Update on $K_{L,S} \rightarrow \pi^{+}\pi^{-}\gamma$

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Status of analysis

- Final data sample
- Central values from fit
  - statistical errors
- Systematic errors, including:
  - errors due to input values
  - errors due to Data/MC mismatch
    - Estimate these using reweighting of cut variable distributions

Introduction

- As a reminder, I'm looking for direct CP violation in $K_{L,S}$ ->$\pi^+\pi^-\gamma$
  - Will appear through additional interference between K_L and K_S in the kaon lifetime plot
  - Will also manifest as interference between Inner Bremsstrahlung and Direct Emission in plot of E_γ
  - Look for both kinds of interference at once
  - Use vacuum beam data to help determine M1 direct emission

Likelihood fitter

- The likelihood function uses the triple differential decay rate (a function of E_{γ},cos θ and τ) which can be found at the end of the talk.
  - Kaon wavefunction generation and propagation follows the treatment in the Monte Carlo and, in the case of the regenerator, the Re(e'/e) fit

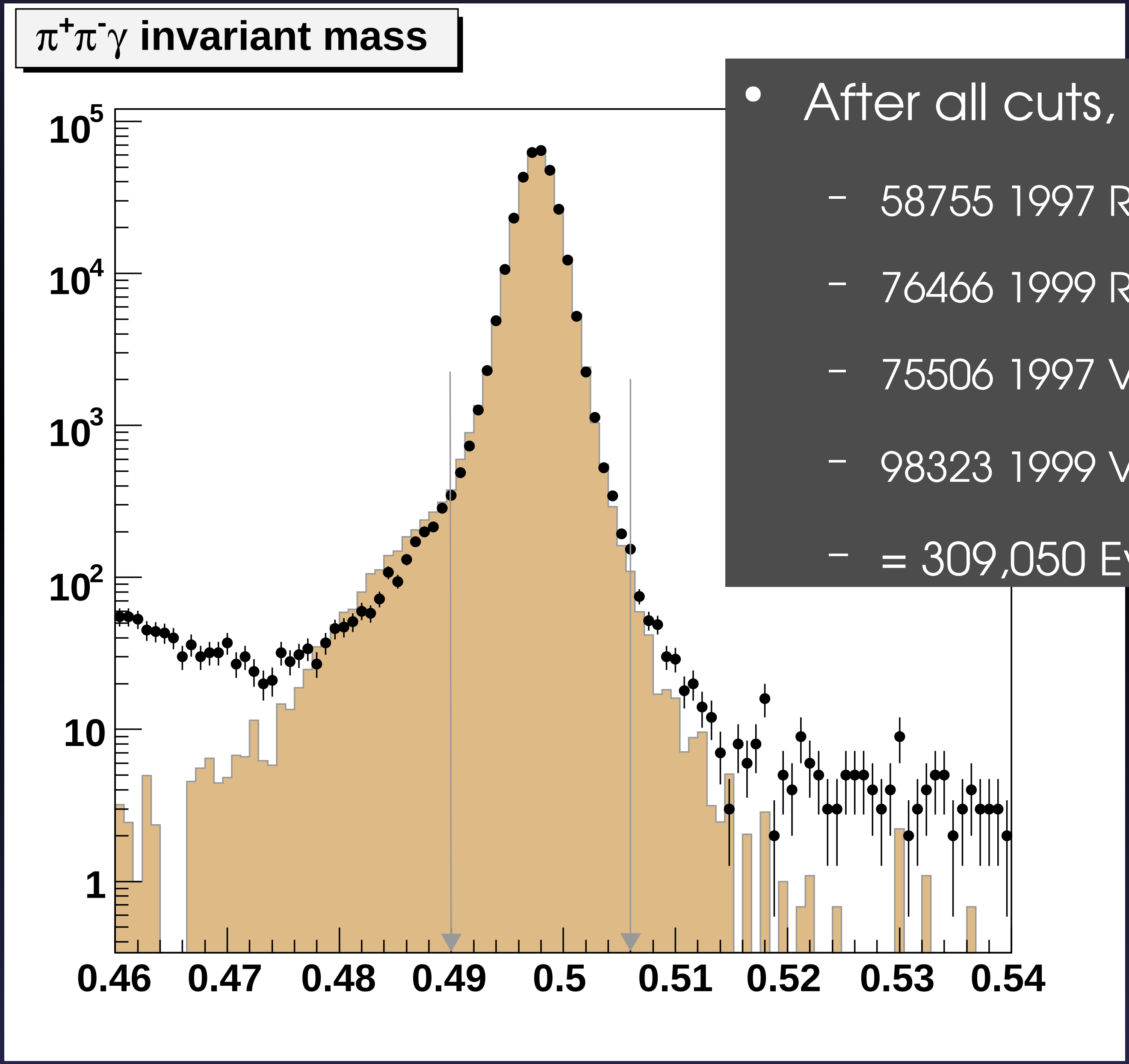
Likelihood Fit

- The likelihood fit uses data from both the regenerator and vacuum beams, from both 1997 and 1999 datasets.
- Uses Minuit to maximize the likelihood function, which can be found at the end of this talk.
  - The likelihood function has been tweaked from last time in order to obtain the correct weighting of the different MC samples

Likelihood Fit

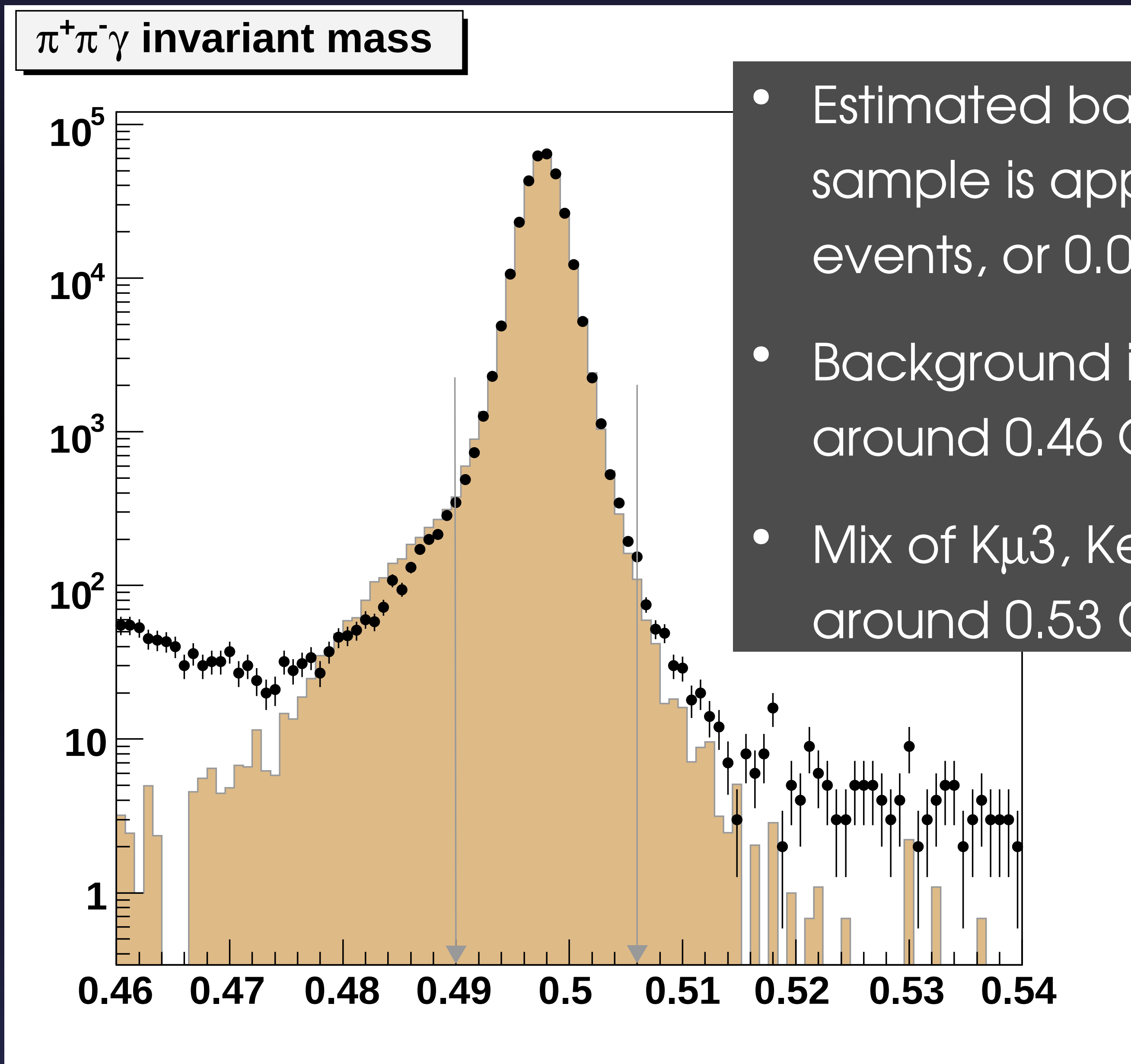
- Input various regenerator and kaon parameters, much like Re(e'/e) fit.
  - ρ, α, η_{+-}, ϕ_{+-}, τ_L, τ_S, $\Delta m_{K,}$,etc
- Float $K_{L,S}$-> $\pi^+\pi^-\gamma$ decay amplitudes:
  - Direct CPV parameter is $\hat{\varepsilon}$
    - amplitude for E1 direct emission
    - Can be used to compute $\eta_{+-\gamma}$
  - g_{E1} is the indirect CPV parameter
  - g_{M1}, a_1/a_2 are usual M1 DE parameters

