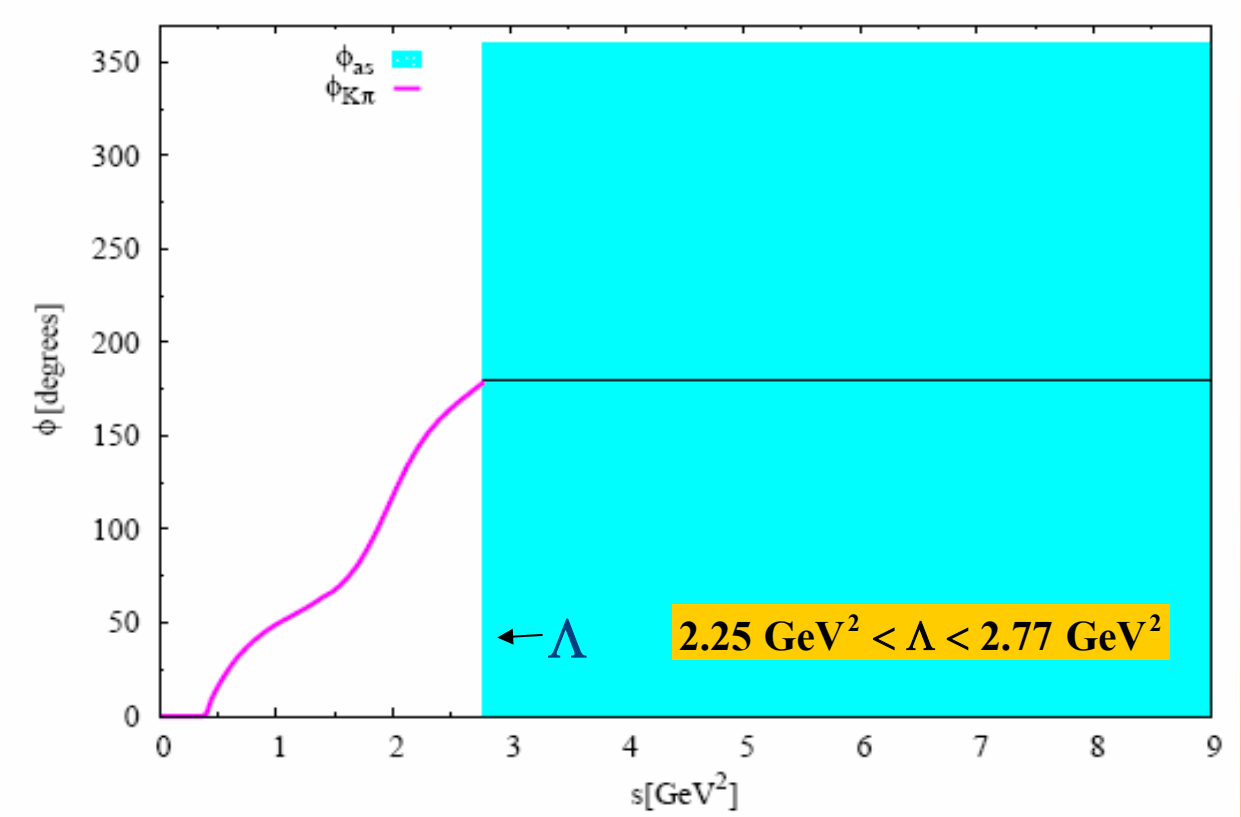
→ ϕ(t) phase of form factor : $\overline{f}(t) = \left|\overline{f}(t)\right| e^{i\phi(t)}$

- Two unknowns: ϕ(s) and lnC=ln($\overline{f}_0$(t=$\Delta_{K\pi}$))

- ϕ(s) a priori unknown but
  → $\overline{f}_0(s) \underset{s\to-\infty}{=} \mathcal{O}(1/s)$ ⇒ For large s, $\phi(s) \to \pi$. Rapid convergence of G(t).

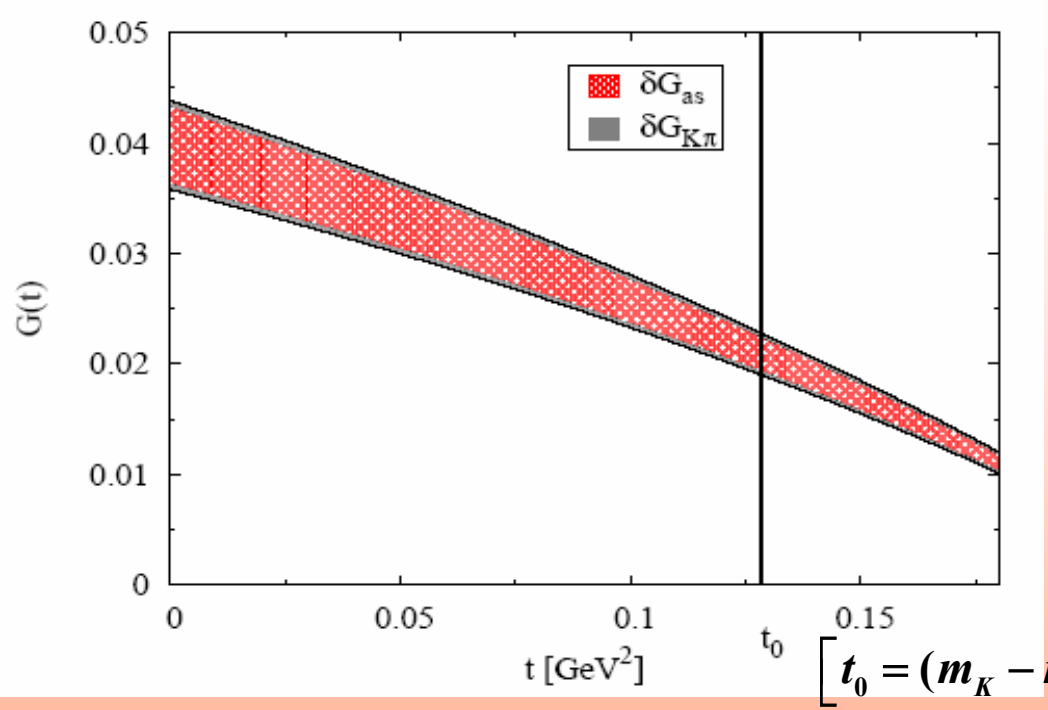
  → At « low energy » $\phi(s) = \delta^{1/2}_{K\pi}(s)$, S wave I=1/2 $K\pi$ scattering phase, [**Watson theorem**] well known : **[Buettiker, Descotes, Moussallam '02]**
  - Experimental input for $1\ \mathrm{GeV} < E < 2.5\ \mathrm{GeV}$ **[Aston et al].**
  - Extrapolation of the phase down to threshold solving the Roy Steiner equations.

- Phase used



- Elastic up to ~1.5 GeV ⇨ $t < \Lambda : \ \phi(t) = \phi_{K\pi}(t) = \delta^{s,\frac{1}{2}}_{\pi,K}(t) \pm \Delta\delta^{s,\frac{1}{2}}_{\pi,K}(t)$

  $t > \Lambda : \ \phi(t) = \phi_{as}(t) = \pi \pm \pi$



$$\left[t_0 = (m_K - m_\pi)^2\right]$$

G(t) with the uncertainties added in quadrature

$$G(0) = 0.0398 \pm 0.0040$$

G(t) does not exceed 20% of the expected value of lnC

lnC~0.20

- Apart from the parameter (lnC) to be determined by the fit, very precise parametrization of the form factor in the physical region.

