Systematics Studies for the $K_L \rightarrow \pi^+\pi^-\gamma$ Analysis

John Shields
University of Virginia
KTeV Collaboration
May 21, 2004

Overview

- Reminder of (approximate) fit results with STAT error
- Results of Systematic Error due to inputs to Matrix Element ($\pi\pi$ phase shifts, η+-)
- Results of Systematic Error due to K_S Contamination
- Results of Systematic Error assigned from Cut Variation
- Future Plans

$K_L \rightarrow \pi^+ \pi^- \gamma$ (approximate) Fit Results

- 3-parameter fit (ge1 assumed non-zero) (**STAT ONLY!**)
  - g_{m1} = 1.19 +/- 0.04
  - a1/a2 = -0.740 +/- 0.007
  - ge1 < 0.12 (90% confidence) (Or: ge1 = 0.03 +/- 0.07)

*__CAVEAT:__ these numbers do not take into account a recently discovered bug in the analysis code involving $\pi^+ \pi^- \gamma$ L3 tag selection (affects ~0.3% of events)

•Central values may shift slightly, but net stat & syst errors are not expected to change significantly

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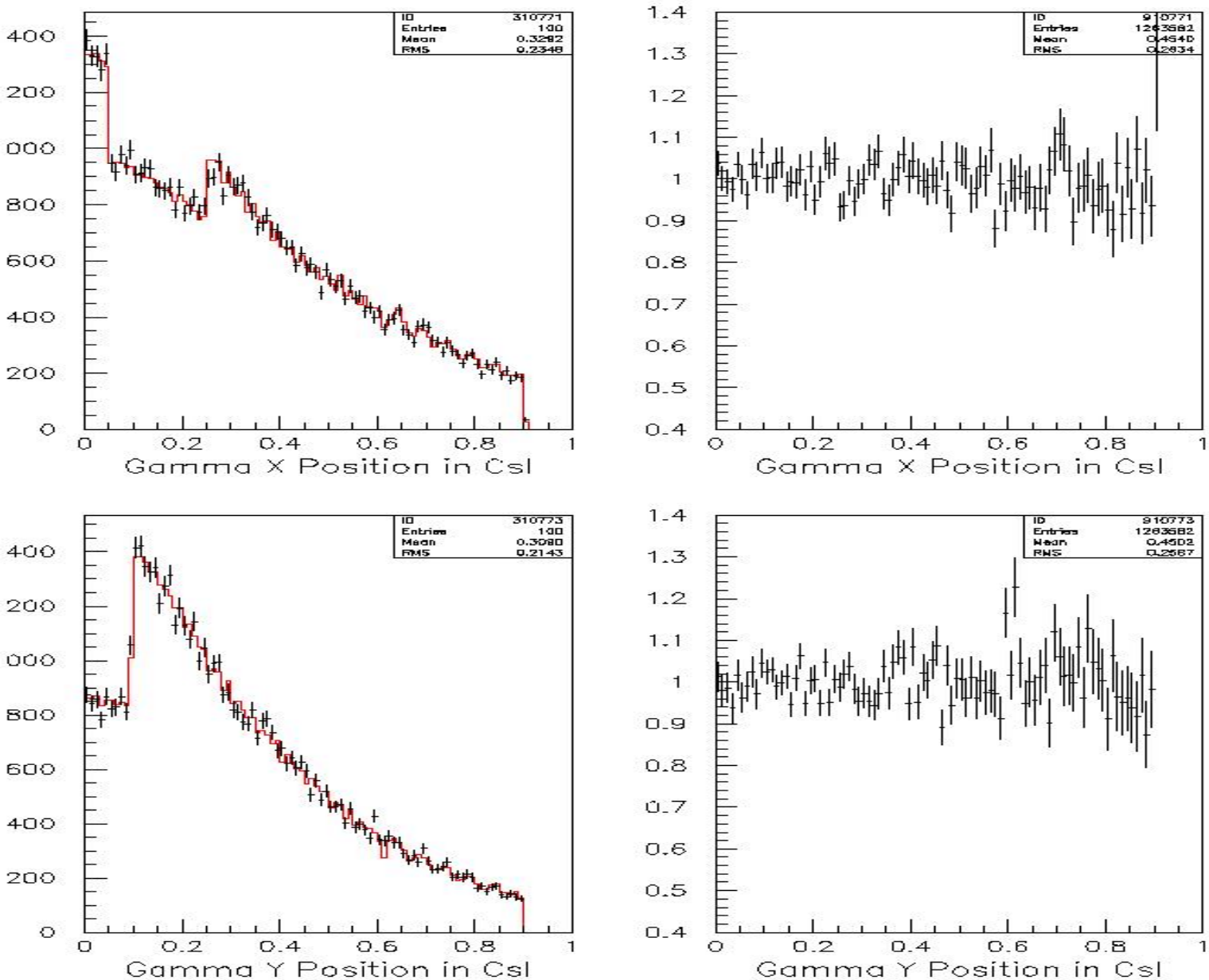
- For comparison:
  - J. Belz, 1997 ($K_L \rightarrow \pi^+ \pi^- \gamma$) : DE/(DE+IB) = 0.68 +/- 0.01
    a1a2 = -0.737 +/- 0.034
  - UVa, 1997 ($K_L \rightarrow \pi^+ \pi^- e^+e^-$) : gm1 = 1.35 +/- 0.17
    a1/a2 = -0.720 +/- 0.028

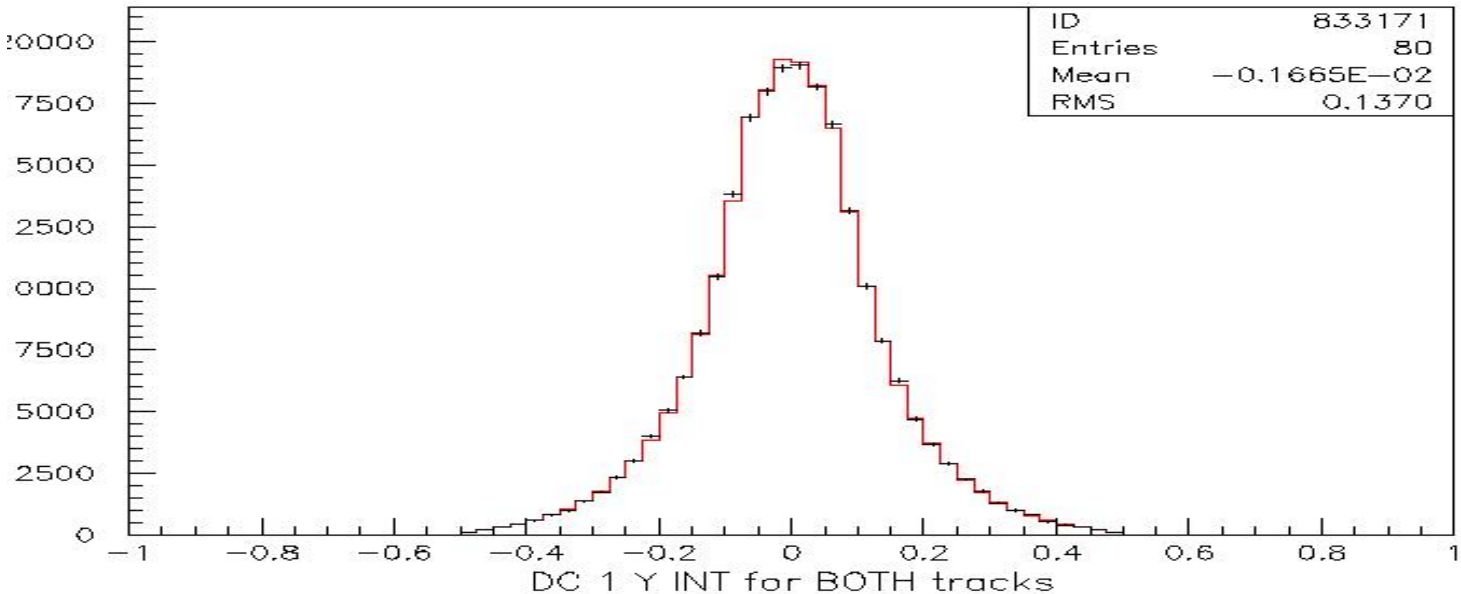
Analysis cuts: $K_L \rightarrow \pi^+\pi^-\gamma$

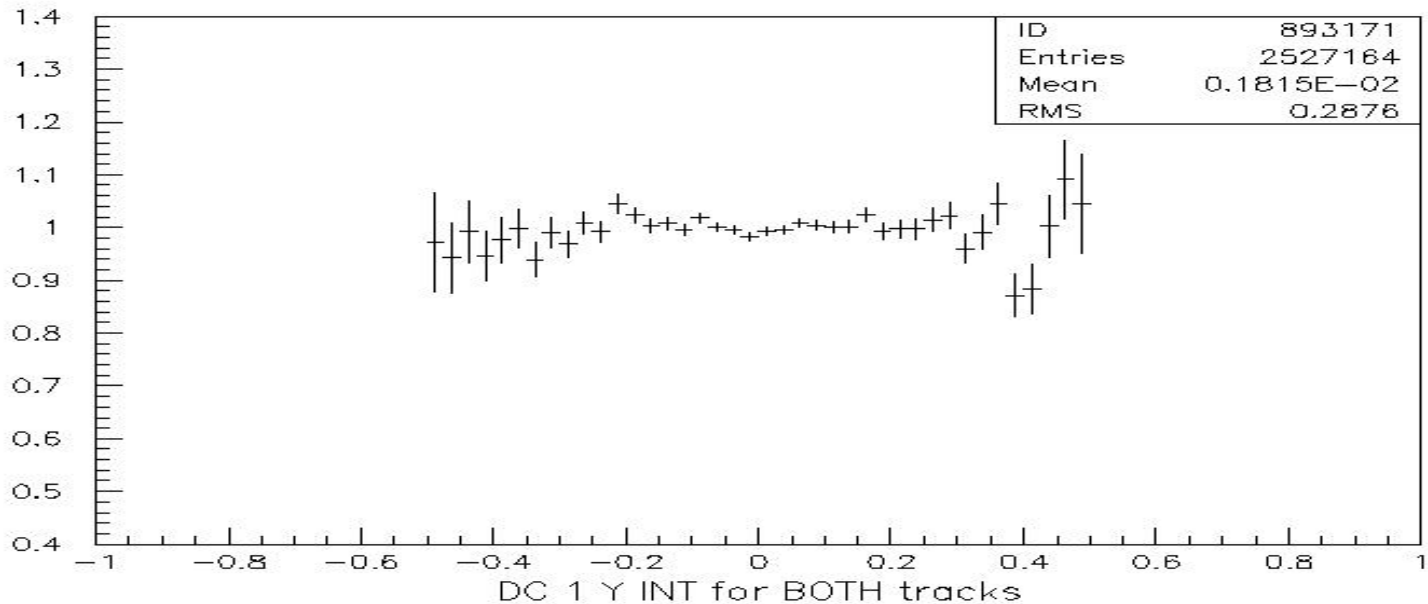
Criterion:	Event in sample if…
• Recon832	Recon832 ok
• L1VER832	istat $\neq 0$
• NTRKS	NTRKS = 2
• Clusters	1 or more non-track clusters
• Magnet offset χ^2	< 50
• Vertex χ^2	< 50
• Vertex Z	$120.0 <$ VTXZ < 158.0
• Track X-separation in CsI	$>$ 3 cm
• Track momentum	$>$ 8 GeV
• π^{+-} E/p	$<$ 0.85 GeV
• Pp0kine	$<$ -0.0055 GeV^2
• γ energy (Lab)	$>$ 1.5 GeV
• γ energy (Center of Mass)	$>$ 20 MeV
• γ-track separation in CsI	$>$ 30 cm
• Fusion χ^2	$<$ 48
• γ CsI pipe block exclusion	smallring $>$ 4.5 cm
• γ CsI outer fiducial cut	seedring $<$ 18.1 cm
• $\pi^+\pi^-$ invariant mass	$<$ 0.492 GeV
• $\pi^+\pi^-\gamma$ momentum	$25.0 < P_{\pi+\pi-\gamma} < 160.0$
• $\pi^+\pi^-\gamma$ P_T^2	$< 2.5 \times 10^{-4}$ GeV^2
• $\pi^+\pi^-\gamma$ invariant mass	$0.48967 < M_{\pi+\pi-\gamma} < 0.50567$