Final Event Sample



- After all cuts, we have:
  - 58755 1997 Regenerator Events
  - 76466 1999 Regenerator Events
  - 75506 1997 Vacuum Beam Events
  - 98323 1999 Vacuum Beam Events
  - = 309,050 Events Total

Final Event Sample



- Estimated background in entire sample is approximately 200 events, or 0.06%
- Background is mostly $\pi^+\pi^-\pi^0$ around 0.46 GeV/c2
- Mix of Kμ3, Ke3 and $\pi^+\pi^-\pi^0$ around 0.53 GeV/c2

Final Selection Cuts

- A listing of all analysis cuts can be found at the end of this talk, however, here are some highlights:
  - Require E_γ > 20MeV in the K's rest frame
    - lowering cut to 6MeV only improves $\hat{\varepsilon}$'s stat error by ~8% , with increased background
  - Also require $M_{\pi\pi} < .477$ GeV/c^2
    - With the above cut on E_γ, we shouldn't see any events with $M_{\pi\pi} > .477$ GeV/c2

More Final Selection Cuts

- Cut on $M_{p\pi}$ to remove Λs
- Cut on $M_{\pi 0}$ to remove π^0, and thus K->$\pi^+\pi^-\pi^0$
- Remove events in which the in-time photon cluster energy is near pedestal, or early energy is significantly greater than pedestal
  - This rejects accidental clusters

Fitter Results

- Running the fit on the entire dataset with nominal cuts we obtain:
  - $\hat{\varepsilon}$ +offset =$(2.47 \pm 0.53)\times10^{-3}$
  - $g_{E1}=(0.000095\pm 0.14)\times10^{-3}$
  - $\widetilde{g_{M1}}=1.138\pm 0.021$
  - $a_1/a_2=-0.7516\pm0.0052$
- Correlations:

	$\hat{\epsilon}$	g_{E1}	$\widetilde{g_{M1}}$	a_1/a_2
$\hat{\epsilon}$	1	-0.028	-0.445	-0.292
g_{E1}	-0.028	1	0.022	0.014
$\widetilde{g_{M1}}$	-0.445	0.022	1	0.978
a_1/a_2	-0.292	0	0.978	1

Comments on Fit Results

- Judging from the statistical error, we should be able to place a new upper limit on g_{E1}
  - Limit from K->$\pi^+\pi^-e^+e^-$: $g_{E1} < 0.03$ (90% C.L.)
  - g_{E1} is the only free parameter for pure K_s decays, hence the strong limit

Comments on Fit Results

- Current result for g_{M1} and a_1/a_2
    - $g_{M1}=1.138 \pm 0.021$
    - $a_1/a_2=-0.7516 \pm 0.0052$
  - Result from $K_L \to \pi\pi\gamma$ (1997 vac only)
    - $g_{M1}=1.198 \pm 0.035 \pm 0.086$
    - $a_1/a_2=-0.738 \pm 0.007 \pm 0.018$ GeV^2/c^2
  - Result from $K_L \to \pi\pi ee$
    - $g_{M1}=1.11 \pm 0.12 \pm 0.08$
    - $a_1/a_2=-0.744 \pm 0.027 \pm 0.032$ GeV^2/c^2