2.4 Dispersive parametrization of the Kπ vector form factor

- In the same way as for the scalar form factor, a dispersion relation with two subtractions for ln(f_+(t)) :
  2 subtraction points at low energy :
  → $\overline{f}_+(0) = 1$
  → $\overline{f}'_+(0) = \Lambda_+ / m_\pi^2$

$$\overline{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}\left(\Lambda_+ + H(t)\right)\right]$$

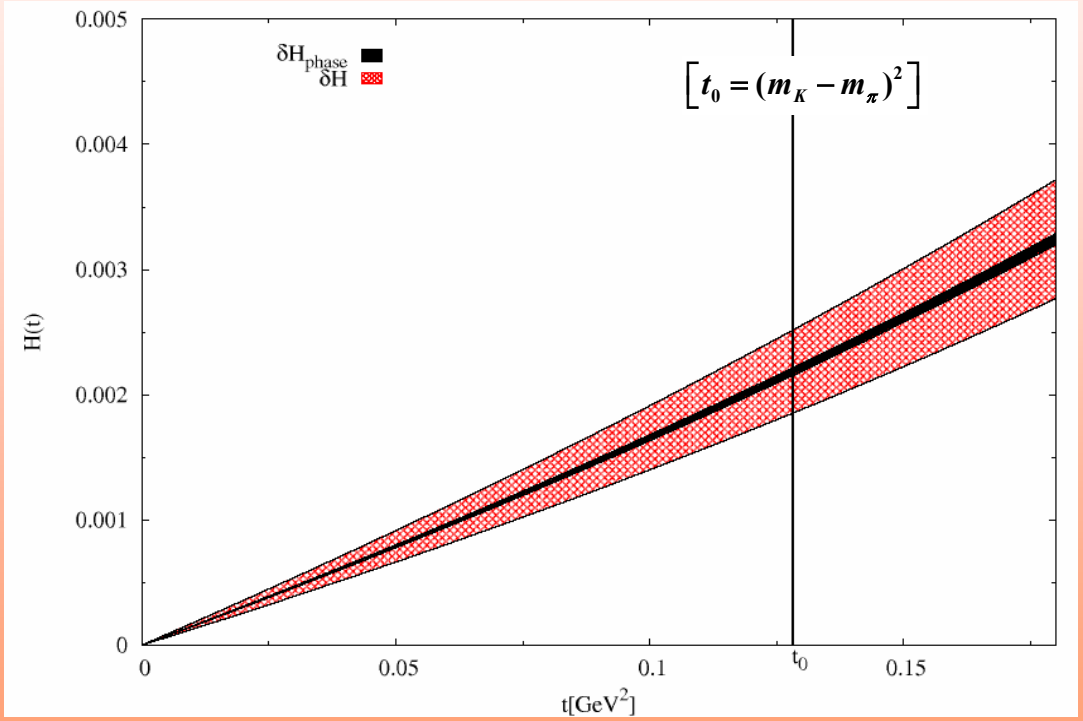
with

$$H(t) = \frac{m_\pi^2 t}{\pi}\int_{t_{\pi K}}^{\infty}\frac{ds}{s^2}\frac{\varphi(s)}{(s-t)}$$

  → φ(t) phase of form factor : $\overline{f}_+(t) = \left|\overline{f}_+(t)\right| e^{i\varphi(t)}$

- φ(s) a priori unknown but
  → $\overline{f}_+(s) \underset{s\to-\infty}{=} \mathcal{O}(1/s)$ ⇨ For large s, φ(s) → π. Rapid convergence of H(t).
  → At « low energy » $\varphi(s) = \delta^{1/2}_{K\pi}(s)$, P wave I=1/2 Kπ scattering phase [**Watson theorem**].

- $K\pi$ scattering phase
  - Experimental input for $0.825\ \text{GeV} < E < 2.5\ \text{GeV}$ **[Aston et al].**
  - Extrapolation of the phase down to threshold complicated ⇨ lack of relevant experimental inputs.
  - Construction of the partial wave amplitude : Breit-Wigner (K*(892)) a la Gounaris-Sakourai (Analyticity, Unitarity and Correct threshold behavior) ⇨ inputs: mass and width of K*(892).



- H(t) precisely known.

3. Dispersive analysis with KTeV data

3.1 Description

- Analysis of the KTeV data using the dispersive representation for the form factors: K_{e3}, $K_{\mu 3}$ and combined fits.

- 2 classes of representations for the vector form factor:
  - 1rst class: parametrizations based on minimal theoretical inputs (more parameters) :
    → Taylor expansion :

$$\overline{f}_{+}^{quad}(t) = 1 + \lambda_{+}^{'} \frac{t}{m_{\pi}^{2}} + \frac{1}{2} \lambda_{+}^{''} \frac{t}{m_{\pi}^{2}}$$

→ R.Hill parametrization [Hill'04]

$$f_{+,0}(t) = f_{+,0}(t_0) \frac{\phi(t_0, t_0, Q^2)}{\phi(t, t_0, Q^2)} \sum_{k=0}^{n} a_k \left(t_0, Q^2\right) z\left(t, t_0\right)^k$$

Conformal mapping: t → z variable with |z|<1

For special ϕ, $\sum_{k=0}^{n} a_k^2$ bounded : use of some high-energy inputs.

3.1 Description

- Analysis of the KTeV data using the dispersive representation for the form factors: K_{e3}, $K_{\mu 3}$ and combined fits.

- 2 classes of representations for the vector form factor:
  - 2nd class: parametrizations relying on established physical inputs, less parameters :
    - → Pole parametrization, one parameter: the pole mass M_V