Photon x,y position in CsI: Data/MC (N_{MC} = 10x data)

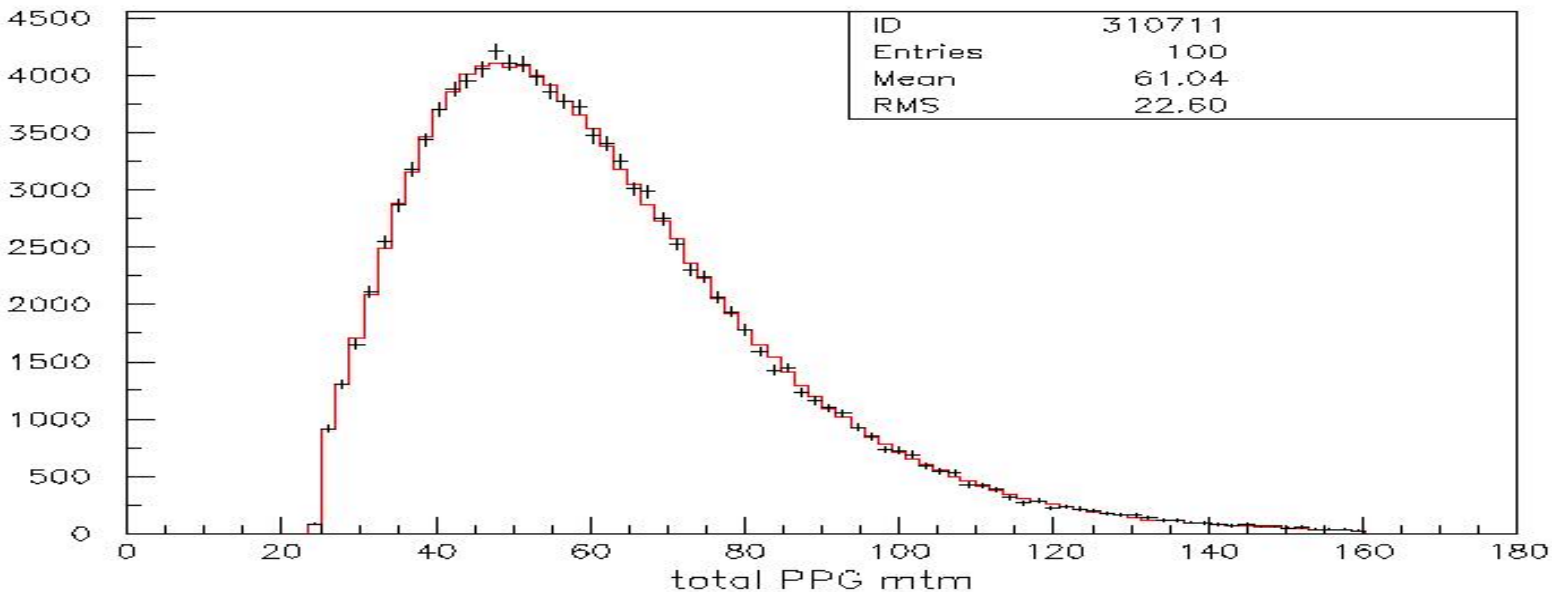


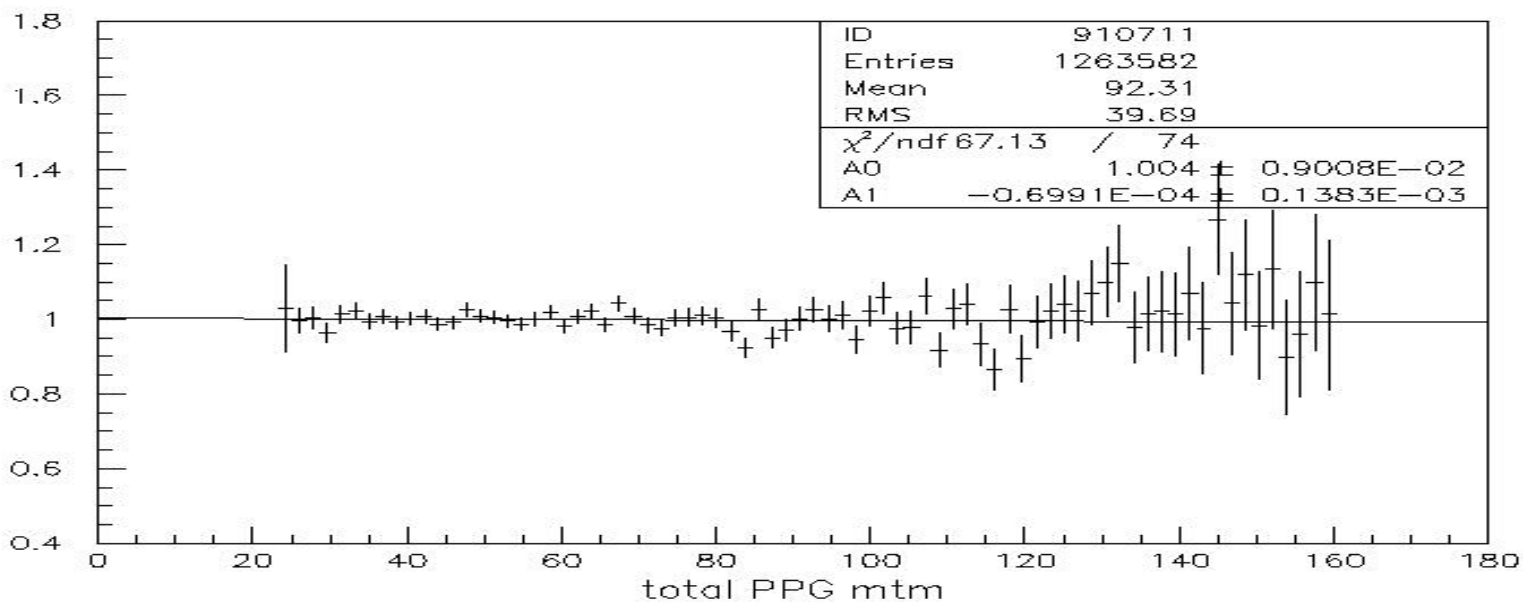
Track position in DC 1





Kaon momentum





Calculation of ππ phase shifts & errors (from Colangelo, 2001)

$$\tan\delta_0^0(M_K) = \sqrt{1-\frac{4M_\pi^2}{M_K^2}}\left[A_0^0 + B_0^0 q^2 + \ldots\right]\left(\frac{4M_\pi^2 - s_0^0}{M_K^2 - s_0^0}\right)$$

$$\tan\delta_1^1(M_{\pi\pi}) = \sqrt{1-\frac{4M_\pi^2}{M_{\pi\pi}^2}}\left[\left(q^2\right)\left(A_1^1 + \ldots\right)\right]\left(\frac{4M_\pi^2 - s_1^1}{M_{\pi\pi}^2 - s_1^1}\right)$$

where:

$s^0_0 = 36.77 M_\pi^2$, $s^1_1 = 30.72 M_\pi^2$

$$q^2(M_X) = \frac{1}{4}\left(\frac{M_X}{M_\pi}\right)^2 - 1$$

$A^0_0 = 0.220$ +/- 0.005

$B^0_0 = 0.268$ +/- 0.006

$A^1_1 = 0.0379$ +/- 0.0005

$\delta^0_0(M_K) = 0.684$ +/- 0.012

$\delta^1_1 = \delta^1_1(M_{\pi\pi})$

Systematic Error due to Inputs (δ^0, δ^1, η_{+-}) to the Monte Carlo

***baseline Monte Carlo has the given variable shifted by 1σ & data refitted**

MC variable	gm1 shift	a1/a2 shift	Ge1 shift
$\delta^0 + \sigma_{\delta 0}$	0.0095	0.00008	0.0348
$\delta^0 - \sigma_{\delta 0}$	0.0014	0.00035	0.0076
$\delta^1 + \sigma_{\delta 1}$	0.0005	0.00262	0.0348
$\delta^1 - \sigma_{\delta 1}$	0.0054	0.00131	0.0097
$\eta_{+-} + \sigma_{\eta +-}$	0.0162	0.00056	0.0121
$\eta_{+-} - \sigma_{\eta +-}$	0.0021	0.00030	0.0217

MC variable	gm1 syst err	a1/a2 syst err	Ge1 syst err
δ^0 ($\pi\pi$ phase shift)	0.0095	0.00035	0.0338
δ^1 ($\pi\pi$ phase shift)	0.0054	0.00026	0.0348
η_{+-}	0.0162	0.00056	0.0217