Systematics Due to Inputs

- Vary fixed input parameters by $\pm 1\sigma$ and rerun fit
- $|\eta_{+-}| = (2.228 \pm 0.010) \times 10^{-3}$
  - $\Delta\hat{\varepsilon} = \pm 2.5 \times 10^{-4}$ $(0.5\ \sigma_{STAT})$
  - $\Delta g_{E1} = -3.2 \times 10^{-10}$ $(\sim 0\ \sigma_{STAT})$
  - $\Delta g_{M1} = \pm .006$ $(0.3\ \sigma_{STAT})$
  - $\Delta a_1/a_2 = \pm 3 \times 10^{-4}$ $(.06\ \sigma_{STAT})$

Systematics Due to Inputs

- Some papers evaluate the strong interaction phase shift δ_0 at $s=M_K$
- Some theorists instead evaluate it at $s= M_{\pi\pi}$
- This is mainly a philosophical issue:
  - Does rescattering occur before or after the emission of the bremsstrahlung γ ?

Systematics Due to Inputs

- We have chosen to evaluate the phase shift at $s=M_K$ however we have also run the fit using $s=M_{\pi\pi}$ and observe a shift in parameters. This is a systematic error. Here are the shifts ($\Delta=\alpha_{\pi\pi} - \alpha_K$):
- $\Delta\hat{\varepsilon} = 7.9\text{x}10^{-4}$ $(1.5\ \sigma_{STAT})$
- Δg_{E1}=-3.2 $\text{x}10^{-9}$ $(\sim 0\ \sigma_{STAT})$
- Δg_{M1}= -0.029 $(1.4\ \sigma_{STAT})$
- $\Delta a_1/a_2$=-0.0063 $(1.2\sigma_{STAT})$

Systematics from Data/MC disagreement

- The likelihood fit uses a large Monte Carlo sample to normalize the likelihood function. Proper normalization then depends on accurate modeling of acceptance
- Any problem with the acceptance will result in a systematic error.

Cut variations as estimates of systematics

- In the past, the acceptance was checked using cut variations:
  - Adjust one cut, apply to data AND MC, rerun the fit, and observe the shift
  - Doing so changes the sample, so this introduces some measure of statistical error in the shift.
  - Not clear where to stop cut adjustments
  - Can sometimes pick up background

Reweighting for systematic estimation

- Instead, we will reweight the Monte Carlo in order to force the data and MC to agree.
  - Data sample is never effected, so no statistical uncertainty is introduced.
    - We can use the pure shift (the difference in estimated parameters before and after the correction) and the error in order to estimate the systematic error due to any problems <u>with each cut variable</u>

Reweighting for systematic estimation

- We could in principle flatten every cut distribution. If the slope is small to begin with, the effect after the correction should also be small
  - Will need to rerun fit once for each cut variable

Reweighting for systematic estimation

- We must be alert to the effect of correlations between cut variables. Flattening one may "un-flatten" another.
    - For example, flattening E_γ will affect $M_{\pi\pi}$
- Estimation is in progress...

Conclusion

- The results so far:
  - $\hat{\varepsilon}$ +offset = $(2.47 \pm 0.53_{stat} \pm \sim 0.83_{syst}) \times 10^{-3}$
  - $g_{E1} = (0.0 \pm 0.14_{stat} \pm \sim 0_{syst}) \times 10^{-3}$
  - $g_{M1} = 1.138 \pm 0.021_{stat} \pm \sim 0.30_{syst}$
  - $a_1/a_2 = -0.7516 \pm 0.0052_{stat} \pm \sim 0.0063_{syst}$

Next Steps

- Finish up Systematic studies
- Find upper limit on g_{E1} using the usual Feldman-Cousins method
- Determine Fit χ^2 for E_γ, $\cos\theta$, z and p_K
  - can generate MC at best fit values without knowing true value of $\hat{\varepsilon}$

Next Steps

- Remove offset from $\hat{\varepsilon}$
- Do Feldman and Cousins method for $\hat{\varepsilon}$ as well
  - Method determines if we generate a central value instead of a upper limit at 90% confidence, in the event of a non-zero estimate for $\hat{\varepsilon}$.
- <u>Finish writing thesis</u>
- **Defend In July !!!!!!!!**