$$\overline{f}_+(t) = \frac{m_V^2}{m_V^2 - t}$$

    - → Dispersive parametrization, one parameter Λ_+

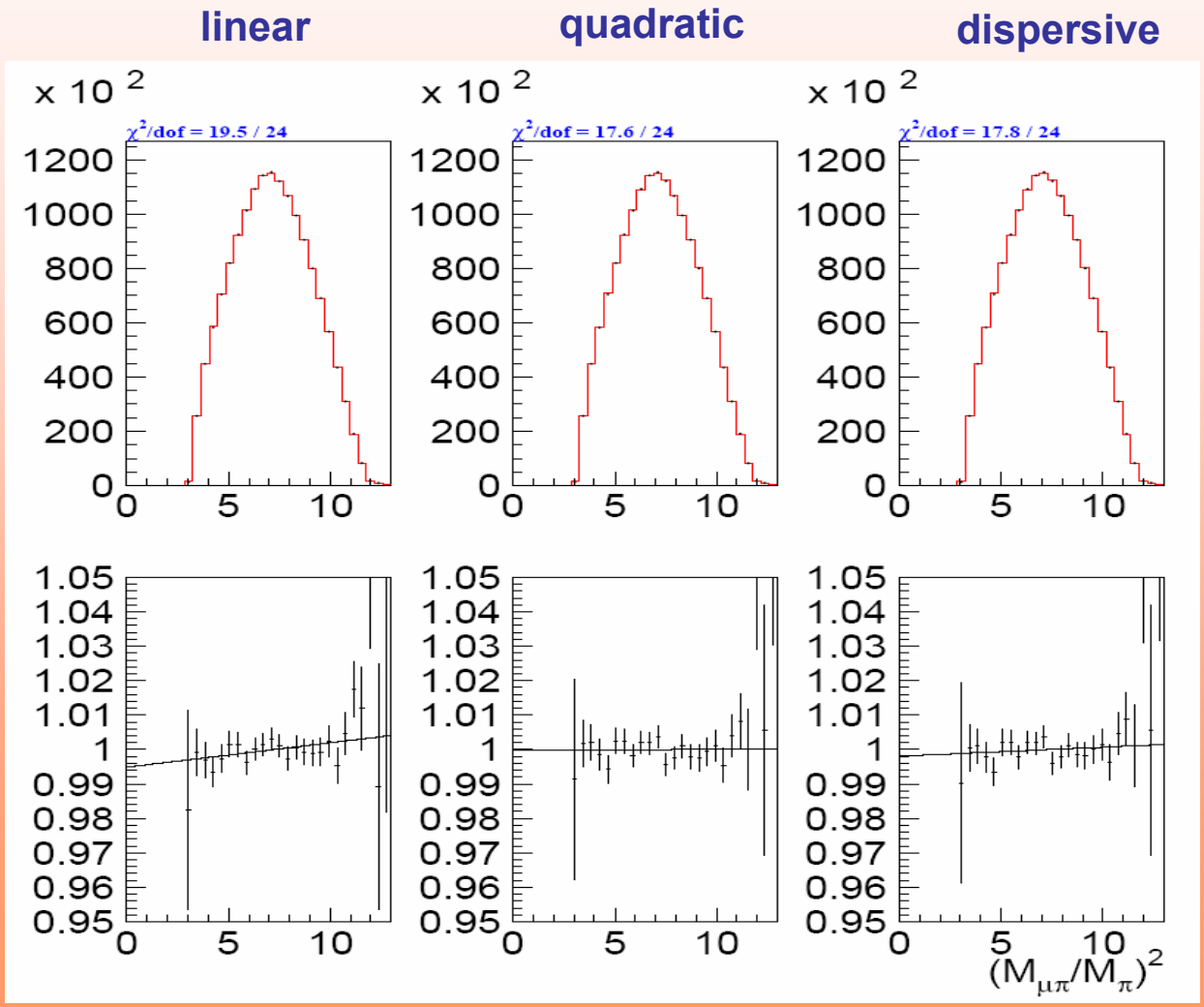
$$\overline{f}_+(t) = \exp\left[\frac{t}{m_\pi^2}\left(\Lambda_+ + H(t)\right)\right]$$

- Dispersive parametrization for the scalar form factor which uses C= $\overline{f}_0(\Delta_{K\pi})$ as the only free parameter to be fitted.

$$\overline{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right]$$

3.1 Description

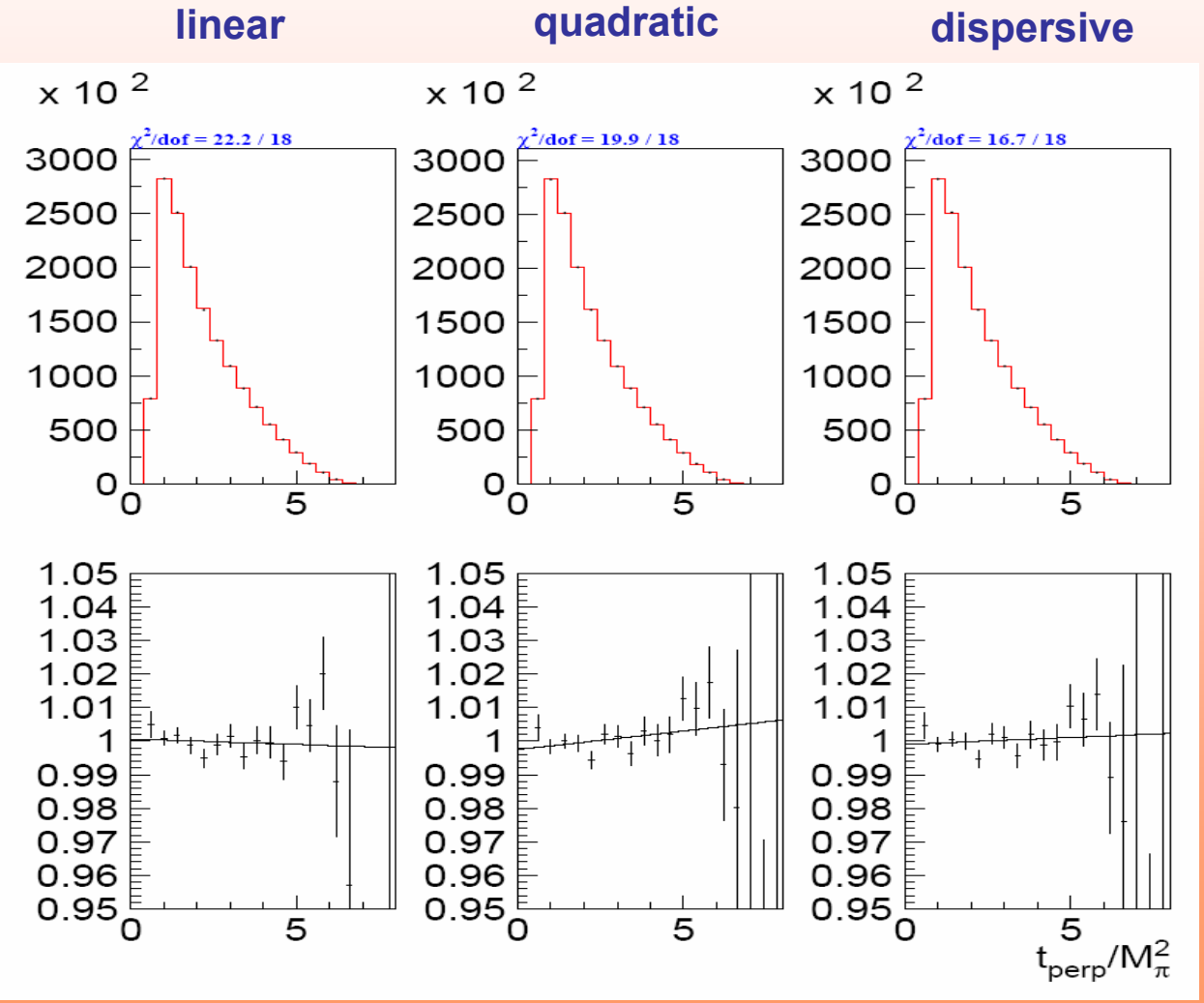
- Results independent of the MC used : fits with old MC [KTeV'04] and with a MC based on the dispersive representations for the form factors.



[S.Glazov]

3.1 Description

- Results independent of the MC used : fits with old MC [KTeV'04] and with a MC based on the dispersive representations for the form factors.