K_S Contamination Study

- Used M. Ronquest's new $\pi^+\pi^-\gamma$ Monte Carlo that can turn K_S, K_L/K_S mixing on & off
- Made 40 data-sized samples of this MC
  - 20 K_L –only samples (differing only by a random seed)
  - 20 samples with full K_L,K_S mixing (differing only by a random seed)
- All 40 samples were fit **IN PLACE OF DATA**, using my usual "baseline" MC and maximum loglikelihood fitting program. Means and std deviations were calculated
- **Net Result: observed shifts btwn K_S, K_L/K_S are within statistical fluctuations**

	gm1	**a1/a2**	**Ge1**
$\lvert \langle X_{KL} \rangle - \langle X_{KL/KS} \rangle \rvert$	0.007	0.0019	0.013
Stat error on $\langle X_{KL} \rangle$	0.009	0.0014	0.013
Stat error on $\langle X_{KL/KS} \rangle$	0.010	0.0021	0.012
K_S contam syst error assigned	0.0	0.0	0.0

- X_{KL} ≡ parameter best-fit for K_L –only sample
- $X_{KL/KS}$ ≡ parameter best-fit for K_L/K_S mixing sample

*note that analysis cuts include a cut at P_{kaon} = 160.0 GeV

Cut Variation Systematics Study

- ✓Analysis job was rerun for each cut choice
- ✓ Cuts were varied one at a time, producing a new subsample (or supersample) of data
- ✓Analysis job & Maximum loglikelihood were recomputed for each subsample
- ✓Error determined using a grid centered on the best-fit values from the loglikelihood. 1 sigma fluctuations were determined as values < 0.5 away from max loglikelihood
- ✓NOTE: the "nominal" data sample & the subsample are not entirely stat independent

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Calculation of the STAT Error on a Shift

Stat error (σ_{shift}) on a given shift S (where $S \equiv X_{subsample} - X_0$) :

$$\sigma^2_{shift} = \sigma^2_{subsample} - \sigma^2_0$$

Where:

$\sigma_{subsample} \equiv$ stat error on the data subsample

$\sigma_0 \equiv$ stat error on the "nominal" data sample

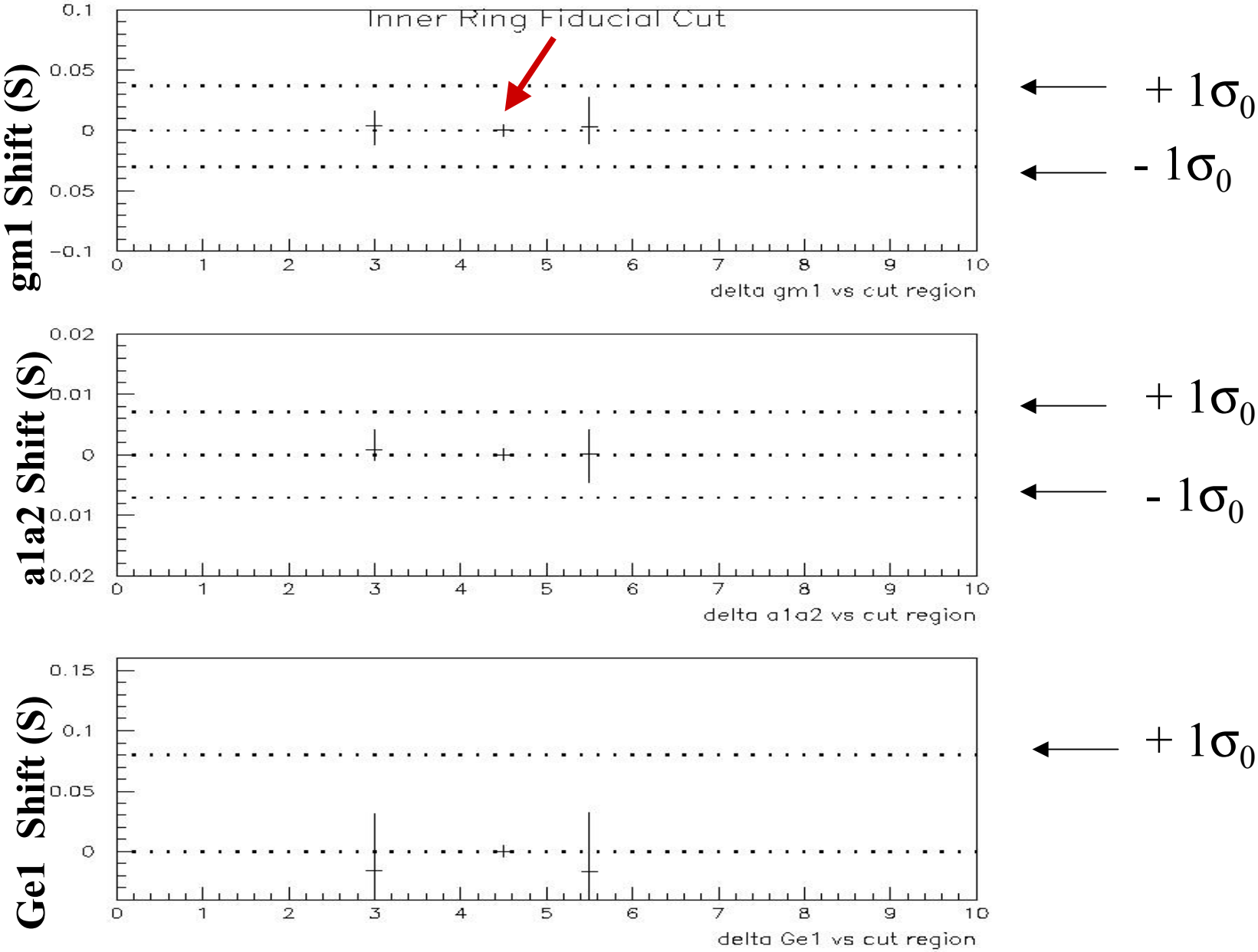
$X \equiv$ parameter best-fit value for the specified cut choice

Method of Assigning a SYSTEMATIC Error to a Shift

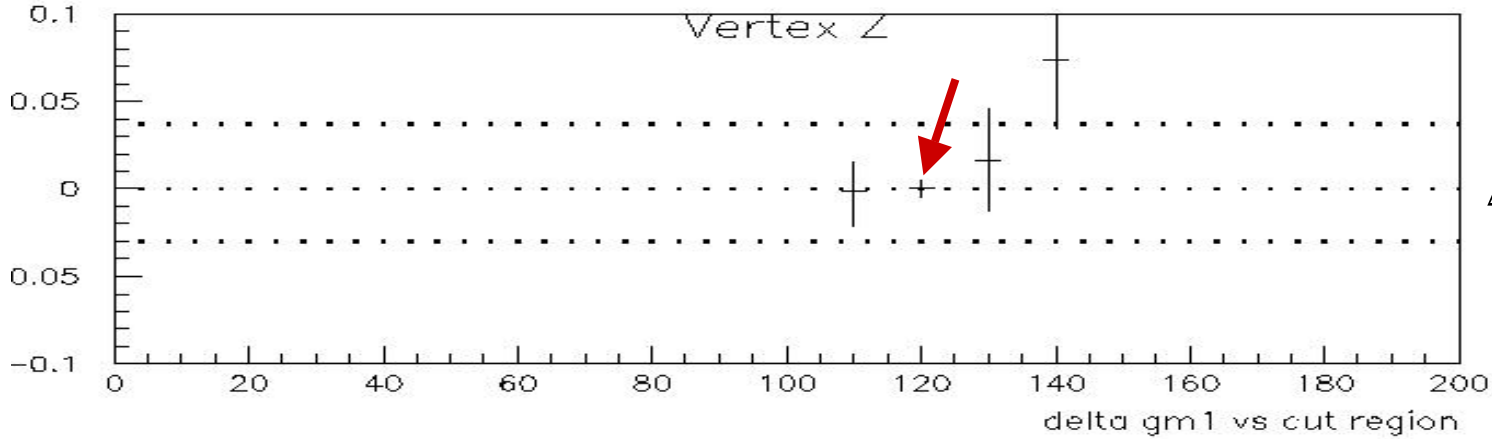
- **Case I: Shift S < σ_{shift}**
  - Within stat fluctuations, there is no evidence for a syst. shift
  - Systematic error $\Delta S = 0$ for this cut
- **Case II: Shift S > σ_{shift}**
  - A systematic error ΔS is assigned according to the "integral prescription" described in the ε'/ε paper:

$$\frac{1}{\sigma_{\text{shift}}\sqrt{2\pi}} = \int_{-\Delta S}^{\Delta S} \exp\left[\frac{-(x-S)^2}{2\sigma_{shift}^2}\right] = 0.683$$

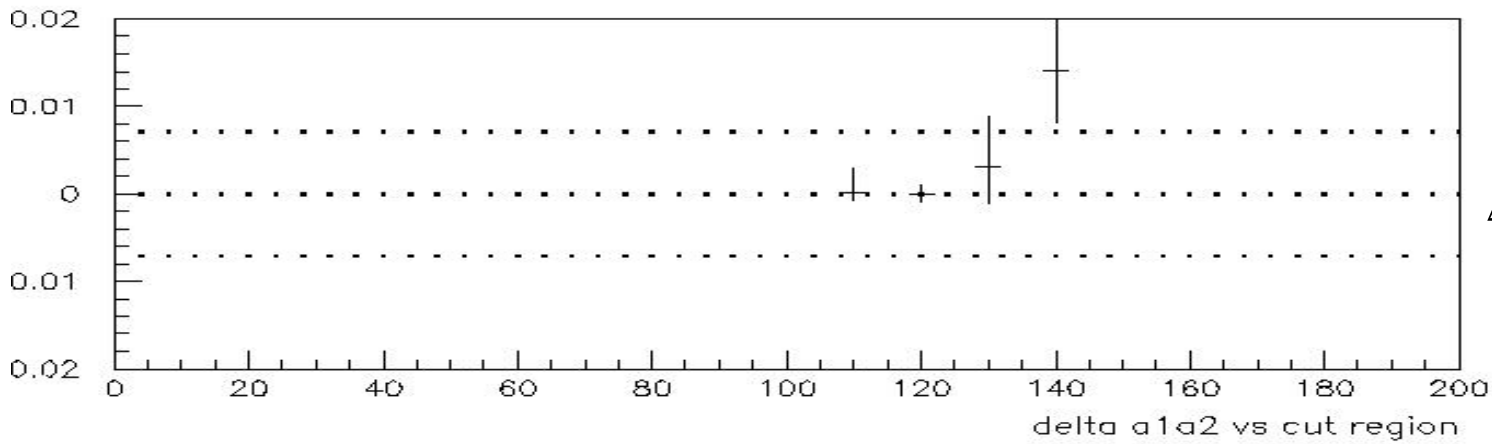
** i.e range (- ΔS , ΔS) includes 68.3% of the mean of a gaussian with mean S & width σ_{shift}



