Radiative Corrections to K_{l3} Decays

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Overview of Presentation

- Radiative Corrections in K_{l3} Decays
- Outline of Previous Studies
- Overview of Calculation
- KL3_GENERATOR and other Event Generators (PHOTOS, KE3ALL).
- Remaining Work

How do Radiative Corrections Affect Kl3 Decays?

1. Correction to the total K_{l3} decay rate:

$$\Gamma_{Kl3} = (1+\delta_T)\,\Gamma_{Kl3}^{born}$$

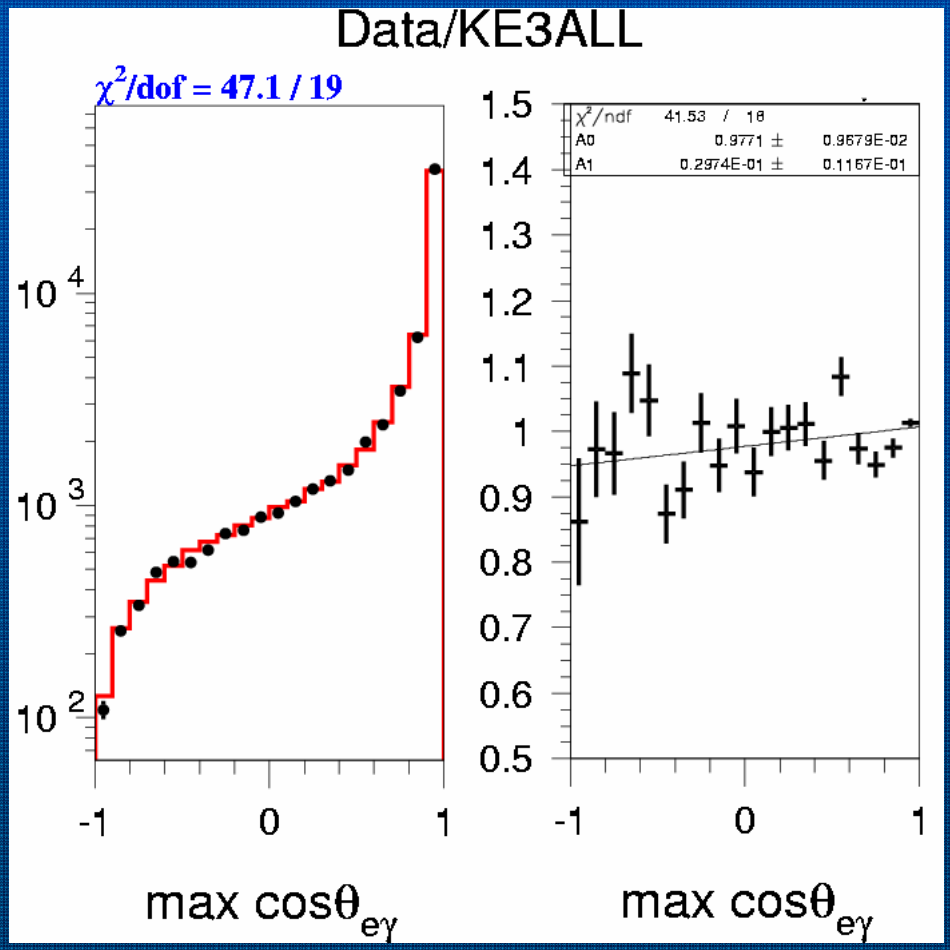
where, $\delta_T = \delta_{SD} + \delta_{LD}$

(LD Correction, δ_{LD}, is ~ 1.5% from previous study)

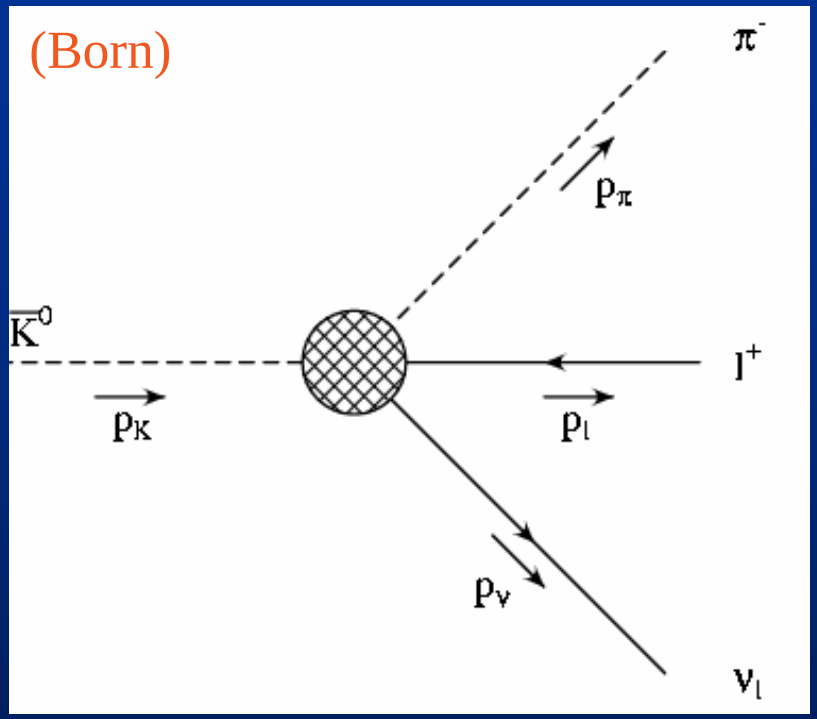
$$\Gamma_{Kl3} = \frac{1}{2M_K}\int\prod_f\left(\frac{d^3p_f}{(2\pi)^3}\frac{1}{\sqrt{2E_f}}\right)\left\langle\left|M_{kl3}\right|^2\right\rangle$$

How do Radiative Corrections Affect Kl3 Decays?