[S.Glazov]

3.1 Description

- Results independent of the MC used : fits with old MC [KTeV'04] and with a MC based on the dispersive representations for the form factors.
- Theoretical uncertainties on G(t) and H(t) negligible.
- Only statistical uncertainties.
- Preliminary results, to be checked, more cross checks are welcome.

3.2 Results for Ke3

Results	PARAMETRIZATION VECTOR FORM FACTOR			
	quadratic	Hill	pole	dispersive
Fit parameters	$\lambda'_+ = 21.67(1.59)$ $\lambda''_+ = 2.87(66)$	a1=1.023(28) a2=0.75(1.58)	M_V=881.03(5.91)	$\Lambda_+ = 25.20(38)$
$\lambda'_+ \times 10^3$ $\lambda''_+ \times 10^3$	21.67(1.59) 2.87(66)	22.69(1.20) 1.93(30)	25.10(41) 1.26(3)	25.20(38) 1.27(4)
Correlation	-0.96	-0.064	-	-
χ^2/dof	62.2/64	62.3/64	66.3/65	66.6/65

- 2 types of results/shapes:
  - Quadratic, Hill : smaller slope, larger curvature. Note for Hill, a_2 badly determined due to the smallness of z (|z|<0.03).
  - Pole, dispersif : larger slope, smaller curvature.

3.3 Results for Kμ3

Results	PARAMETRIZATION VECTOR FF/SCALAR FF			
	quad/dispers	Hill/dispersive	pole/dispersive	dispersive/dispersive
Fit parameters v_i	$\lambda'_+ = 17.5(3.4)$ $\lambda''_+ = 4.3(1.4)$	a1=1.057(63) a2=3.906(3.178)	$M_V = 890.00(13.00)$	$\Lambda_+ = 24.57(83)$
$\lambda'_+ \times 10^3$	17.5(3.4)	20.00(2.60)	24.59(72)	24.57(83)
$\lambda''_+ \times 10^3$	4.3(1.4)	2.5(6)	1.21(7)	1.19(4)
$\ln C$	0.169(16)	0.170(16)	0.1944(93)	0.1947(91)
$\lambda_0 \times 10^3$	11.03(1.37)	11.11(1.37)	13.20(79)	13.22(78)
$\lambda'_0 \times 10^3$	0.54(3)	0.54(3)	0.59(2)	0.59(2)
$\rho(v_i, \ln C)$	0.707 -0.819	0.477 -0.766	0.588 -	-0.557 -
χ^2/dof	189.2/235	189.3/235	193.1/236	193/236

- Interpretation of the dependence of lnC on $\overline{f}_+(t)$:
  - Quadratic or Hill parametrization: No existence of K* assumed, small slope, larger curvature for $\overline{f}_+(t)$, small lnC (lnC~0.170)
  - Pole or dispersive parametrization: Existence of K* assumed ⇨ constrains the correlations between slope and curvature: larger slope, smaller curvature, larger lnC (lnC~0.195)
  - Strong influence of the vector form factor curvature. Preference for dispersive parametrization ⇨ imposes constraints on the curvature on physical grounds.

3.4 Combined Results between Kμ3 and Ke3 for dispersive result

$\Lambda_+ = 0.0250 \pm 0.0034$ and $\ln C = 0.191 \pm 0.012$

- To be compared with $C_{SM} = B\exp + \Delta_{CT}$

$$= 1.2439 \pm 0.0042 + \Delta_{CT}$$

$$\ln C_{SM} = 0.2183(34) + \Delta_{CT} / B_{\exp}$$

- lnC smaller than the SM prediction whatever the parametrization used for $\overline{f}_+(t)$ With $\Delta_{CT} = (-3.5 \pm 8).10^{-3}$ $\ln C_{KTeV}$ at ~1.5 σ.

- Difference can be explained as a signature of exchange of new particules (W_R, H^+) at high energy.

4. Applications

4.1 Prediction of $R_{\mu/e}$: test of consistency

- From the measurements of lnC and Λ_+ ⇨ phase space integrals:

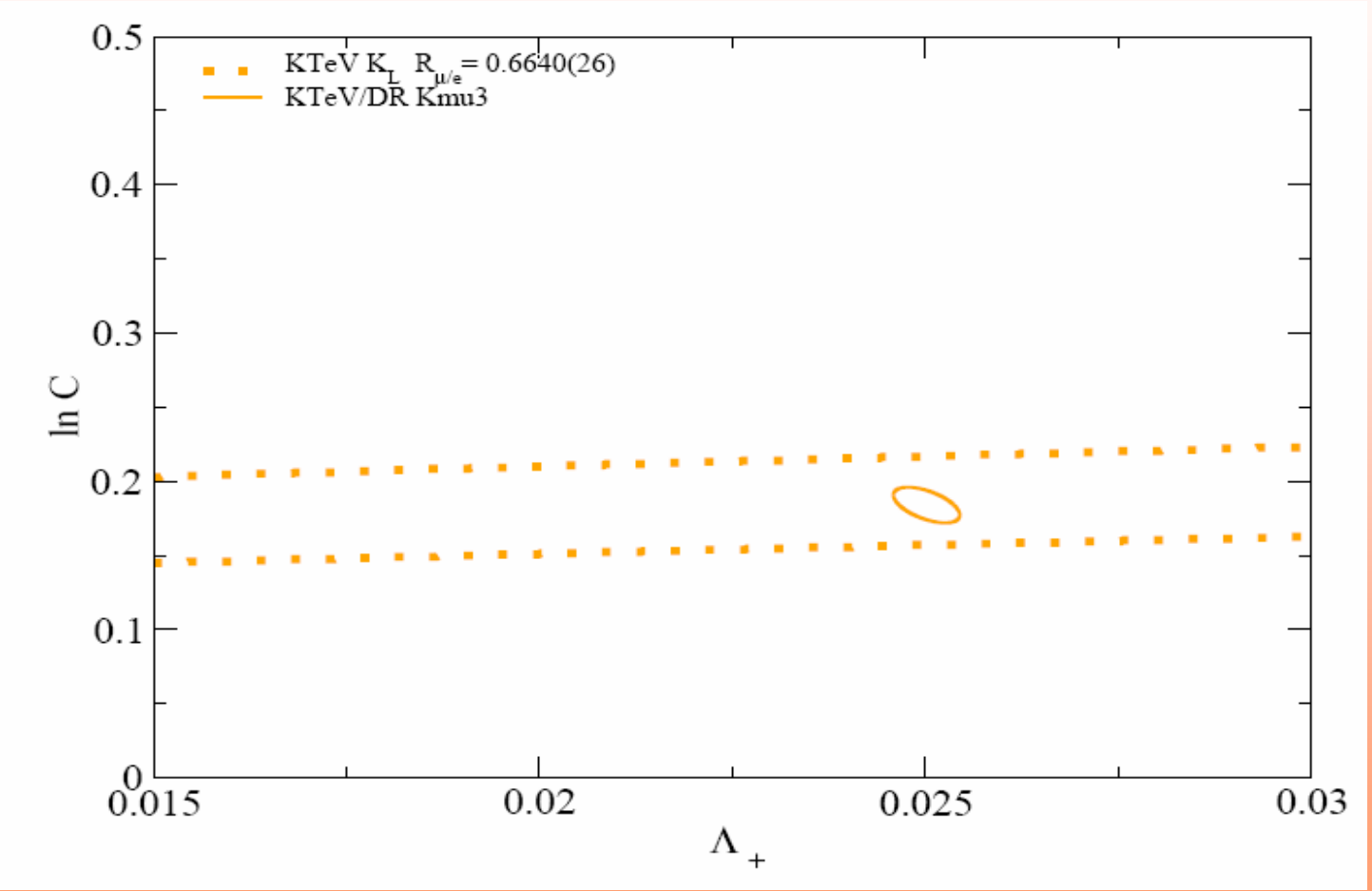
$$I^l_{K^{+/0}} = \int dt \frac{1}{m^8_{K^{+/0}}} \lambda^{3/2} F\left(t, \overline{f}_+(t), \overline{f}_0(t)\right)$$

