Study of Systematic Effects in the Maximum Likelihood Fit in $K_L \to \pi^+\pi^-e^+e^-$ Analysis

Alexander Golossanov
University of Virginia

August 02, 2003*

KTeV Collaboration Meeting
Fermilab†

*Last compilation August 1, 2003

†In this document (if you have access to the internet) you can click on any text in *purple color* for additional information

1. What Am I Going to Talk About?

- ✓ Study of systematic effects in the re-weighting due to the choice of initial parameters for MC sample.
- ✓ Study of the background systematics.
- ✓ Study of other systematics with cut variation. Cut on the kaon energy shows large walk in the estimated values.
- ✓ Results and estimation of correlations between parameters.

2. Remind the Notation

- Matrix element $\mu = \mu(x; \alpha, \beta)$ represents **our model** for $K_L \to \pi^+\pi^-e^+e^-$ decay, where

$$x = (\phi, cos\theta_{e^+}, cos\theta_{\pi^-}, M_{\pi\pi}, M_{ee}), \qquad \alpha = \left(\frac{a_1}{a_2}; g_{M_1}; g_{CR}; g_{E1}\right)$$

  and β defines all other parameters.

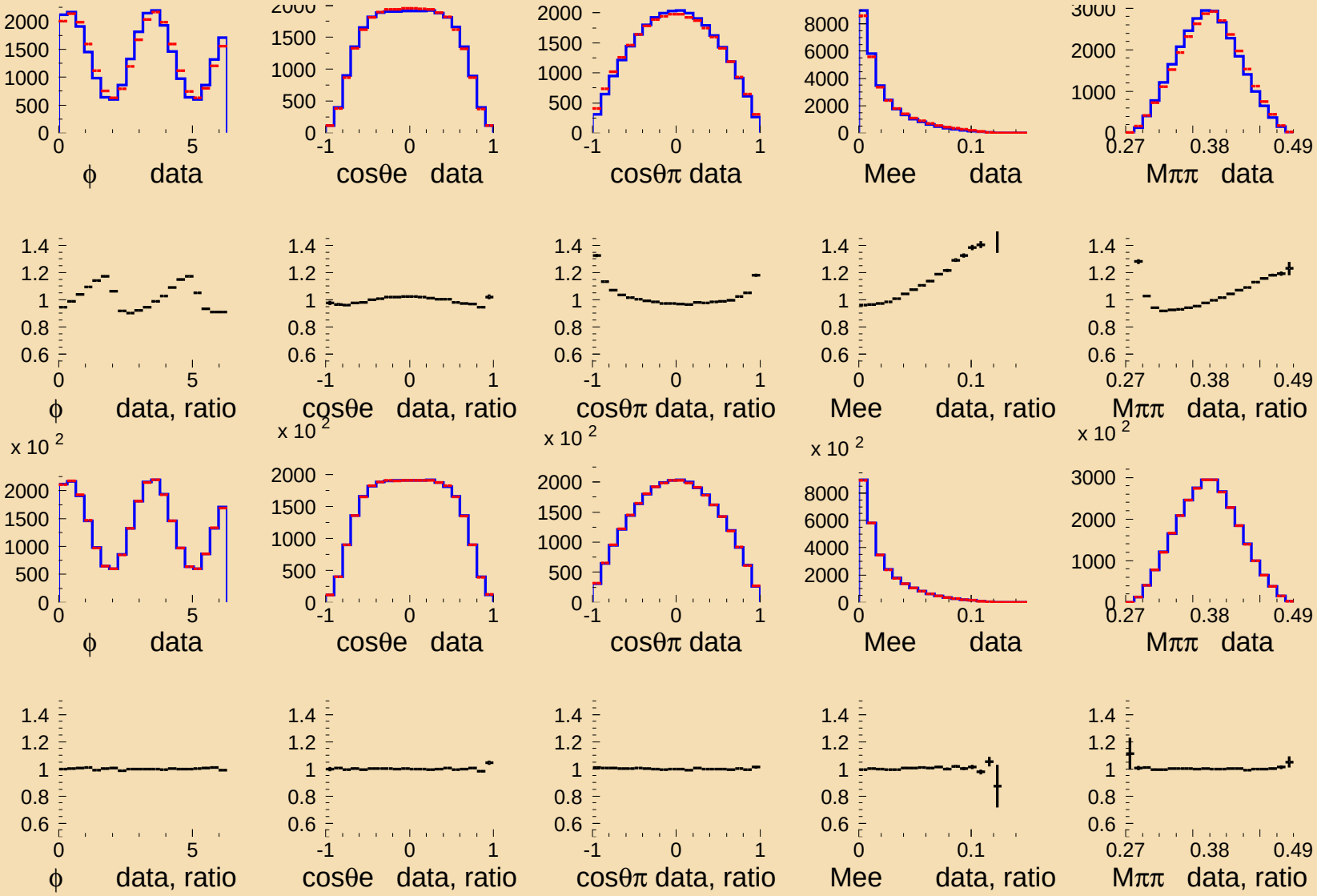
- We use the method of **maximum likelihood** with re-weighting to extract the parameters α. Logarithm of the likelihood function $\mathcal{L}$ can be written as follows:

$$log\mathcal{L}(\alpha) = \sum_{i=1}^{N_d} log\mu(x_i; \alpha) - N_d \cdot log \sum_{j=1}^{N_{mc}} \frac{\mu(x_j; \alpha)}{\mu(x_j; \alpha_0)} + f(x, \alpha_0)$$

  where f does not depend on α.