More Next Steps

- Generate long write-up and begin to address godparent comments
  - We'll need a committee soon.
- Float quadrapole (E2,M2) amplitudes for both K_L and K_S
  - A referee for the 1997 vac only paper wanted this done
- Perhaps take a closer look at $K \to \pi^+\pi^-\gamma\gamma$
  - Sehgal has expressed interest in this

Extra Slides

Analysis Cuts

Cut Variable	Keep Event If...
Kaon Mass	$0.48967\ GeV/c^2 < M_{\pi^+\pi^-\gamma} < 0.50567\ GeV/c^2$
P_T^2w.r.t Regenerator	$P_T^2 < 2.5 \times 10^{-4}\ GeV^2/c^2$
Kaon Momentum	40.0 GeV/c $< P_{\pi^+\pi^-\gamma} <$ 160.0 GeV/c
Photon Energy in Lab Frame	$E_\gamma^* > 1.5$GeV
Photon Energy in Kaon Rest Frame, From Calorimeter	20.0MeV $< E_\gamma^* <$ 175.0 MeV
Photon Energy in Kaon Rest Frame, From Kinematics	20.0MeV $< E_\gamma^* <$ 175.0 MeV
$\pi\pi$ Invariant Mass, Implied From Above Cut	$0.2711\ GeV/c^2 < M_{\pi\pi} < 0.4772\ GeV/c^2$
Shape χ^2 For Photon Cluster	$\chi^2 < 48$
Outer Fiducial Cut For Photon Cluster	ISEEDRING $<$ 18.1 cm
Inner Fiducial Cut For Photon Cluster	ISMLRING2 $>$ 4.5 cm
Photon/Track Separation at CsI	d $>$ 30 cm
Number of CsI clusters	NCLUS ≥ 3
pp0kin w.r.t. Target	$-0.10\ GeV^2/c^2 < P^2_{\pi^0} < -0.0055\ GeV^2/c^2$
L3 pp0kin	passes
Z vertex	125.5 m $<$ VTXZ $<$ 158.0 m
E/p	E/p $<$ 0.85
Track Momentum	TRKP $>$ 8.0 GeV
Vertex χ^2	VTXCHI $<$ 50.0
Magnet Offset χ^2	TRKOCHI $<$ 50.0
Track x separation at CsI	$\Delta x > 3.0$ cm
Track y separation at CsI	$\Delta y > 3.0$ cm
Total track separation at CsI	$\Delta r > 20.0$ cm
Number of Tracks	NTRK $= 2$
$\Lambda \to p\pi$ invariant mass	$M_{p\pi} < 1.112 GeV/c^2$ or $M_{p\pi} > 1.119 GeV/c^2$
Early energy in photon cluster	ADCSI_EARLY $<$ 150 counts
In-time energy in photon cluster	ADCSI_INTIM $>$ 115 counts
Photon/Upstream Track Projection at CsI	d $>$ 2.0cm distance
Reconstruction Routines	Return no errors
Veto Cuts	All pass
Level 1 Trigger Verification	Event passes
Fiducial Cuts	All pass
Number of Photon Candidates That Pass ALL Cuts	$N_{COMBINATIONS} = 1$ ONLY

Likelihood function

(E_γ, cosθ, z, p_K)

decay rate

$$\log \mathcal{L}(\vec{\alpha}) = \sum_{i=1}^{N_D^{97VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{97REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha})$$

fit parameters

$$+ \sum_{i=1}^{N_D^{99VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{99REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha})$$

initial guess parameters

$$- \left(N_D^{97VAC} + N_D^{97REG} + N_D^{99VAC} + N_D^{99REG}\right)$$

$$\times \log \left[N_D^{97VAC} \frac{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} + N_D^{97REG} \frac{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} \right.$$

$$\left. + N_D^{99VAC} \frac{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} + N_D^{99REG} \frac{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} \right]$$

generation parameters

Likelihood function

Likelihood sum over data events

$$
\begin{aligned}
\log \mathcal{L}(\vec{\alpha}) = & \boxed{\sum_{i=1}^{N_D^{97VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{97REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{99VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{99REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha})} \\
& - \left(N_D^{97VAC} + N_D^{97REG} + N_D^{99VAC} + N_D^{99REG}\right) \\
& \times \log \left[N_D^{97VAC} \frac{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} + N_D^{97REG} \frac{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} \right. \\
& \left. + N_D^{99VAC} \frac{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} + N_D^{99REG} \frac{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} \right]
\end{aligned}
$$