- Then we can deduce $R_{\mu/e}$: $$R_{\mu/e} = \frac{\Gamma_{K^{+/0}\mu 3}}{\Gamma_{K^{+/0}e3}} = \frac{\left(1+2\Delta^{EM}_{K^{+/0}\mu}\right)}{\left(1+2\Delta^{EM}_{K^{+/0}e}\right)} \frac{I^{\mu}_{K^{+/0}}}{I^{e}_{K^{+/0}}}$$ (universality μ/e)

	K_{e3} only	$K_{\mu 3}$ only	$K_{e3}+K_{\mu 3}$
I^e_K	0.15450(21)	0.15416(47)	0.15442(19)
I^μ_K	-	0.10207(24)	0.10215(14)
I^μ_K/I^e_K		0.6621(12)	0.6615(10)
$R_{\mu/e}$	-	0.6650(30)	0.6645(30)

- To be compared with the measured $R_{\mu/e}$: $R^{exp}_{\mu/e} = 0.6640 \pm 0.0016 \pm 0.0022$

- $R_{\mu/e}$= f(lnC, Λ_+), but dependence on Λ_+ almost cancels by taking the ratio.
- One can represents the results of the fit and $R_{\mu/e}$ in the plane (lnC, Λ_+) :



4.2 Test of the SM + Lattice results on F_K/F_π, $f_+(0)$

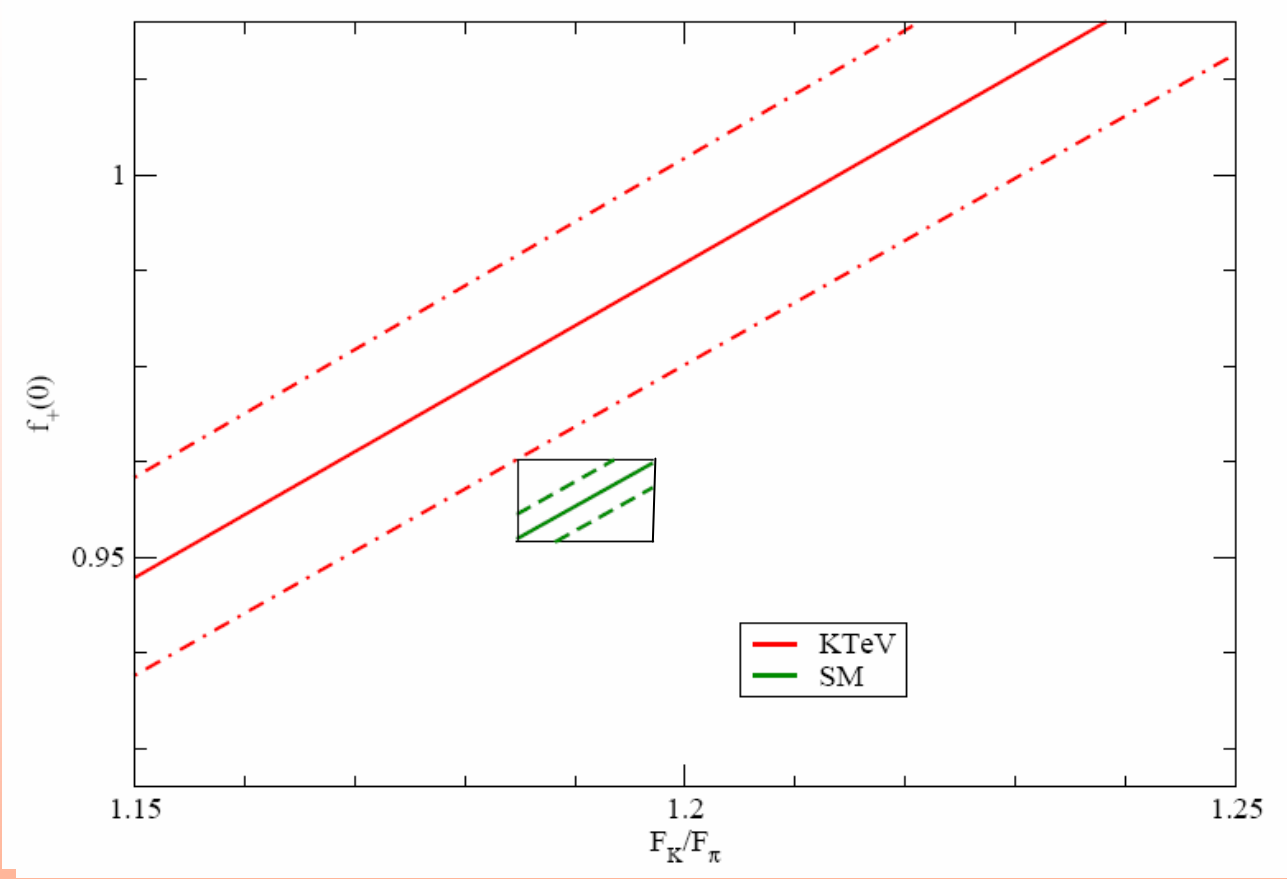
- SM case: F_K/F_π, $f_+(0)$ are known from experiments :
  - Unitarity of V_{CKM}: $|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1$
  - V_{ud} from $0^+ \rightarrow 0^+$ superallowed β decays: $V_{ud} = 0.97418(26)$ **[Towner&Hardy'07]**
  - V_{us} from unitarity : $|V_{us}|^2 = 1 - |V_{ud}|^2$

$$\frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))} \sim \left|\frac{F_K \mathcal{V}_{us}}{F_\pi \mathcal{V}_{ud}}\right| \Rightarrow \frac{\hat{F}_K}{\hat{F}_\pi} = 1.191 \pm 0.006$$

$$\Gamma(K^0 \to \pi^+ e^- \nu) \sim |f_+(0)\ \mathcal{V}_{us}| \Rightarrow \hat{f}_+(0) = 0.9574 \pm 0.0052$$

- If Physics Beyond the Standard Model, then F_π, F_K/F_π, $f_+(0)$ unknown : for instance, modification of EW couplings.

- Lattice simulation try to determine these fundamental quantities independently of the knowledge of EW couplings $\Rightarrow$ we consider the N_f=2+1 results.