2. Radiative effects (*bremsstrahlung*) necessary to study Monte Carlo acceptance.



Basic *Lowest Order* K_{l3} Decay (Born):



Form Factor Vocabulary

$$f_2(t) = f_+(t) - f_-(t)$$

$$f_0(t) = f_+(t) + \frac{t}{M_K^2 - M_\pi^2} f_-(t)$$

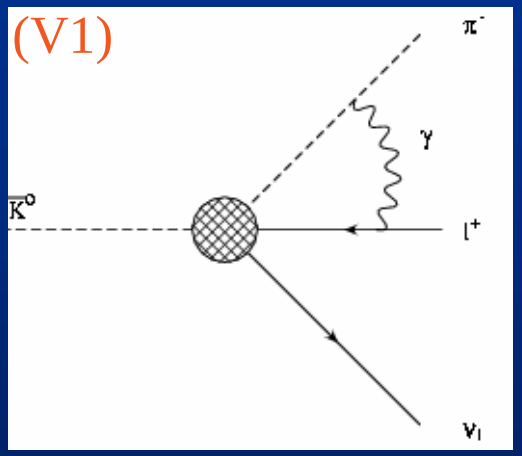
where, $t = (p_K - p_\pi)^2$

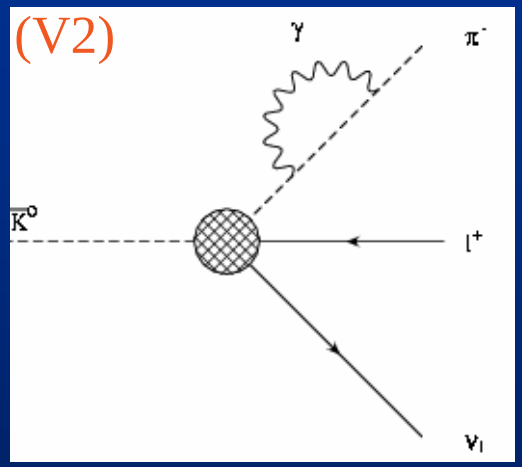
Matrix Element:

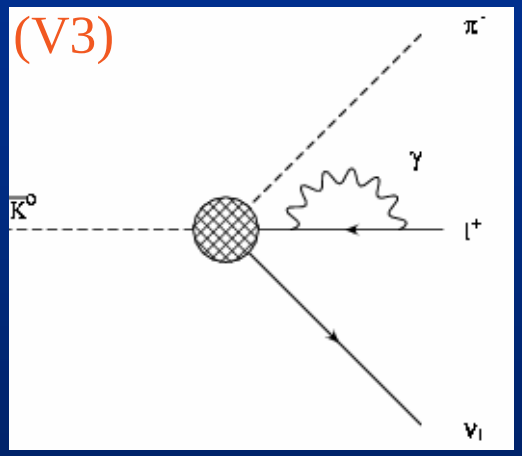
$$iM^{Born} = i\sqrt{2}G_F V_{us}^* \times \bar{u}(p_\nu)\mathrm{P_R}[2p_{K,\mu}\gamma^\mu f_+(t) + m_l f_2(t)]v(p_l)$$

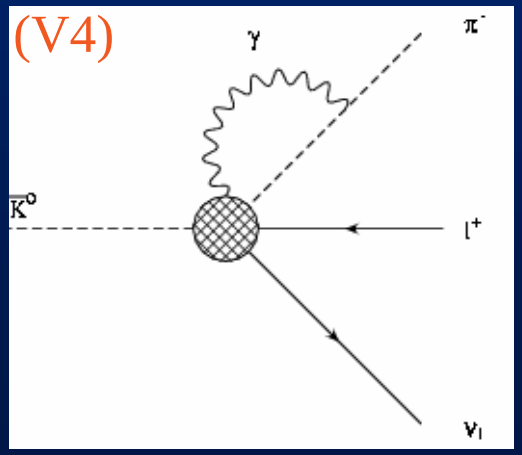
Feynman Diagrams for Radiative Corrections

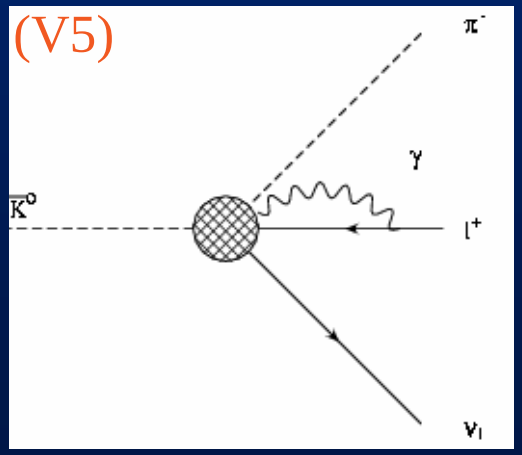
Virtual Corrections:



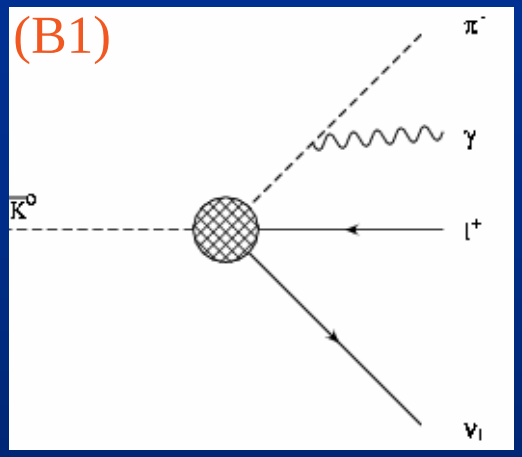




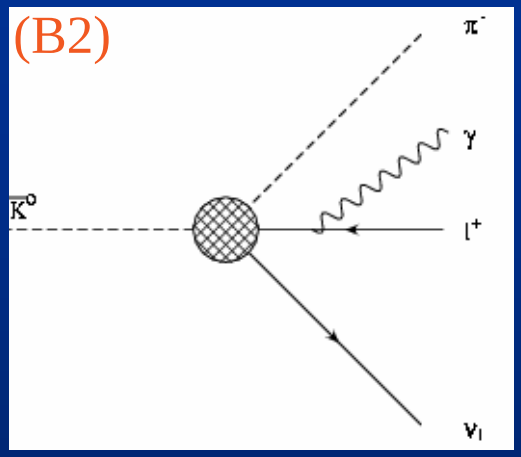




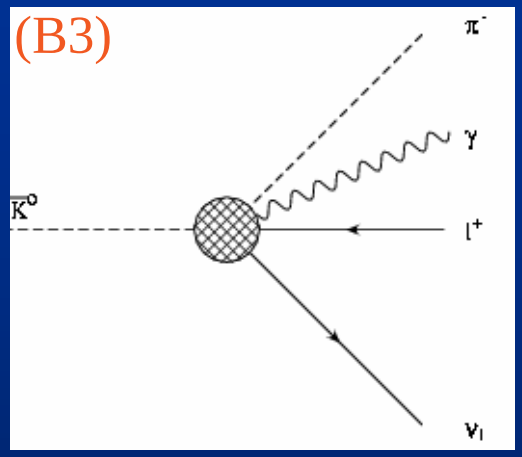
Bremsstrahlung Correction:



Radiation from Pion (π)



Radiation from Lepton (l)



Radiation from Eff. Vertex

**All 8 diagrams plus Born Diagram will contribute to the Decay Rate, $\Gamma(K_{l3})$.

Previous Studies:

E.Ginsberg. Physical Review 171, (1968).
(+ Corrections in Additional Errata).

H.Fearing, E.Fischbach, and J.Smith. PR D2, (1970).

V.Cirigliano, et. al. (hep-ph/0110153).
V.Cirigliano, et. al. (hep-ph/0401173).

V.Bytev, et. al. (hep-ph/0210049).

Ginsberg Study:

- This study is the standard resource for the LD correction (δ_{LD}) to the Born Decay Rate.
- Paper considers both *Virtual* and *Bremsstrahlung* corrections to neutral K_{e3} decays (later paper looks at neutral $K_{\mu 3}$).
- Final result in second errata is that $\delta_{LD} \sim 1.5\%$ for K_{e3}.

- Numerous errors which are corrected in later *errata* and papers (additional errors found in Cirigliano, et. al.).
- Pre- Standard Model Calculation (*check assumptions*).
- For Bremsstrahlung diagrams, Ginsberg integrates over the final-state photon; therefore, not readily applicable to MC acceptance studies.
- Neglects effect of t-dependence of the K-π hadronic form factors ($f_+(t)$ = constant & $f_-(t)$ = constant) – This can change results for both Virtual and Bremsstrahlung results.

Fearing, Fischbach, Smith (FFS) Study:

- A very nice resource for *bremsstrahlung* contribution to radiative decays.
- Allows for *t*-dependence in the K-π hadronic form factors.
- Good discussion of "Structure Dependent Terms" for the effective 'K^0-π-l-ν' vertex *(will discuss later).*
- Parentage of the KTeV in-house generator KE3ALL.

- Does not consider Virtual Corrections to K_{l3} decays (no δ calculation).
- Though it allows for *t*-dependence is form factors, FFS do not consider effects beyond a linear model.

$$f_X(t) = f_X(0)\left[1 + \frac{\lambda_X}{m_\pi^2} t\right]$$

where, X= +, 0

- Does not reproduce some distributions (e.g. $\cos(\theta^*_{l\gamma})$) observed in data (results are reasonable though).

Cirigliano et. al. Study:

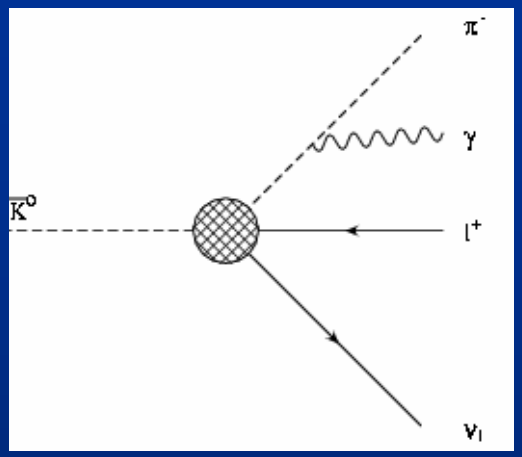
- Calculates radiative corrections to the decay width of the K_{e3} decay (short and long distance contributions).
- Finds and corrects additional errors in the Ginsberg paper.
- *Derives* the hadronic form factors from Chiral Perturbation Theory (χPT).