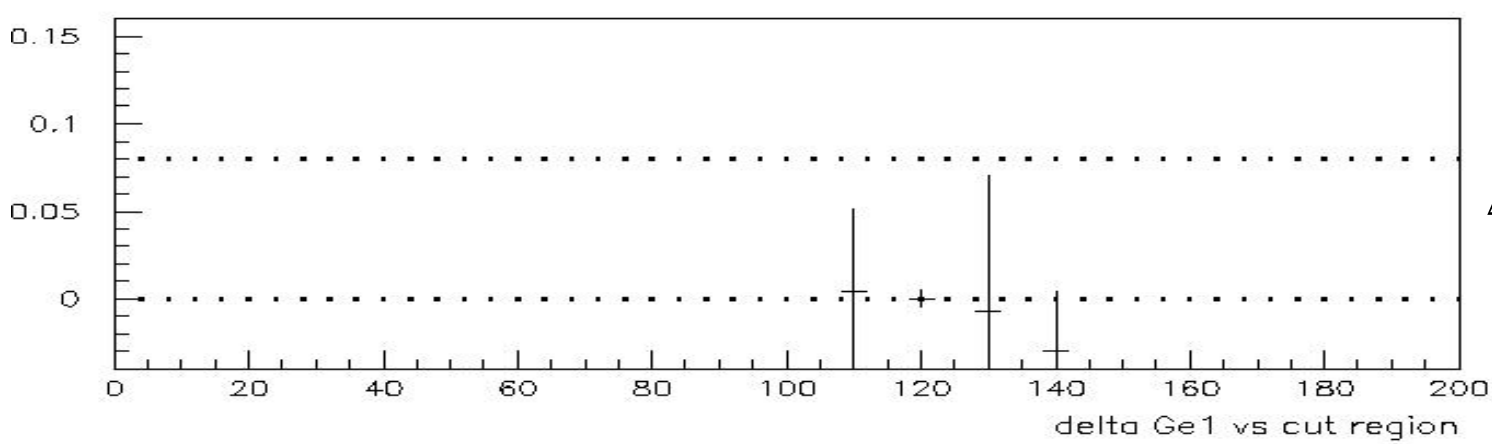
- **error bars on the data points are errors on the SHIFT (σ_S)**
- **note that $S < \sigma_S$ in all 3 plots. Hence $\Delta S = 0$ for all plots**



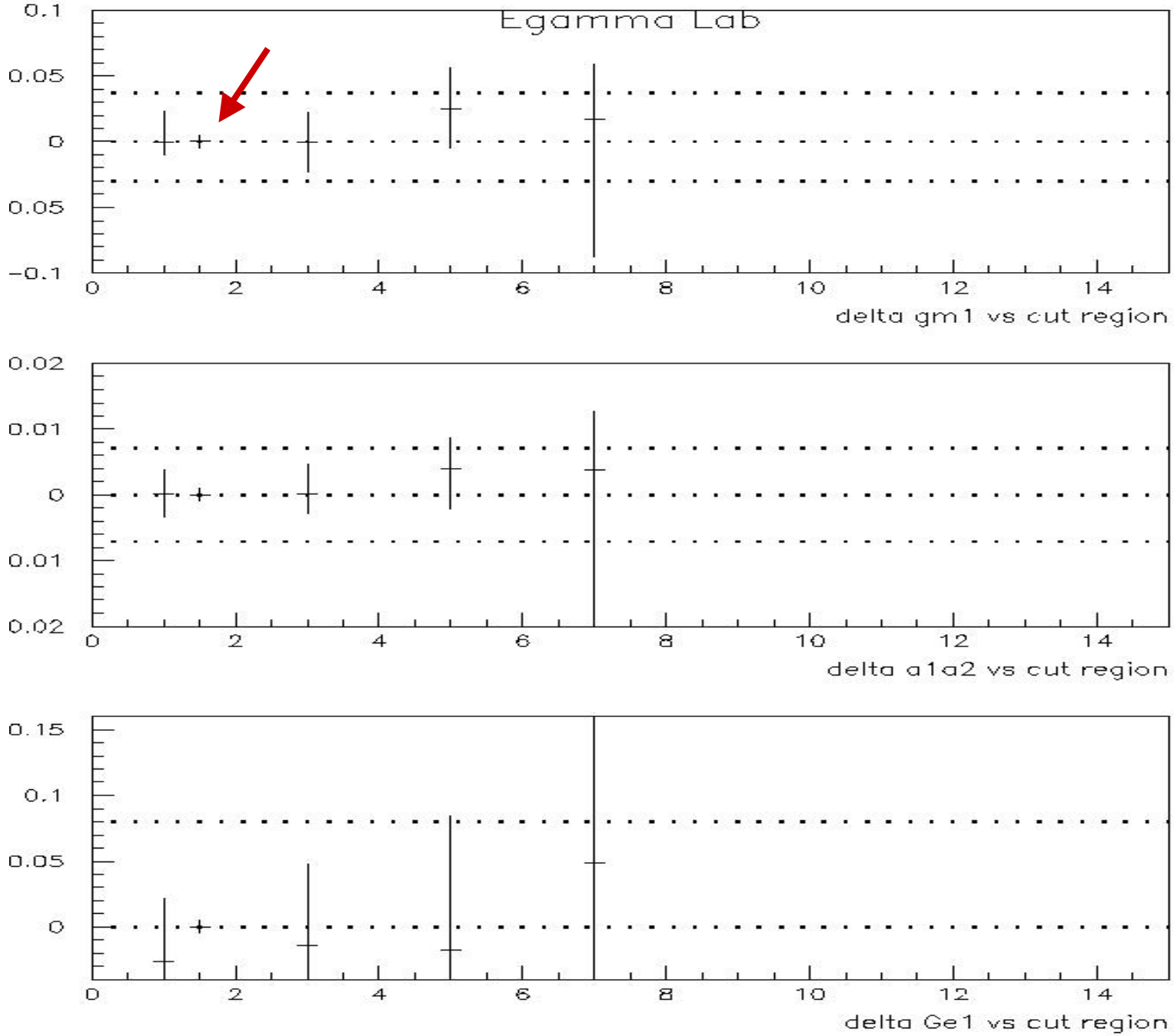
$\Delta S_{gm1} = 0.093$



$\Delta S_{a1a2} = 0.0177$



$\Delta S_{ge1} = 0.0$



$\Delta S = 0$, all plots

systematic	σ_{gm1}	σ_{a1a2}	σ_{Ge1}
background	0.03	0.004	0.09
Kaon slope	0.003	0.0004	0.005
Fusion chi2	0.0	0.0	0.0
Inner ring	0.0	0.0	0.0
Outer ring	0.0	0.0	0.0
Mpipi	0.0	0.0	0.0
pt2	0.0	0.0	0.0
pp0kine	0.0	0.0051	0.0
P kaon	0.072	0.015	0.0
Track-γ sep	0.0	0.004	0.0
Time (run no.)	0.0	0.0	0.0
Vertex Z	0.093	0.0177	0.0
Egamma lab	0.0	0.0	0.0
E/p	0.018	0.0	0.0
Kshort Contam	0.0	0.0	0.0
Pion Showers	0.0	0.0	0.0
η_{+-} in ME	0.016	0.00056	0.0217
δ^0 phase in ME	0.0095	0.00035	0.0338
δ^1 phase in ME	0.0054	0.00262	0.0348

Summary of Systematic Errors

variable	Stat error	Sys Error
gm1	0.037	0.126
a1/a2	0.0071	0.0244
Ge1	0.070	0.105

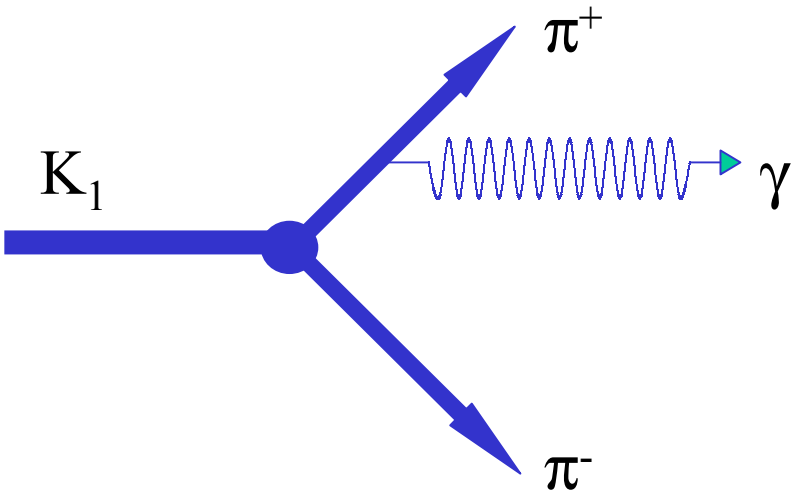
variable	σ_{quad}
Gm1	0.131
a1/a2	0.0254
Ge1	0.131

Summary and Future Plans

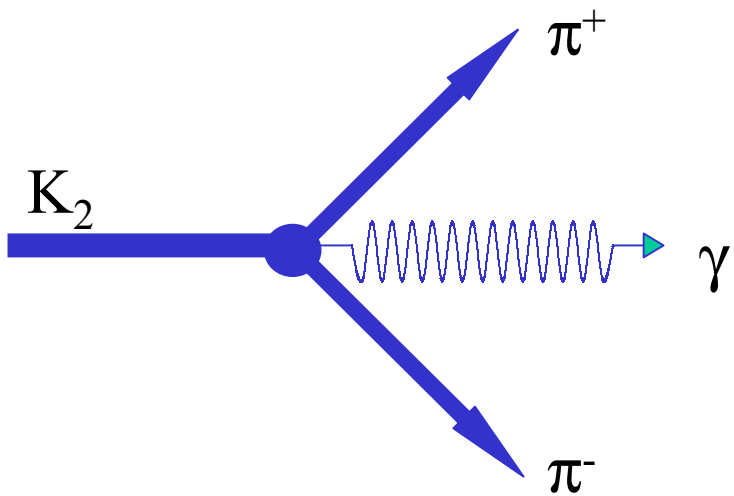
- Matrix element input (pion phase, η+-), K_S contamination, & cut variation studies complete
- Data will be refit to account for a small (~0.3%) correction to the L3 tag selection.
- NET systematic error is expected to be completed within the next few weeks
  - (only major systematic remaining: syst error on the likelihood fitting/reweighting method)
- Exact Ge1 upper limit will be determined via the method of Feldman & Cousins
- Work on the Long Writeup draft has begun
  - Should be available in the June/July timeframe
- **GRADUATE: Aug/Sept, 2004**