Black rectangle: SM constraint on F_K/F_π and $f_+(0)$

$$\frac{\hat{F}_K}{\hat{F}_\pi} = 1.191(6)$$

$$\hat{f}_+(0) = 0.9574(52)$$

The green band :

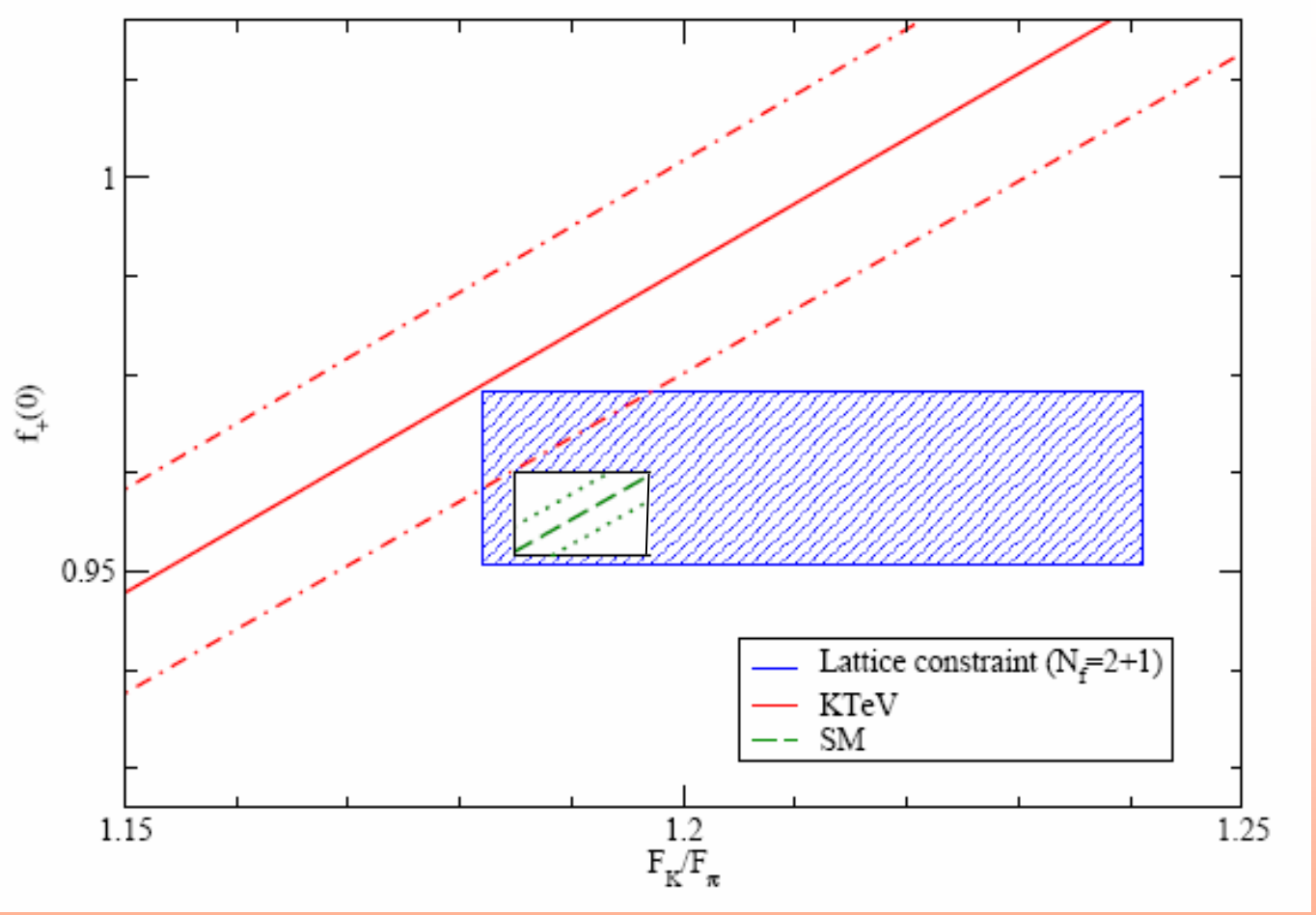
$$B_{\exp} = \left.\frac{F_K}{F_\pi f_+^{K^0}(0)}\right|_{\exp}$$

- Measurement of KTeV ⇨ prediction for $F_K/F_\pi/f_+(0)$ via the CT theorem

$$\frac{F_K}{F_\pi f_+^{K^0}(0)} = C - \Delta_{CT}$$

we take $\Delta_{CT} = (-3.5 \pm 8).10^{-3}$

⇨ The red band : KTeV results are at ~1.5σ from the SM



- KTeV result consistent with lattice ⇨ favours a value for F_K/F_π on the lower side of the lattice results and a value of $f_+(0)$ in the upper side of the lattice results in agreement with the recent lattice results:

  $F_K / F_\pi = 1.189(7)$ **[HBQCD-UKQCD]** and $f_+(0) = 0.9644(49)$ **[RBC-UKQCD]**

- Systematic uncertainties on the lattice difficult to estimate.

5. Conclusion

- Callan-Treiman theorem offers the possibility to test the SM: 2 different measurements of $F_K/F_\pi/f_+(0)$ (by B_{exp} measurements and by $C=f_0(\Delta_{K\pi})$ measurement).

- To that aim, introduction of a new parametrization for the form factors: a dispersive one relying on physical inputs ⇨ constraints on the curvature and higher order terms. Allows to determine the shape of form factors.

- One free parameter lnC to be measured.