Likelihood function

normalization factor -
uses MC events

$$
\begin{aligned}
\log \mathcal{L}(\vec{\alpha}) = & \sum_{i=1}^{N_D^{97VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{97REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) \\
& + \sum_{i=1}^{N_D^{99VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{99REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) \\
& - \left(N_D^{97VAC} + N_D^{97REG} + N_D^{99VAC} + N_D^{99REG}\right) \\
& \times \log \left[N_D^{97VAC} \frac{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} + N_D^{97REG} \frac{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} \right. \\
& \left. + N_D^{99VAC} \frac{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} + N_D^{99REG} \frac{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} \right]
\end{aligned}
$$

Likelihood function

Weighting Factor for each MC sample ---

this ensures that the relative weight is correct

$$
\begin{aligned}
\log \mathcal{L}(\vec{\alpha}) = & \sum_{i=1}^{N_D^{97VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{97REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) \\
& + \sum_{i=1}^{N_D^{99VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{99REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) \\
& - (N_D^{97VAC} + N_D^{97REG} + N_D^{99VAC} + N_D^{99REG}) \\
& \times \log \left[N_D^{97VAC} \frac{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} + N_D^{97REG} \frac{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} \right. \\
& \left. + N_D^{99VAC} \frac{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} + N_D^{99REG} \frac{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} \right]
\end{aligned}
$$

Likelihood function

Weighting Factor for each MC sample ---

this ensures that the relative weight is correct

$$\log \mathcal{L}(\vec{\alpha}) = \sum_{i=1}^{N_D^{97VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{97REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha})$$

This allows us to work with arbitrary amounts of Monte Carlo in each subsample.

$$- \left(N_D^{97VAC} + N_D^{97REG} + N_D^{99VAC} + N_D^{99REG}\right)$$

$$\times \log \left[N_D^{97VAC} \frac{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} + N_D^{97REG} \frac{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} \right.$$

$$\left. + N_D^{99VAC} \frac{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} + N_D^{99REG} \frac{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}}{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha}_0)}} \right]$$

Likelihood function

Weighting Factor for each MC sample ---

this ensures that the relative weight is correct

$$\log \mathcal{L}(\vec{\alpha}) = \sum^{N_D^{97VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum^{N_D^{97REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha})$$

This term is computed at the beginning of maximization, and is then held constant

$$+ \sum_{i=1} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1} \log \mathcal{D}(\vec{x}_i;\vec{\alpha})$$

$$- \left(N_D^{97VAC} + N_D^{97REG} + N_D^{99VAC} + N_D^{99REG}\right)$$

$$\times \log \left[N_D^{97VAC} \frac{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}}{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}} + N_D^{97REG} \frac{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}}{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}} \right.$$

$$\left. + N_D^{99VAC} \frac{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}}{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}} + N_D^{99REG} \frac{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}}{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}} \right]$$

Likelihood function

This term is responsible for describing how the normalization changes as the fit parameters are adjusted

$$
\begin{aligned}
\log \mathcal{L}(\vec{\alpha}) = & \sum_{i=1} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) \\
& + \sum_{i=1}^{N_D^{99VAC}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) + \sum_{i=1}^{N_D^{99REG}} \log \mathcal{D}(\vec{x}_i;\vec{\alpha}) \\
& - \left(N_D^{97VAC} + N_D^{97REG} + N_D^{99VAC} + N_D^{99REG}\right) \\
& \times \log \left[N_D^{97VAC} \frac{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}}{\sum_{i=1}^{N_{MC}^{97VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}} + N_D^{97REG} \frac{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}}{\sum_{i=1}^{N_{MC}^{97REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}} \right. \\
& \left. + N_D^{99VAC} \frac{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}}{\sum_{i=1}^{N_{MC}^{99VAC}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}} + N_D^{99REG} \frac{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha})}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}}{\sum_{i=1}^{N_{MC}^{99REG}} \frac{\mathcal{D}(\vec{x}_i;\vec{\alpha}_g)}{\mathcal{D}(\vec{x}_i;\vec{\alpha_0})}} \right]
\end{aligned}
$$