(end of presentation)

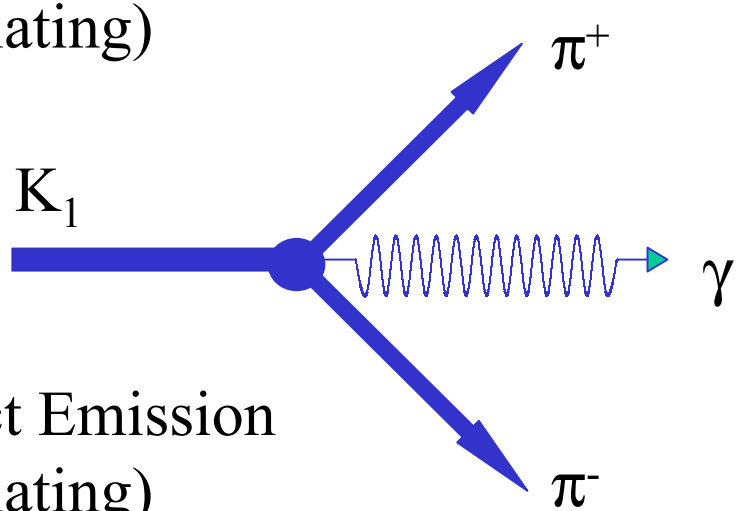
$K_L \rightarrow \pi^+\pi^-\gamma$ Decay



Inner Bremsstrahlung
(CP Violating)



M1 Direct Emission
(CP conserving)



E1 Direct Emission
(CP Violating)

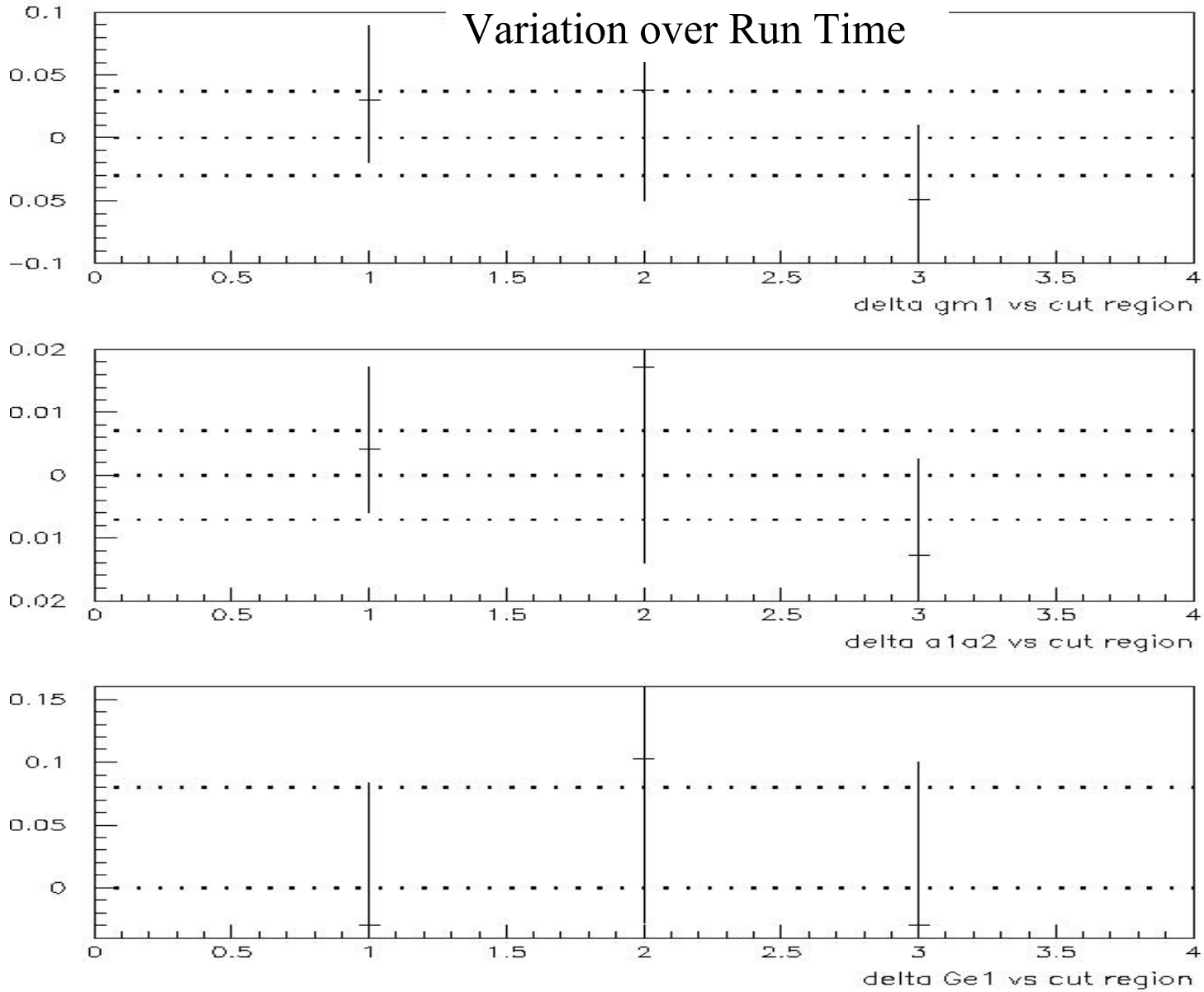
$K_L \to \pi^+\pi^-\gamma$ Differential Decay Rate, Sehgal Model

$$\frac{d\Gamma}{d\omega\, d\cos\theta} = (const)\left[\left|\xi_E\right|^2 + \left|\xi_M\right|^2\right]\omega^3\beta^3\left(1 - \frac{2\omega}{M_K}\right)\sin^2\theta$$

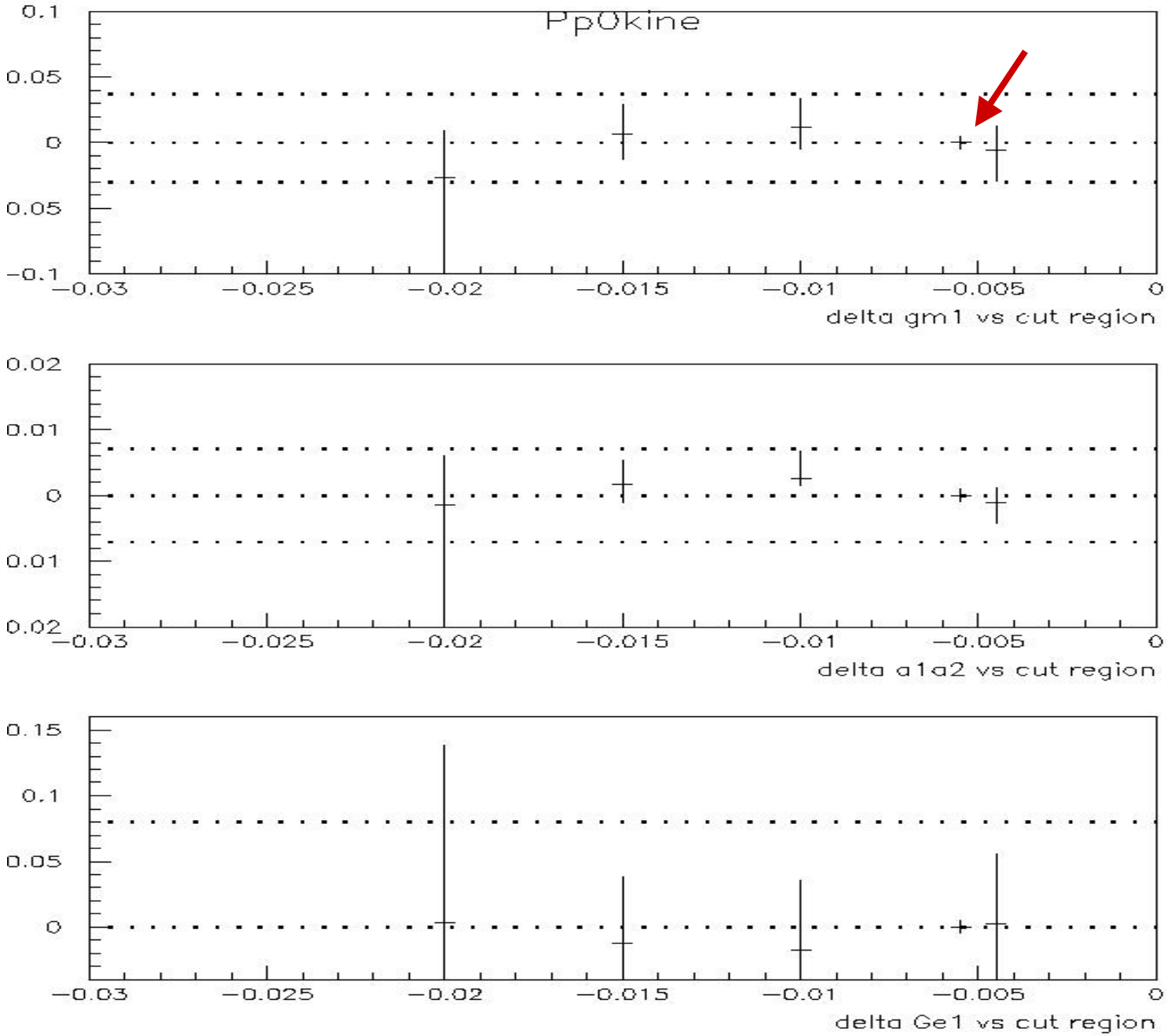
where:

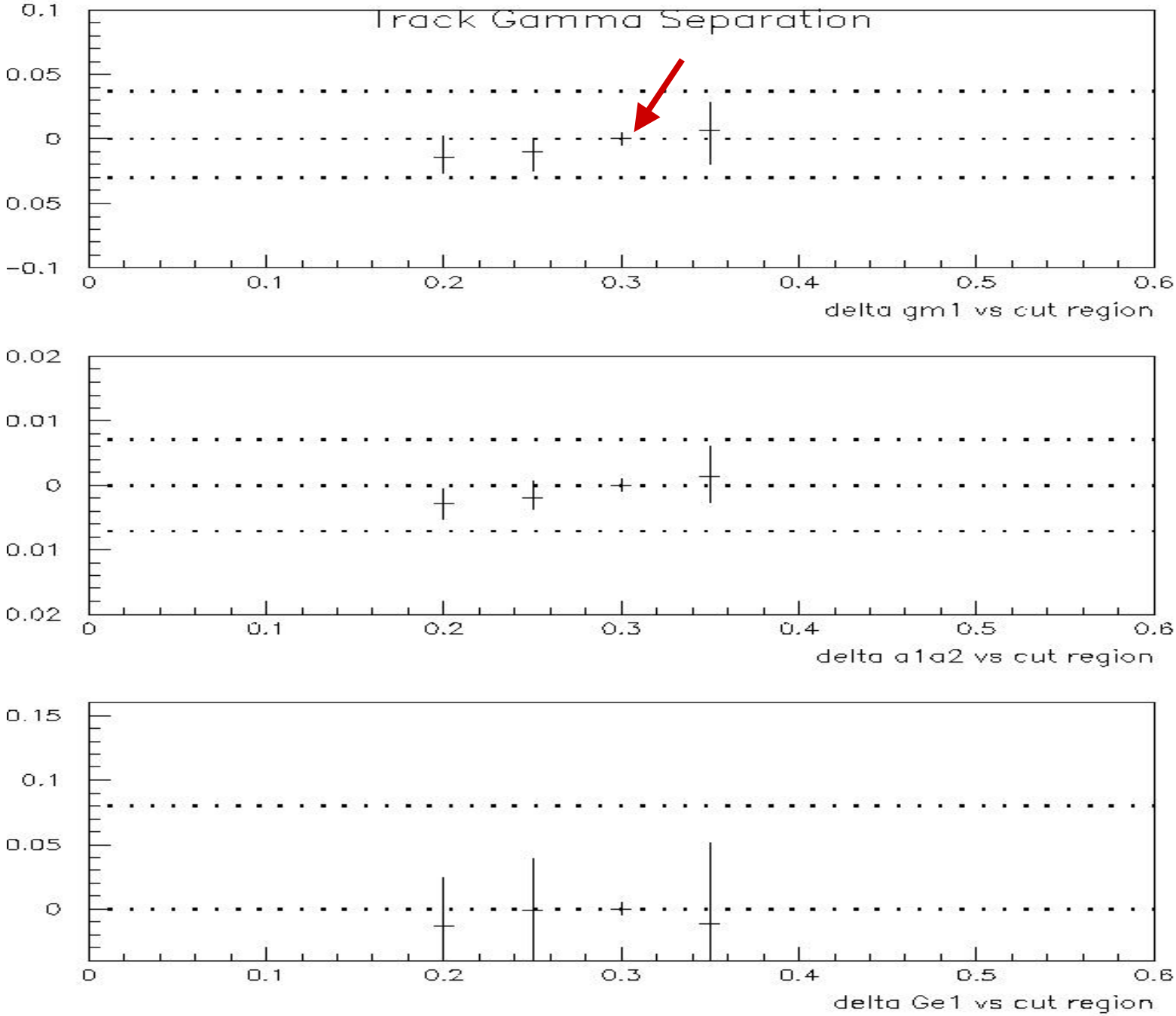
$$\left|\xi_E\right| = C_2\left|\frac{\eta_{+-}}{\omega^2\left(1-\beta^2\cos^2\theta\right)}\right| + g_{e1}$$

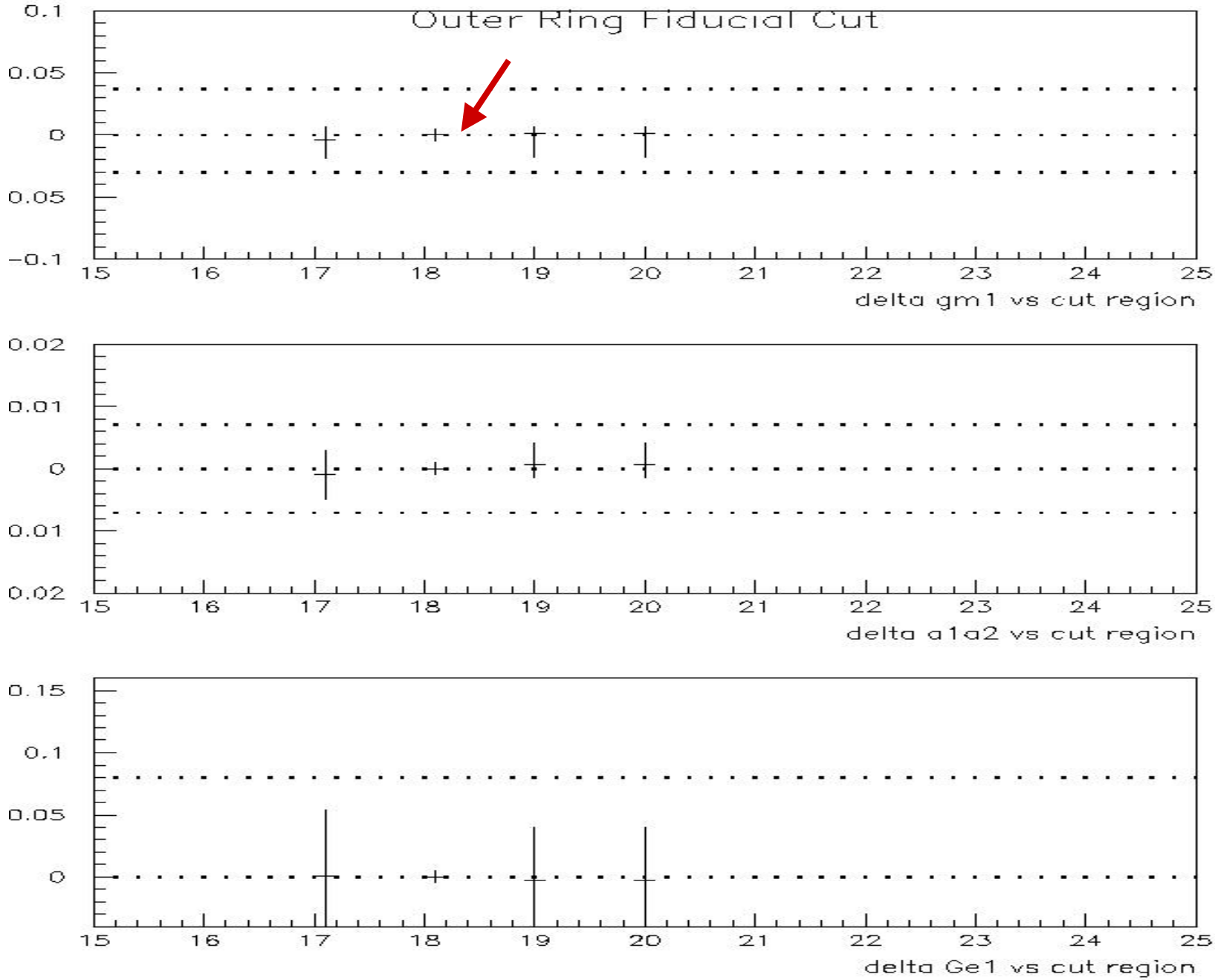
$$\left|\xi_M\right| = \tilde{g}_{m1}\left[1 + \frac{a_1/a_2}{\left(M^2_\rho - M^2_K\right) + 2M_K E^*_\gamma}\right]$$

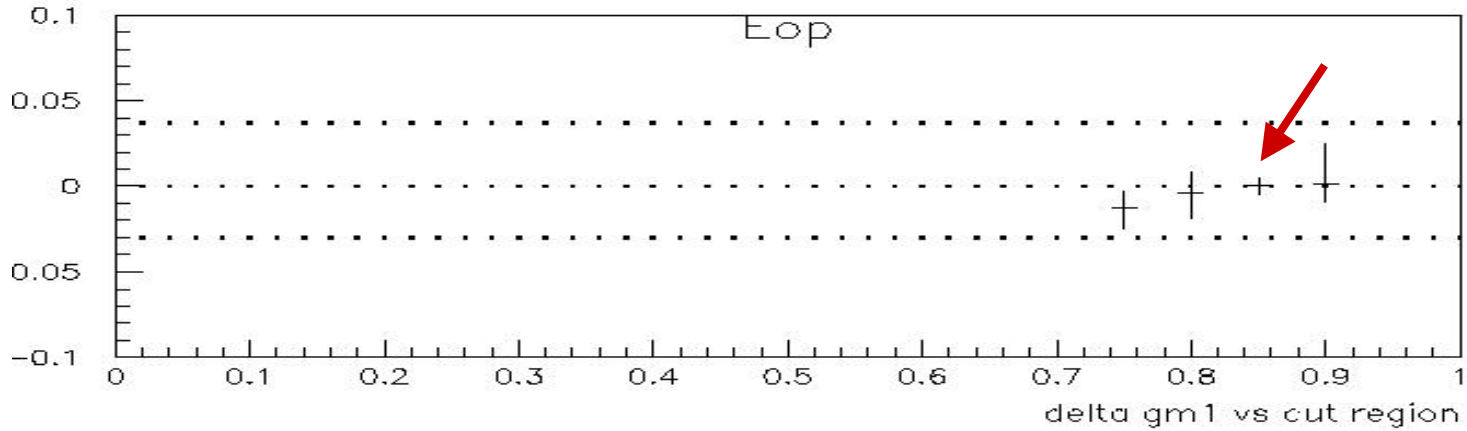


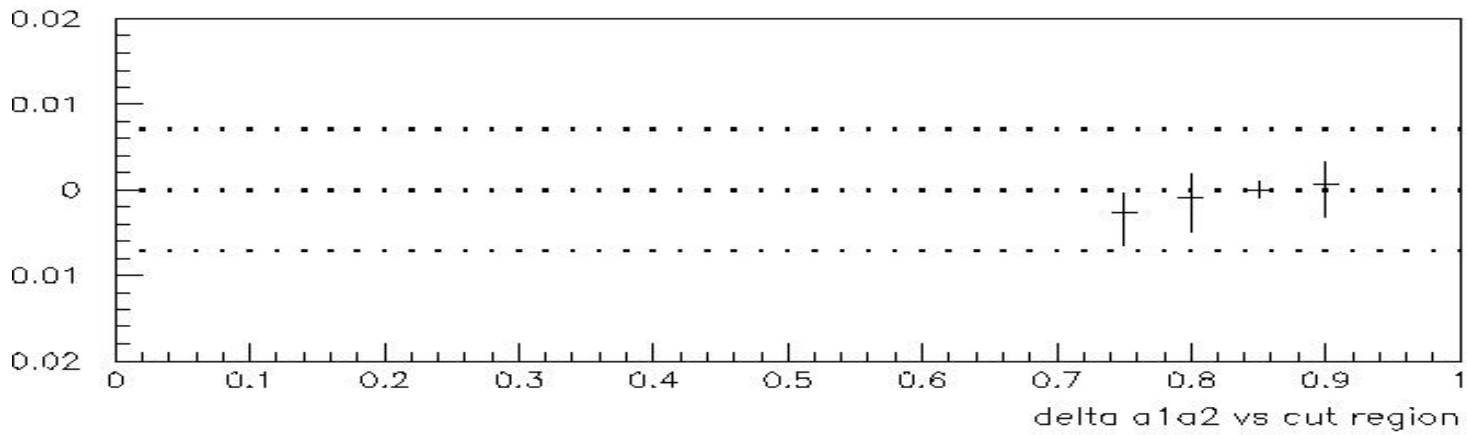
*Run divided into 3 time periods → data pts **<u>Stat Independent</u>**!