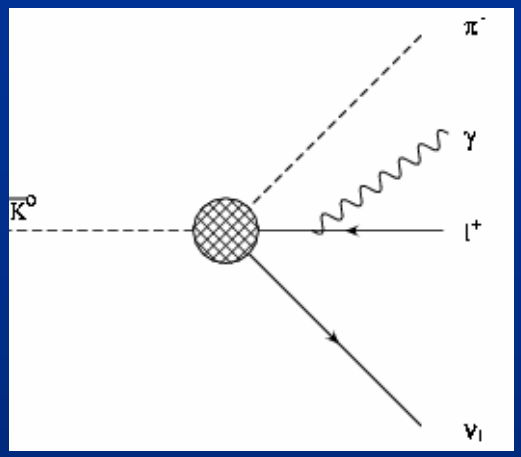
- Neglects mass of lepton; therefore, only applicable to K_{e3} decays.
- Takes the contribution of the other form factor, $f_0(t)$, to be zero.
- Long distance contribution closely follows the work and approach of Ginsberg.
- Result is integrated over final state photon 4-momenta – not readily applicable to MC acceptance studies.
- χPT calculation of form factors does not match data – we need a consistent, phenomenological model of the form factors.

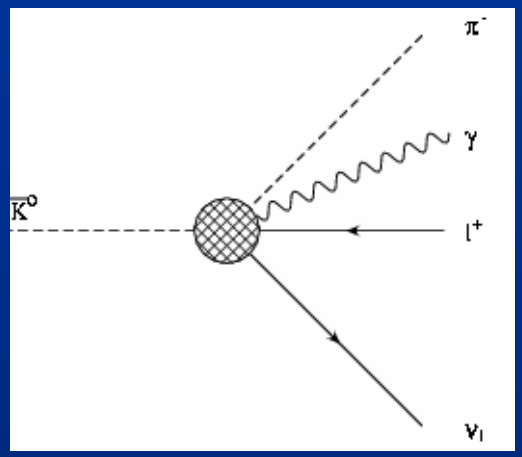
What we need:

- ü Calculation of radiative corrections to K_{l3} decays that addresses the needs of experiment.
- ü Calculation that takes into consideration that photons are in the final state.
- ü Calculation the analyzes the effects of the form factors and radiative corrections to them.
- ü From a practical standpoint – an event generation tool.

Overview of bremsstrahlung calculation:







<u>Important Issues to Consider:</u>

- How do we treat the pion? (*point particle*)
- What is "t" for radiative events?
- How do we treat the K-π-l-ν Vertex? Some of the external particles are no longer on shell!
- What is the vertex for when the photon radiates off the vertex?

What is “t” for radiative Kl3 Decays?

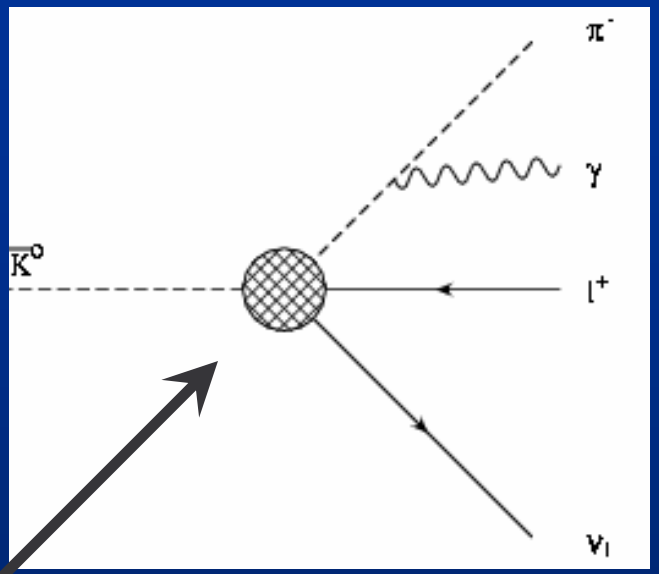
- For the Born Matrix element, we have:

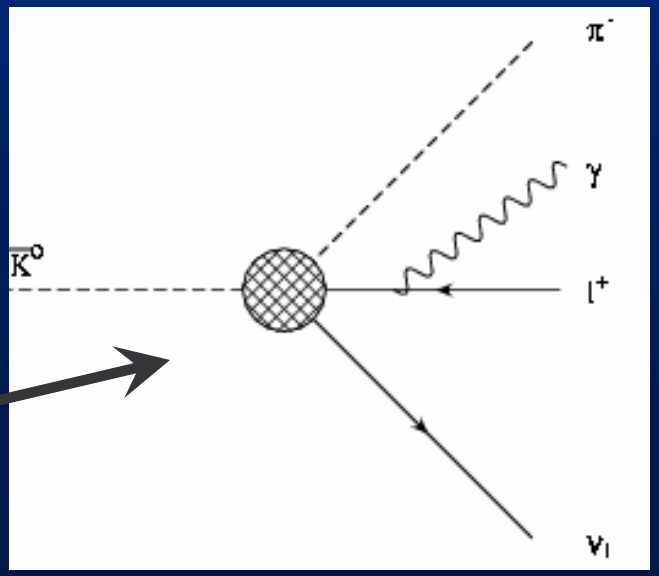
$$t = (p_K - p_\pi)^2 = (p_l + p_\nu)^2$$