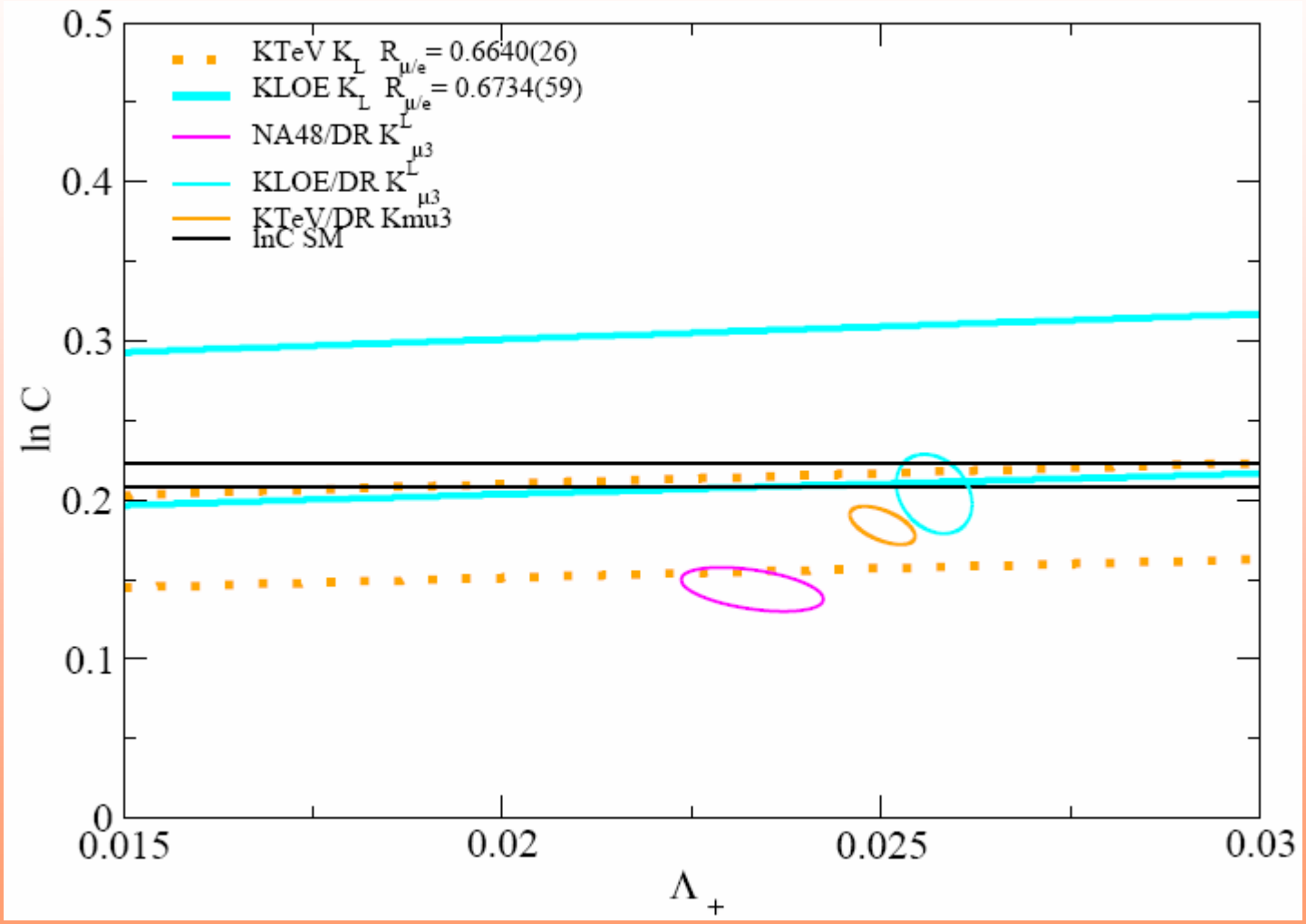
- Dispersive fit with KTeV data: preliminary result at least, 1-2σ deviation with the SM prediction depending on the value of Δ_{CT} (NA48 reports 4-5σ in the same direction).

- Test of consistency with the lattice results.

- Some checks done but further suggestions and other checks are welcome.

6. Additional slides

- With the other experiments+ the SM prediction



4.2 Extraction of $f_+^{K^0}(0)|V_{us}|$

$$\frac{\Gamma_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW}\left(1 + 2\Delta^{EM}_{K^{+/0}l}\right)\left|f_+^{K^{+/0}}(0)V_{us}\right|^2 I^l_{K^{+/0}}$$

Kaon life time

EM Radiative Corrections

$$I_K^l = \int dt \frac{1}{m_K^8}\lambda^{3/2} F\left(t, \hat{f}_+(t), f(t)\right)$$

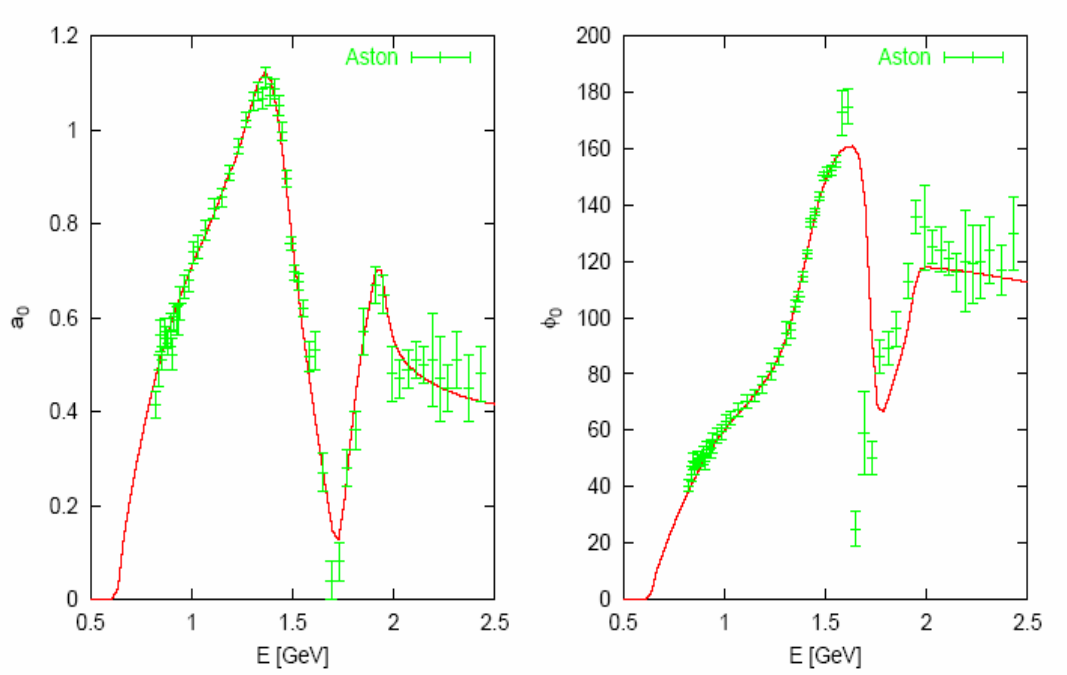
- From the calculation of I_K^l, the measurements of τ_K, $\Gamma(K_{e3})$ or $\Gamma(K_{\mu 3})$

⇨ $f_+^{K^0}(0)|V_{us}|$

	K_{e3} only	$K_{\mu 3}$ only	$K_{e3}+K_{\mu 3}$
$\lvert f_+(0)V_{us}\rvert^e$	0.21630(58)	0.21654(66)	0.21636(58)
$\lvert f_+(0)V_{us}\rvert^\mu$	-	0.21682(65)	0.21674(62)