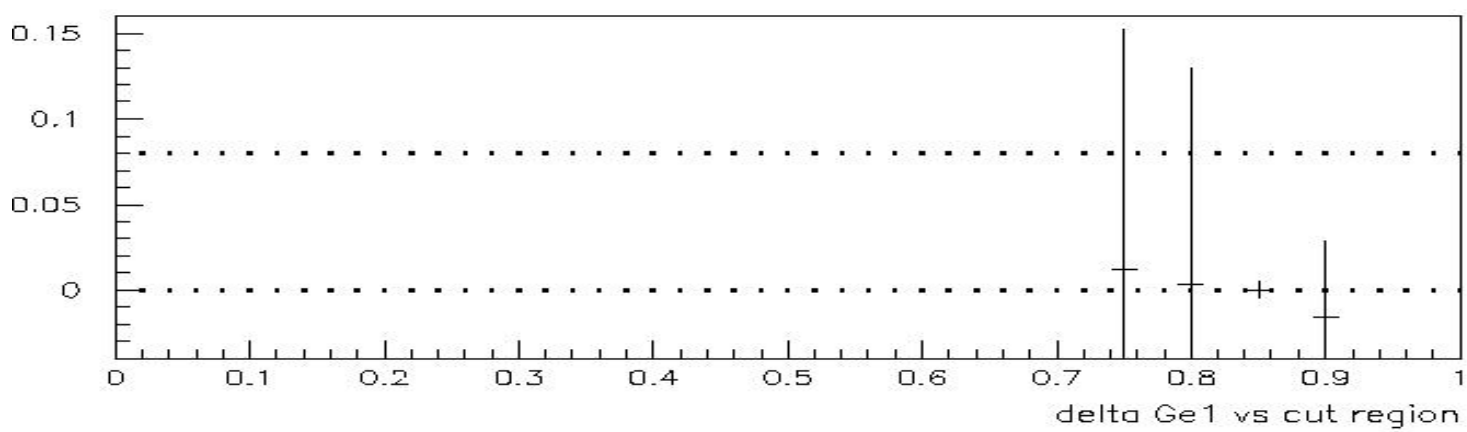

PpOkine
0.1
0.05
0
0.05
−0.1
−0.03
−0.025
−0.02
−0.015
−0.01
−0.005
0
delta gm1 vs cut region
0.02
0.01
0
0.01
0.02
−0.03
−0.025
−0.02
−0.015
−0.01
−0.005
0
delta a1a2 vs cut region
0.15
0.1
0.05
0
−0.03
−0.025
−0.02
−0.015
−0.01
−0.005
0
delta Ge1 vs cut region

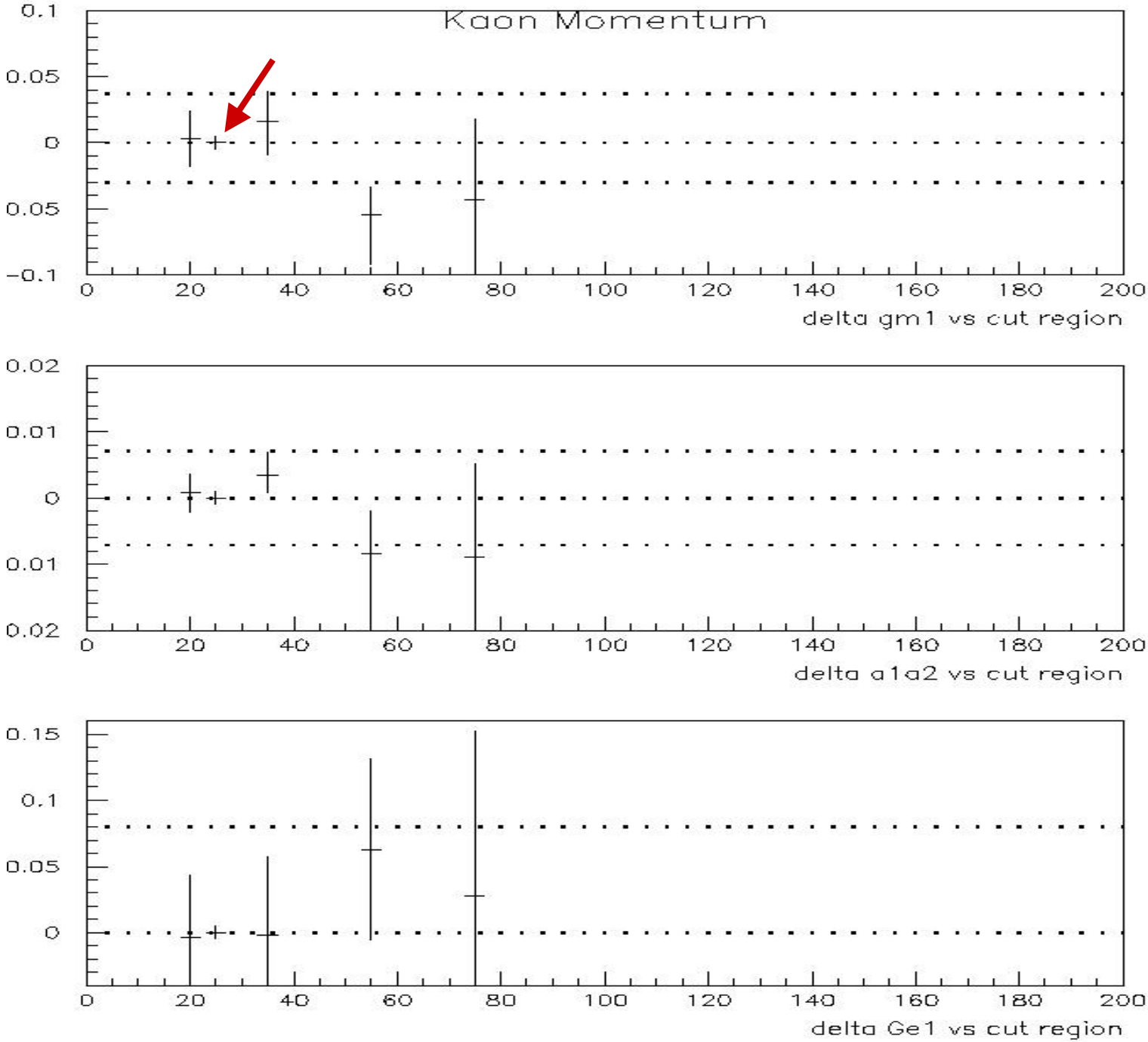

Track Gamma Separation
0.1
0.05
0
0.05
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
delta gm1 vs cut region
0.02
0.01
0
0.01
0.02
0
0.1
0.2
0.3
0.4
0.5
0.6
delta a1a2 vs cut region
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5
0.6
delta Ge1 vs cut region

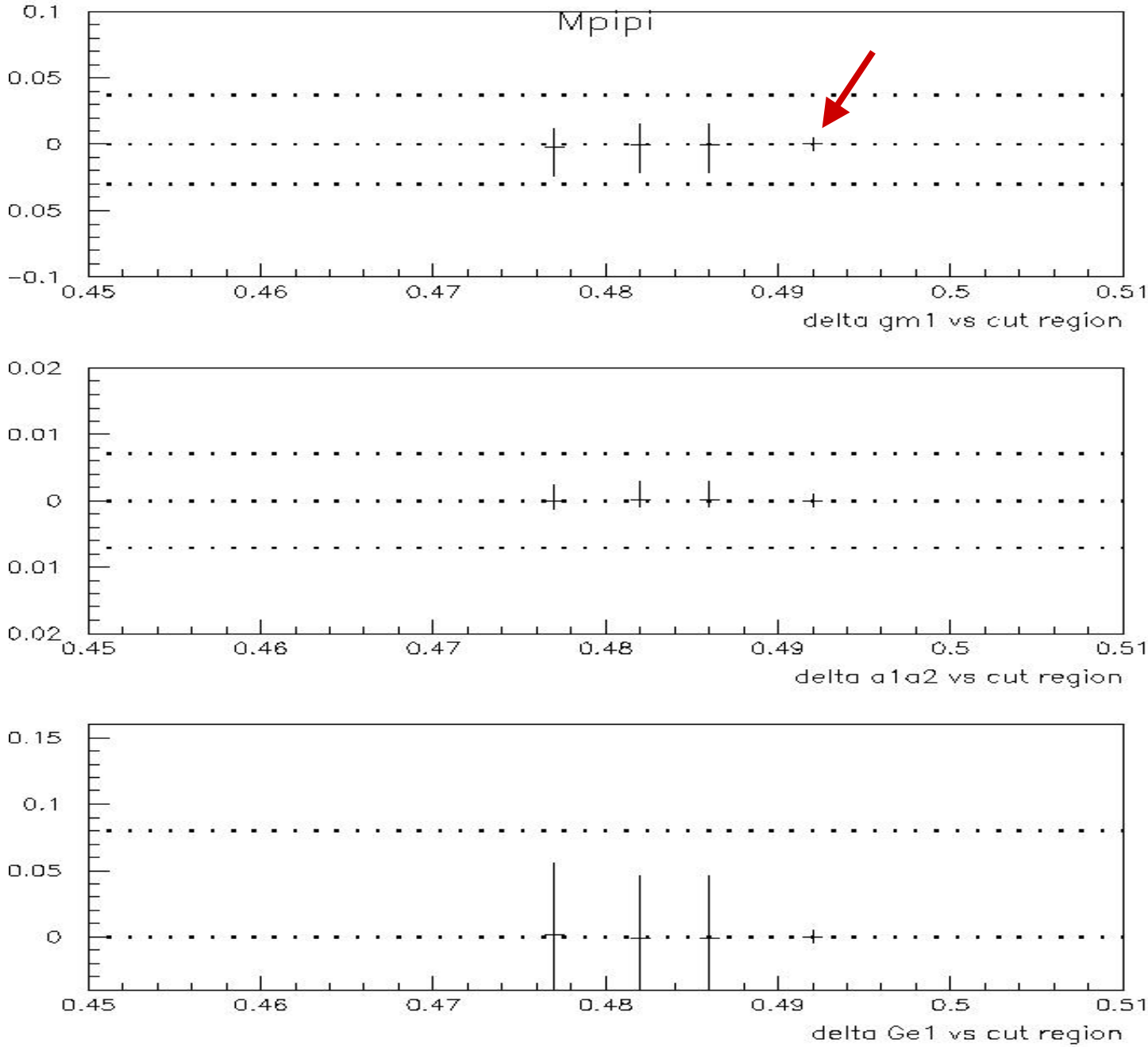

Outer Ring Fiducial Cut
0.1
0.05
0
0.05
-0.1
15
16
17
18
19
20
21
22
23
24
25
delta gm1 vs cut region
0.02
0.01
0
0.01
0.02
delta a1a2 vs cut region
0.15
0.1
0.05
0
delta Ge1 vs cut region