- For radiative events there is an ambiguity! The form of “t” depends on where the photon is radiated:

$$t_1 = (p_K - (p_\pi + k))^2 = (p_l + p_\nu)^2$$

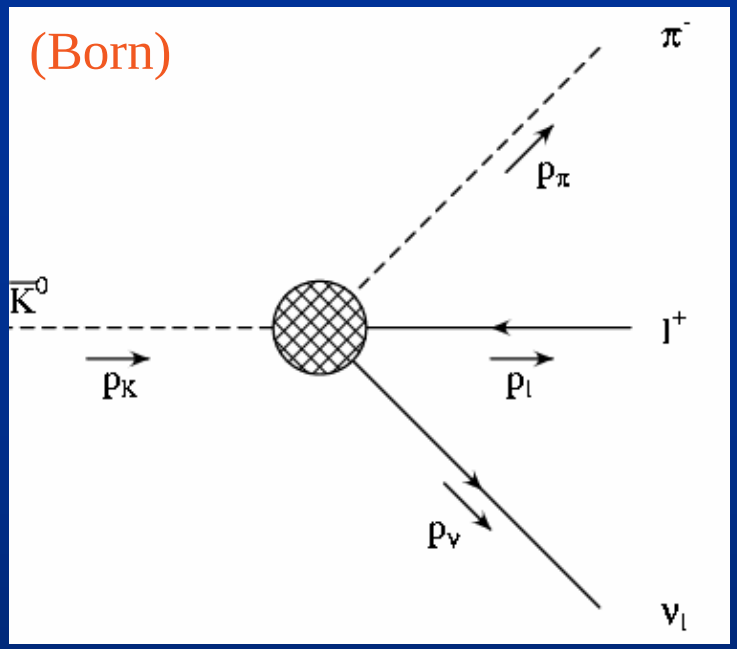
$$t_2 = (p_K - p_\pi)^2 = (p_l + p_\nu + k)^2$$





The K^0-π^--l^+-ν Vertex:

- Use born-level diagram to obtain a Feynman Rule.
- Assume this holds for when the pion or lepton are off-shell.
- From earlier slide, effective vertex is then,



$$\mathrm{EV}_1 = i\sqrt{2}G_F V_{us}^* \mathrm{P_R}[2p_{K,\mu}\gamma^\mu f_+(t) + m_l f_2(t)]$$

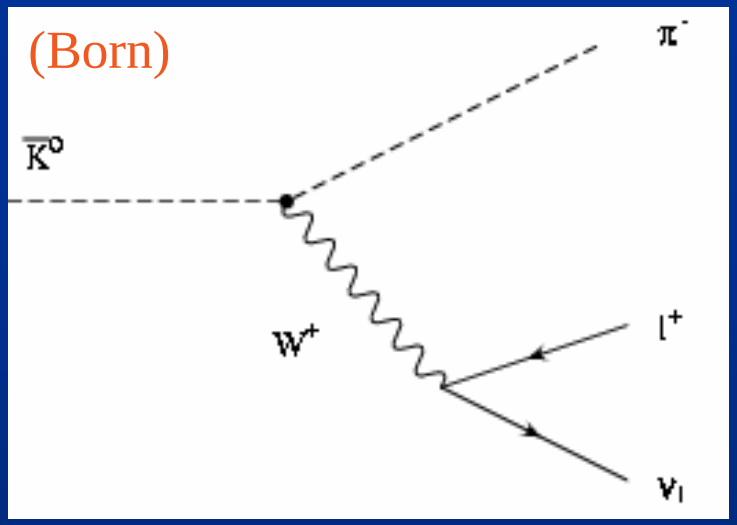
This is what all the previous studies use!

Is this unique? Does this make sense?

From a “Modern” perspective (Standard Model) Energy/Momentum Conservation and the Dirac Equation have been used.

The K^0-π^--l^+-ν Vertex (v2):

- There are many equivalent versions of this "Feynman Rule" that give same "Born-level" result.
- The difference between versions are "Structure Dependent" Terms.



$$\mathrm{EV}_2 = i\sqrt{2}G_F V_{us}^* \mathrm{P_R}[(p_K + p_\pi)_\mu \gamma^\mu f_+(t) + (p_K - p_\pi)_\mu \gamma^\mu f_-(t)]$$

where the form factors come from the hadronic matrix element:

$$\langle \pi^-(p_\pi) | \bar{s}\gamma^\mu \mathrm{P_L} d | K^0(p_K) \rangle = (p_K + p_\pi)^\mu f_+(t) + (p_K - p_\pi)^\mu f_-(t)$$

Illustrate that EV1 and EV2 are Equivalent for Born Diagram

$$\bar{u}(p_\nu)[\mathrm{EV}_2]v(p_l) =$$

$$i\sqrt{2}G_F V_{us}^* \bar{u}(p_\nu)\mathrm{P_R}[(\not{p}_K + \not{p}_\pi) f_+(t) + (\not{p}_K - \not{p}_\pi) f_-(t)]v(p_l)$$

** use Energy Momentum Conservation **

$$= i\sqrt{2}G_F V_{us}^* \bar{u}(p_\nu)\mathrm{P_R}[(2\not{p}_K - \not{p}_\nu - \not{p}_l) f_+(t) + (\not{p}_\nu + \not{p}_l) f_-(t)]v(p_l)$$

** use Dirac Equations for ν and l **

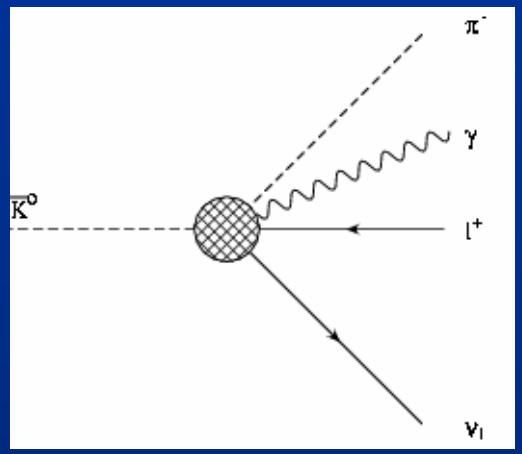
$$= i\sqrt{2}G_F V_{us}^* \bar{u}(p_\nu)\mathrm{P_R}[(2\not{p}_K + m_l) f_+(t) - m_l f_-(t)]v(p_l)$$

$$= i\sqrt{2}G_F V_{us}^* \bar{u}(p_\nu)\mathrm{P_R}[2\not{p}_K f_+(t) + m_l f_2(t)]v(p_l)$$