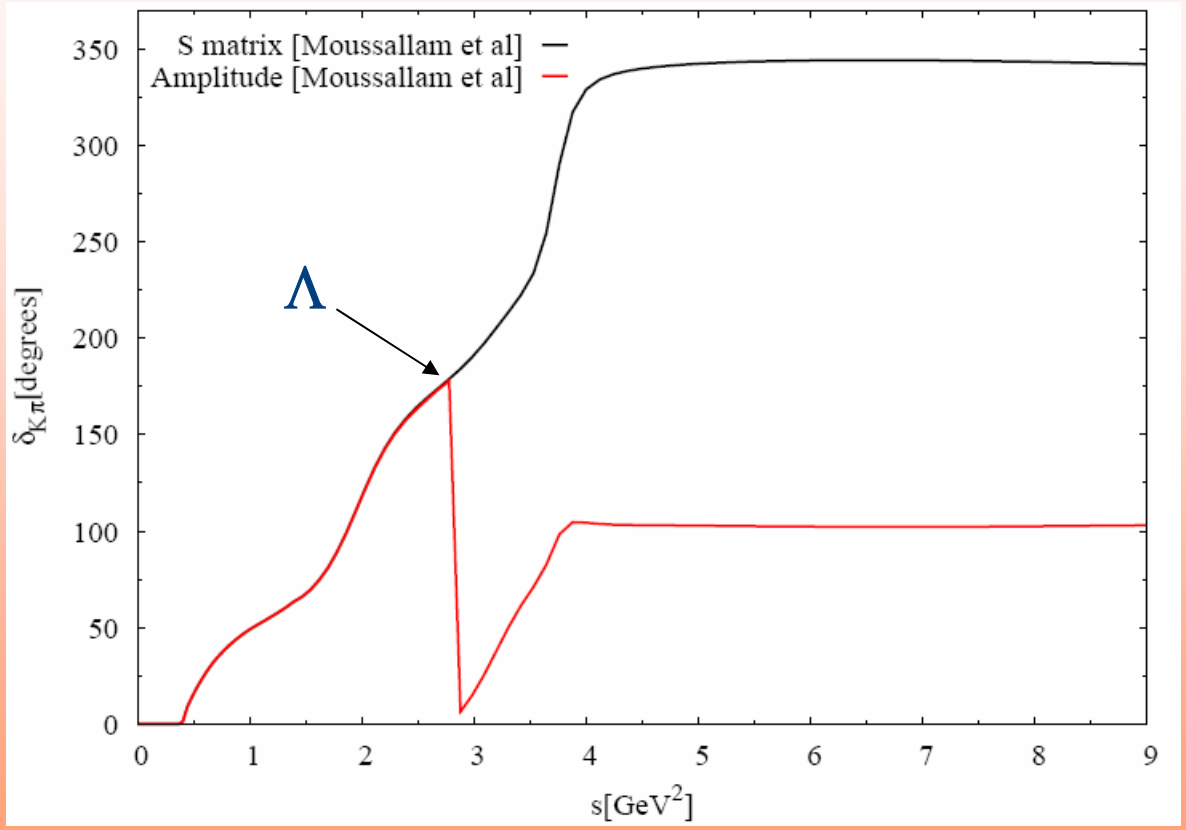
$K\pi$ scattering phase [Buettiker, Descotes, Moussallam '02]

- Experimental input for **$1\ \mathrm{GeV} < E < 2.5\ \mathrm{GeV}$**
  - $\pi K \to \pi K$
    [Eastbrook et al] & [Aston et al]
  - $\pi\pi \to K\overline{K}$
    [Cohen et al] & [Etkin et al]



- Solving Roy-Steiner equations : Analyticity, Unitarity and Crossing ⇒ set of six coupled equations in terms of 2 parameters $a_{1/2}$ and $a_{3/2}$. 4 S,P waves from $\pi K \to \pi K$; 2 S, P waves from $\pi\pi \to K\overline{K}$.
- $E > 2.5\ \mathrm{GeV}$: Regge Phenomenology.

$K\pi$ scattering phase [Buettiker, Descotes, Moussallam '02]



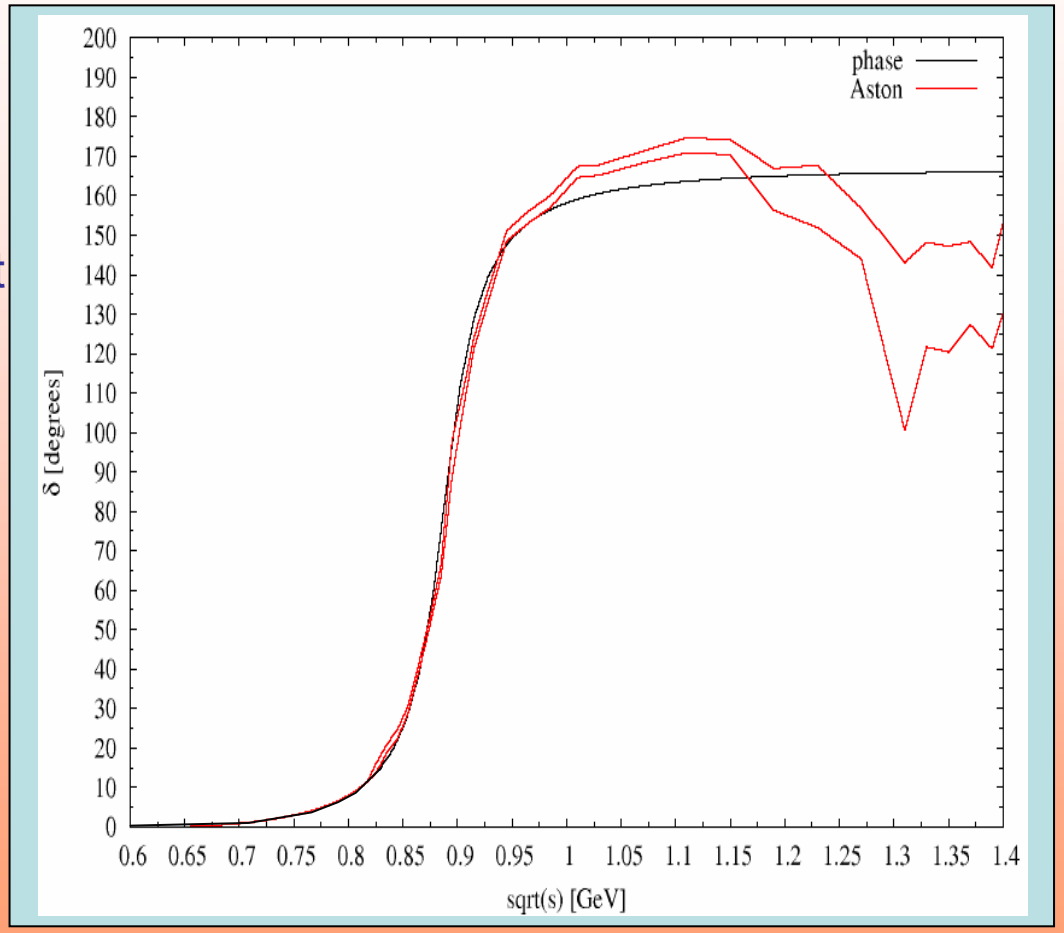
$K\pi$ scattering phase

- Experimental input for $0.825\ \mathrm{GeV} < E < 2.5\ \mathrm{GeV}$ $\pi K \to \pi K$ **[Aston et al].**

- Extrapolation of the phase down to the threshold complicated ⇨ lack of relevant experimental inputs.

- Construction of the partial wave amplitude : Breit-Wigner (K*(892)) a la Gounaris-Sakourai (Analyticity, Unitarity and Correct threshold behavior)

- Reproduce the value of p-wave scattering length.



4.2 Test of the SM

- 2 different measurements of $F_K/F_\pi/f_+(0)$
  - measurement of $C=f_0(\Delta_{K\pi})$ in $K^L_{\mu 3}$ +Callan-Treiman relation

$$\frac{F_K}{F_\pi f_+^{K^0}(0)} = C - \Delta_{CT}$$ with $$\Delta_{CT} = (-3.5 \pm 8).10^{-3}$$

  - Direct measurement from B_{exp}.

- If difference, then a signature of exchange of new particules (W_R, H^+) at high energy.