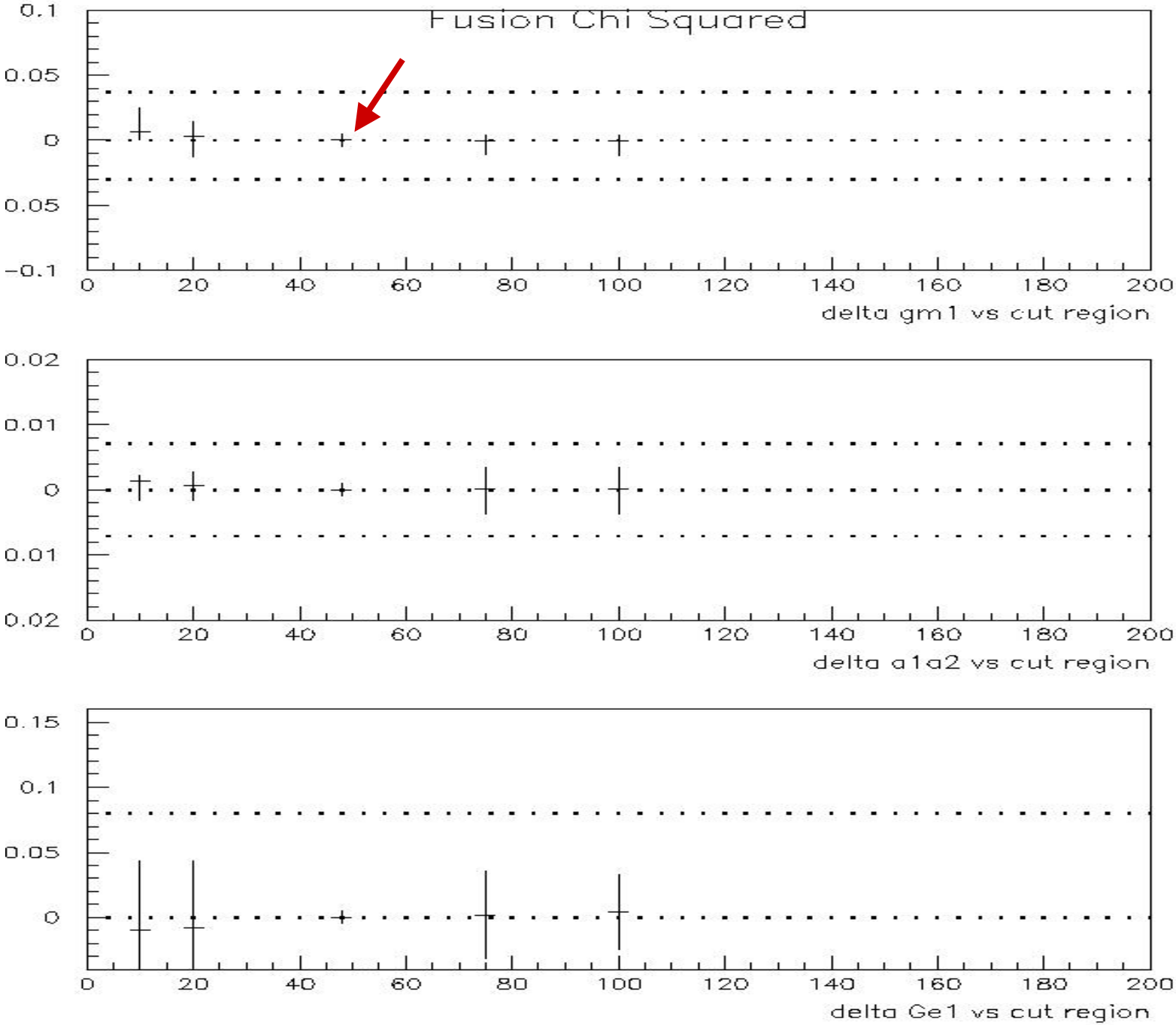

Eop
0.1
0.05
0
0.05
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
delta gm1 vs cut region

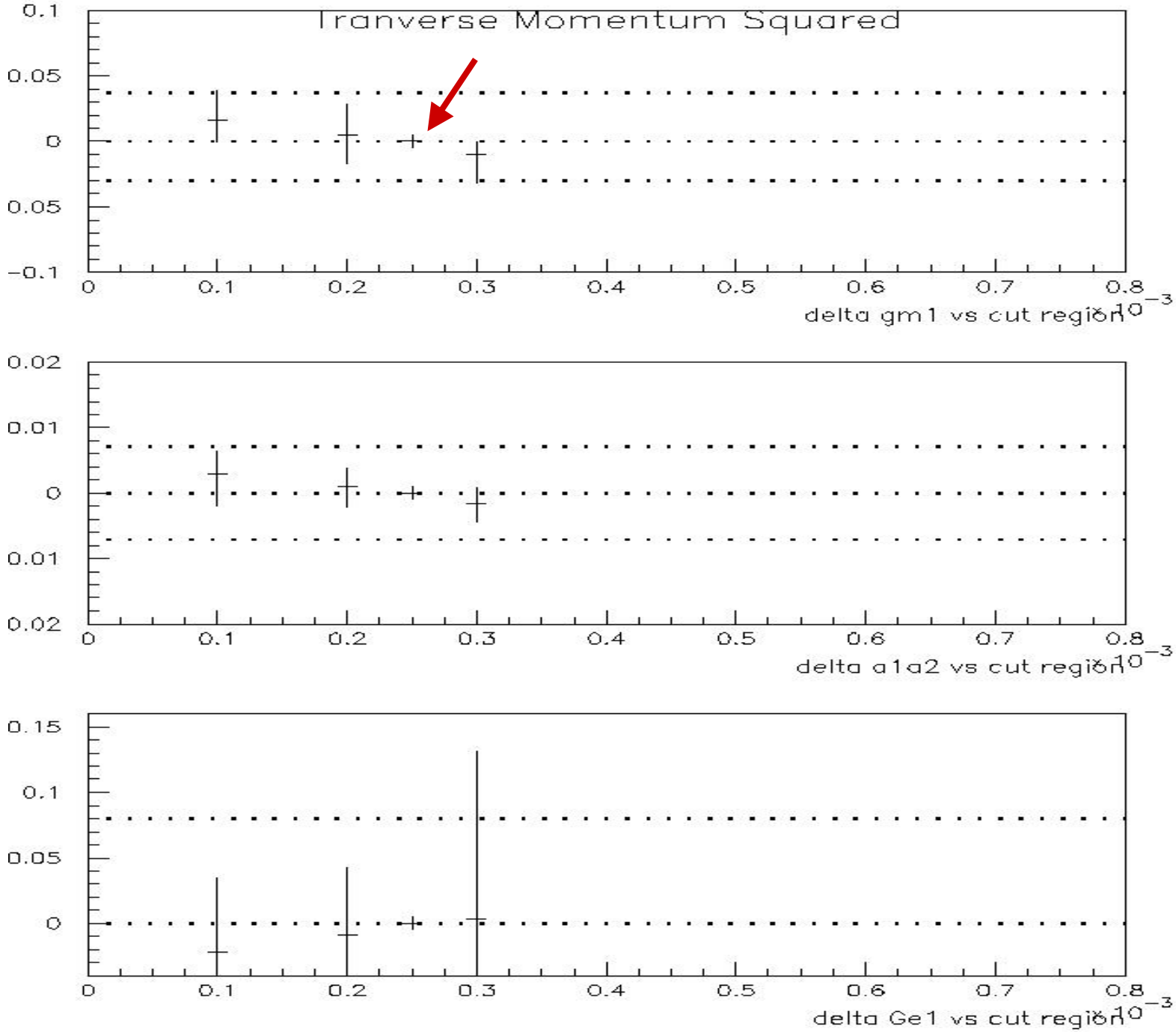

0.02
0.01
0
0.01
0.02
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
delta a1a2 vs cut region


0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
delta Ge1 vs cut region


Kaon Momentum
0.1
0.05
0
0.05
−0.1
0
20
40
60
80
100
120
140
160
180
200
delta gm1 vs cut region
0.02
0.01
0
0.01
0.02
delta a1a2 vs cut region
0.15
0.1
0.05
0
delta Ge1 vs cut region


Mpipi
0.1
0.05
0
0.05
−0.1
0.45
0.46
0.47
0.48
0.49
0.5
0.51
delta gm1 vs cut region
0.02
0.01
0
0.01
0.02
0.45
0.46
0.47
0.48
0.49
0.5
0.51
delta a1a2 vs cut region
0.15
0.1
0.05
0
0.45
0.46
0.47
0.48
0.49
0.5
0.51
delta Ge1 vs cut region


Fusion Chi Squared
0.1
0.05
0
0.05
-0.1
0
20
40
60
80
100
120
140
160
180
200
delta gm1 vs cut region
0.02
0.01
0
0.01
0.02
delta a1a2 vs cut region
0.15
0.1
0.05
0
delta Ge1 vs cut region


Tranverse Momentum Squared
0.1
0.05
0
0.05
−0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
10^{-3}
delta gm1 vs cut region
0.02
0.01
0
0.01
0.02
delta a1a2 vs cut region
0.15
0.1
0.05
0
delta Ge1 vs cut region