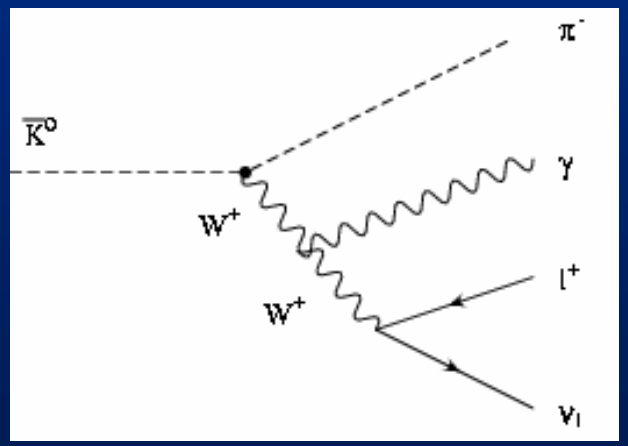
$$= \bar{u}(p_\nu)[\mathrm{EV}_1]v(p_l)$$

Photon Radiation from the Effective Vertex:

- In the literature (Ginsberg and FFS), the form of this interaction is ambiguous.
- Both authors claim this vertex is a consequence of the "Ward Identity", however, this statement appears false.



- A consistent use of the Ward Identity does not fully constrain the form of the vertex (as claimed above).
- Treating this diagram as a radiation from the $W^{+/-}$ boson produces an interaction consistent with the Ward Identity and reveals that this graph is suppressed by a factor of (M_K^2/M_W^2) over the pion and lepton radiation graphs.



Bremsstrahlung Matrix Element:

$$iM^{Brem} = ie\sqrt{2}G_F V_{us}^{*}\overline{u}(p_\nu)P_R\{$$

$$[f_+(t_1)+m_l f_2(t_1)]\times\left[\frac{\varepsilon^*(k)\cdot p_\pi}{k\cdot p_\pi}\right]$$

$$+[f_+(t_2)+m_l f_2(t_2)]\times\left[-\frac{\varepsilon^*(k)\cdot p_l}{k\cdot p_l}+\frac{\not{\varepsilon}^*(k)\not{p}_l}{2k\cdot p_l}\right]$$

$$+ f_2(t_2)\not{\varepsilon}^* \quad \}v(p_l)$$

Brem. From Pion

Brem. From Lepton

New Brem. From Lepton Term

$$t_1 = (p_K - p_\pi - k)^2$$

$$t_2 = (p_K - p_\pi)^2$$

K_{l3} Event Generators

- PHOTOS – *General* algorithm for photon radiative corrections in decays (Barberio & Was).
- KE3ALL – KTeV in-house event generator implementing FFS bremsstrahlung calculation.
- KL3_GENERATOR – My event generator implementing can use EV_1 and EV_2 based bremsstrahlung calculation and KE3ALL matrix element.

K_{l3} Event Generator Functionality:

	Photos	KE3ALL	KL3_GENERATOR
Does $K_{\mu 3}$	Y	N	Y
Form Factor Model	None	Linear	Const., Linear, Quadratic, Pole
Includes $f_0(t)$	N	N	Y
Virtual Corrections	Y	Y	Y
Calculates δ_{LD}	N	N	Coming Soon

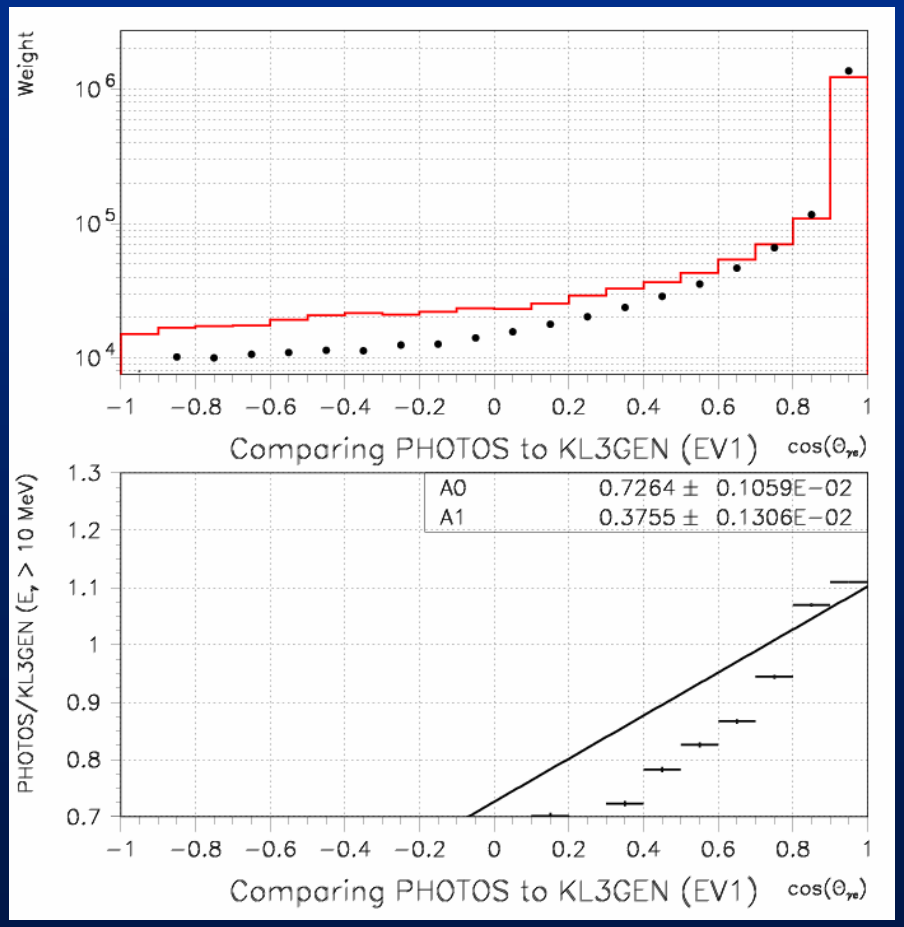
KL3_GENERATOR Features:

- Standalone Event Generator for Neutral K_{l3} Events (writes unweighted events to file).
- Program written in FORTRAN and utilizes the following packages:
  - § cernlib (plotting, etc.)
  - § lapack (linear algebra)
  - § looptools (scalar and tensor loop integrals)
  - § vegas (MC phase space integration).
- Tested on both *Linux* and *Unix/Alpha* architectures.
- Implement cross checks:
  - § Matrix Elements Numerically Computed by different methods: analytic expression for squared/summed matrix element (checked with *FORM*) **and** numerical matrix element calculation (spinors, Dirac matrices, etc.) **and** compare with KE3ALL ME routine.
  - § Numerically check cancellation of infrared divergences.

$\cos(\theta^*_{l\gamma})$ Comparisons (v1)

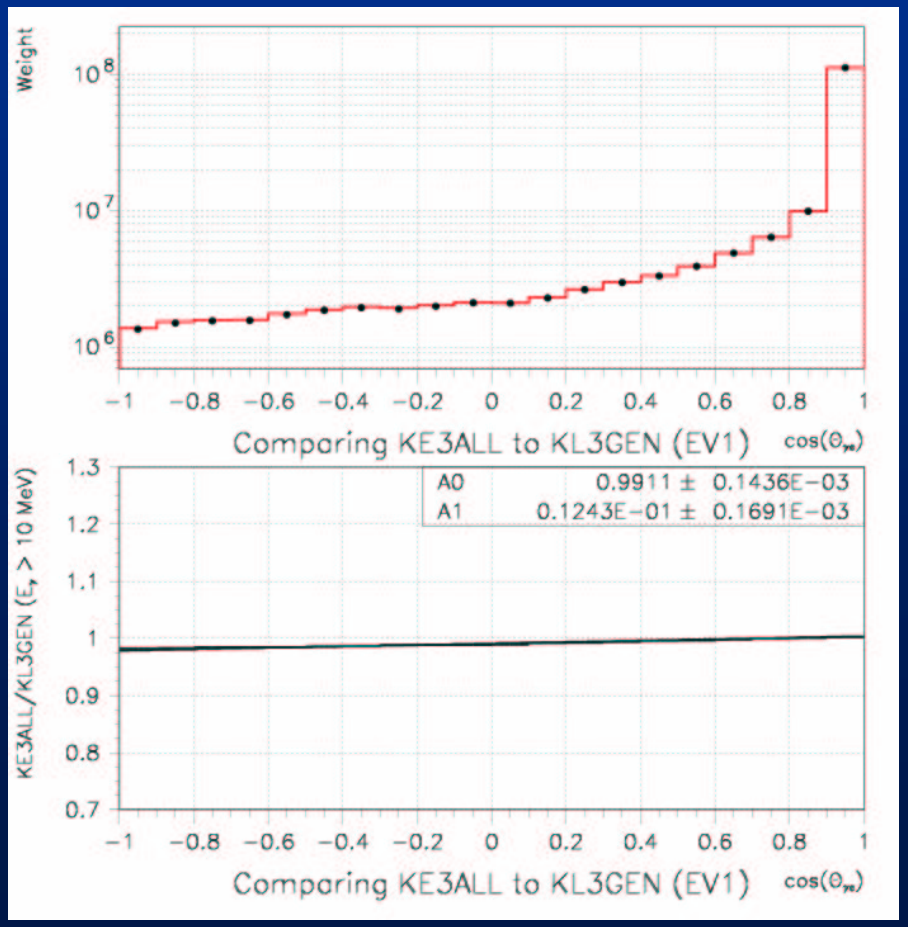
Photos versus KL3_GENERATOR

(dots) (red line)