Decay Rate

- In the likelihood function, D =

$$\frac{dN}{d\tau\, dE_\gamma\, d\cos(\theta)} = N_K \left[|\rho^2| \left[\frac{d\Gamma_{K_S \to \pi^+\pi^-\gamma}}{dE_\gamma\, d\cos(\theta)} \right] e^{-\frac{\tau}{\tau_s}} + \left[\frac{d\Gamma_{K_L \to \pi^+\pi^-\gamma}}{dE_\gamma\, d\cos\theta} \right] e^{-\frac{\tau}{\tau_L}} \right.$$

$$\left. + 2\,Re \left[\rho \frac{d\gamma^*_{LS}}{dE_\gamma\, d\cos(\theta)} e^{i\Delta m_K \tau} \right] e^{-\left(\frac{1}{\tau_L} + \frac{1}{\tau_S}\right)\frac{\tau}{2}} \right]$$

where

$$\frac{d\gamma_{LS}}{dE_\gamma\, d\cos(\theta)} \propto \left[E_{IB}(K_L) + E_{DE}(K_L) \right] * \left[E^*_{IB}(K_S) + E^*_{DE}(K_S) \right] + M(K_L)\, M^*(K_S)$$

$$\frac{d\Gamma_{K_L \to \pi^+\pi^-\gamma}}{dE_\gamma\, d\cos(\theta)} \propto |E_{IB}(K_L) + E_{DE}(K_L)|^2 + |M(K_L)|^2$$

$$\frac{d\Gamma_{K_S \to \pi^+\pi^-\gamma}}{dE_\gamma\, d\cos(\theta)} \propto |E_{IB}(K_S) + E_{DE}(K_S)|^2$$

Decay Amplitudes

$$E_{IB}(K_S) = \left(4\frac{M_K^{\;2}}{E_\gamma^2}\right)\frac{e^{i\delta_0}}{1-\beta^2\cos^2(\theta)}$$

CP Conserving

$$E_{IB}(K_L) = \left(4\frac{M_K^{\;2}}{E_\gamma^2}\right)\frac{\overbrace{\eta_{+\,-}}^{\epsilon+\epsilon'}\,e^{i\delta_0}}{1-\beta^2\cos^2(\theta)}$$

CP Violating

$$M(K_S) = i\,\epsilon\,g_{M1}\left(\frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2E_\gamma M_K} + 1\right)e^{i\delta_1}$$

CP Violating

$$M(K_L) = i\,g_{M1}\left(\frac{a_1/a_2}{M_\rho^2 - M_K^2 + 2E_\gamma M_K} + 1\right)e^{i\delta_1}$$

CP Conserving

$$E_{DE}(K_S) = \frac{g_{E1}}{\epsilon}\,e^{i(\delta_1+\phi_\epsilon)}$$

CP Conserving

$$E_{DE}(K_L) = \underbrace{g_{E1}\,e^{i(\delta_1+\phi_\epsilon)}}_{\text{Indirect CP Violating Term}} + \underbrace{i\,16\,\hat{\epsilon}\,e^{i\delta_1}}_{\text{Direct CP Violating Term}}$$

CP Violating

- Define:

$$\widetilde{\eta_{+-\gamma}} = \eta_{+-} + \left[\hat{\epsilon} + \underbrace{\frac{i}{16}\frac{\epsilon'}{\epsilon} g_{E1}}_{\text{Upper Limit} \sim \epsilon'} e^{i\phi_\epsilon} \right] e^{i\left(\delta_1 - \delta_0 + \frac{\pi}{2}\right)} \left(2\frac{E_\gamma}{M_K}\right)^2 \left(1 - \beta^2 \cos^2(\theta)\right)$$

$$\widetilde{\eta_{+-\gamma}} = \eta_{+-} + \widetilde{\epsilon'_{+-\gamma}}$$

- We can also define

$$\eta_{+-\gamma} = \eta_{+-} + \epsilon'_{+-\gamma}$$

$$\epsilon'_{+-\gamma} = \frac{1}{\Gamma_{K_S \to \pi^+ \pi^- \gamma}} \int d[PS]\, \tilde{\epsilon}'_{+-\gamma} \left| E_{IB}(K_S) + E_{DE}(K_S) \right|^2$$

which is the connection between the "new" and "old" direct CP violation parameters for this mode. Given one, the other can be computed.