KE3ALL versus KL3_GENERATOR

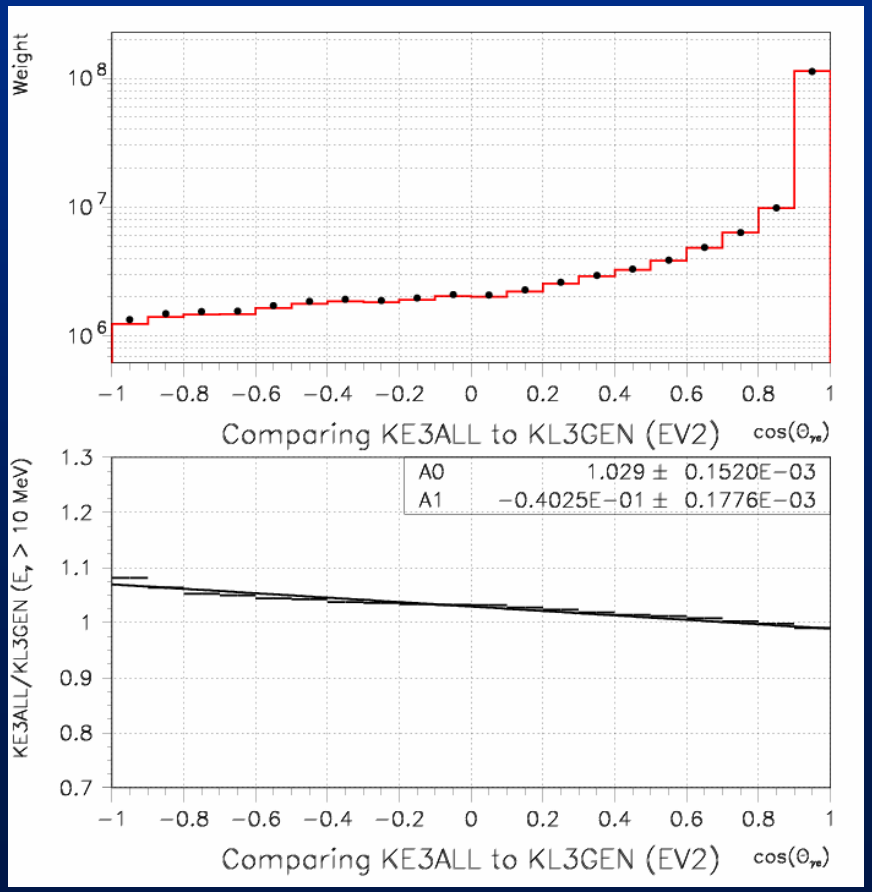
(dots) (red line)

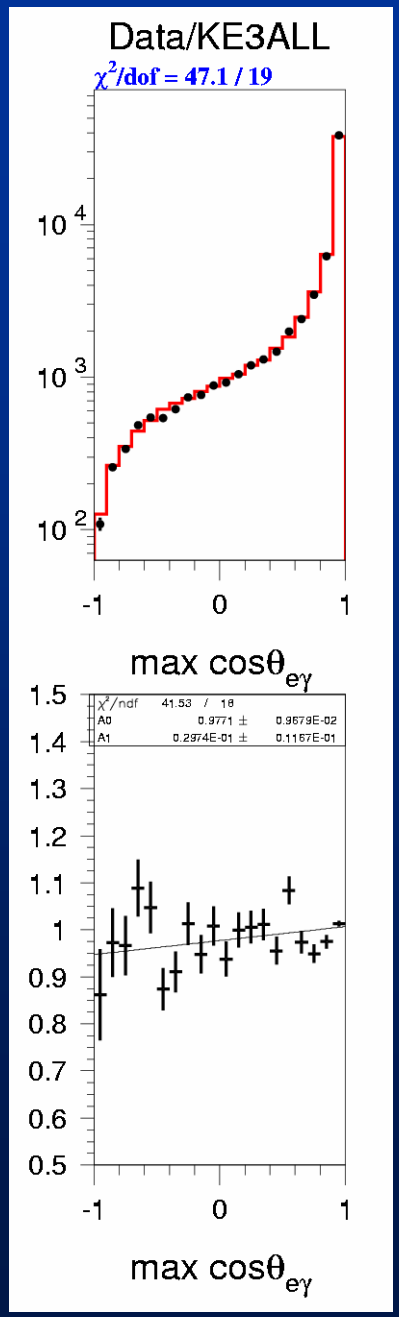


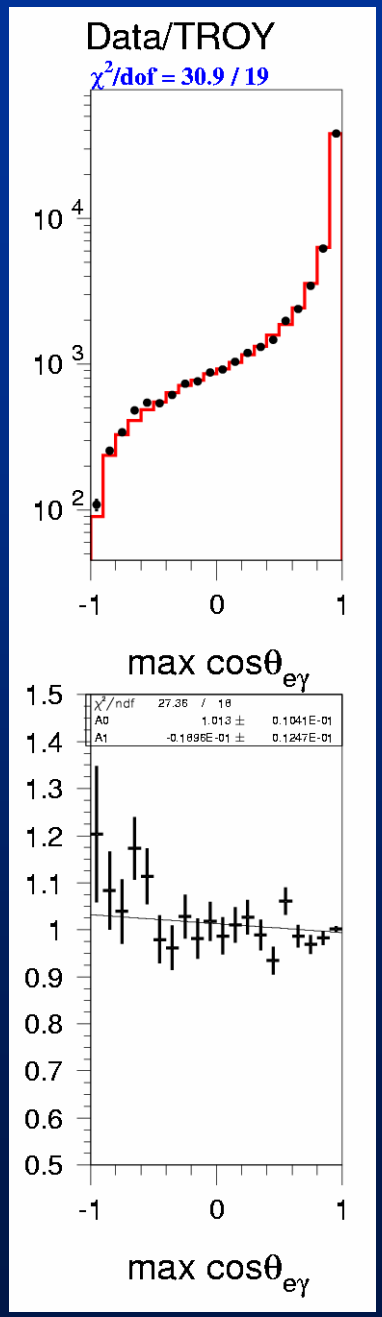
$\cos(\theta^*_{l\gamma})$ Comparisons (v2)

KE3ALL versus KL3_GENERATOR

(dots) (red line)







Remaining Work

- Complete Cross-Checks on the Matrix Elements.
- Complete implementation of Virtual Corrections in KL3_GENERATOR. This is used for event generation and δ_{LD} calculation.
- Analyze effect of different form factor models on radiative correction parameter δ_{LD}.