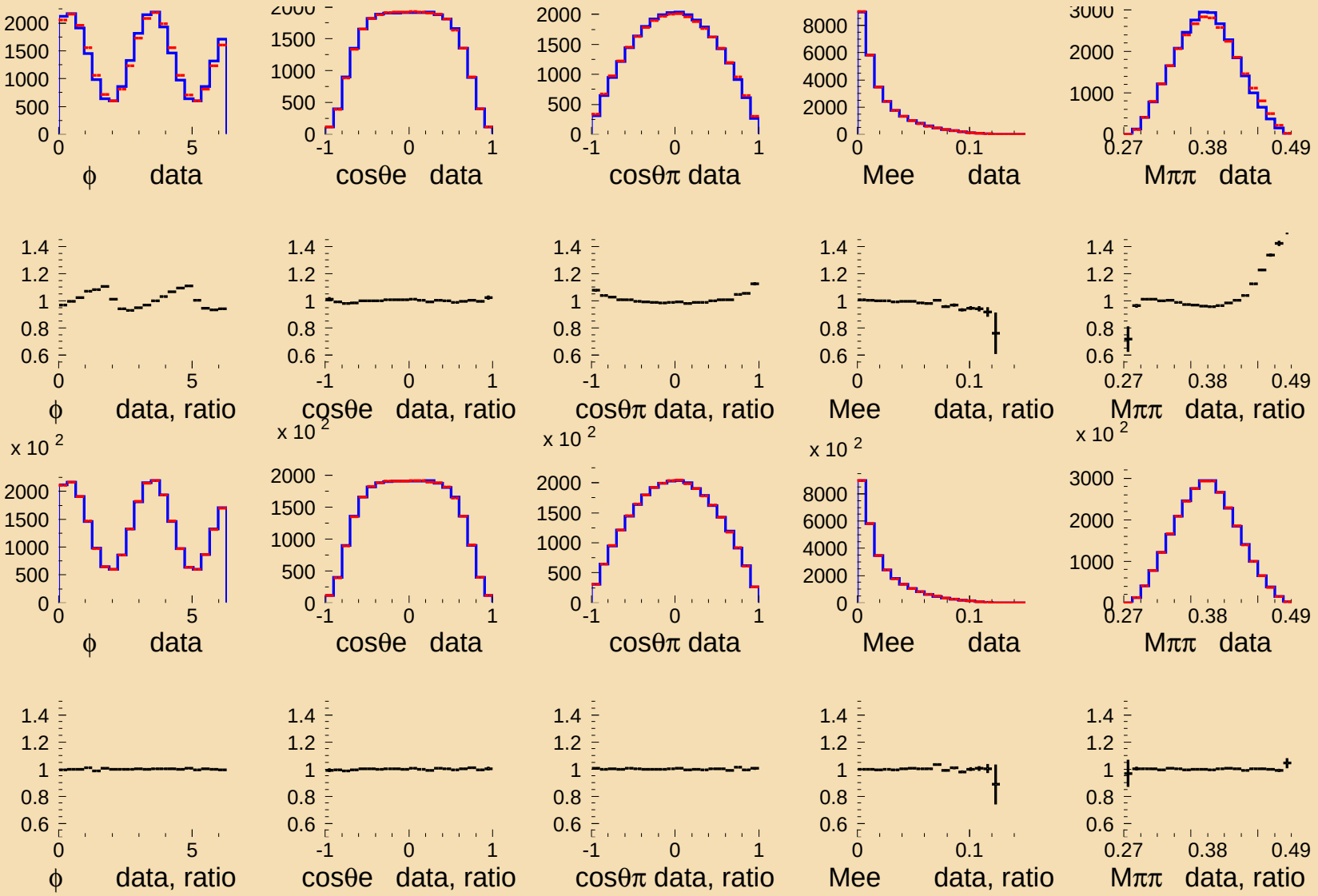
- We generate **one large MC sample** for a certain initial choice of $\alpha = \alpha_0$ and then re-weight it to obtain $\mathcal{L}(\alpha)$ for all other values of α.

3. Vary α_0 to Check Re-weighting: MC re-weighted to MC′



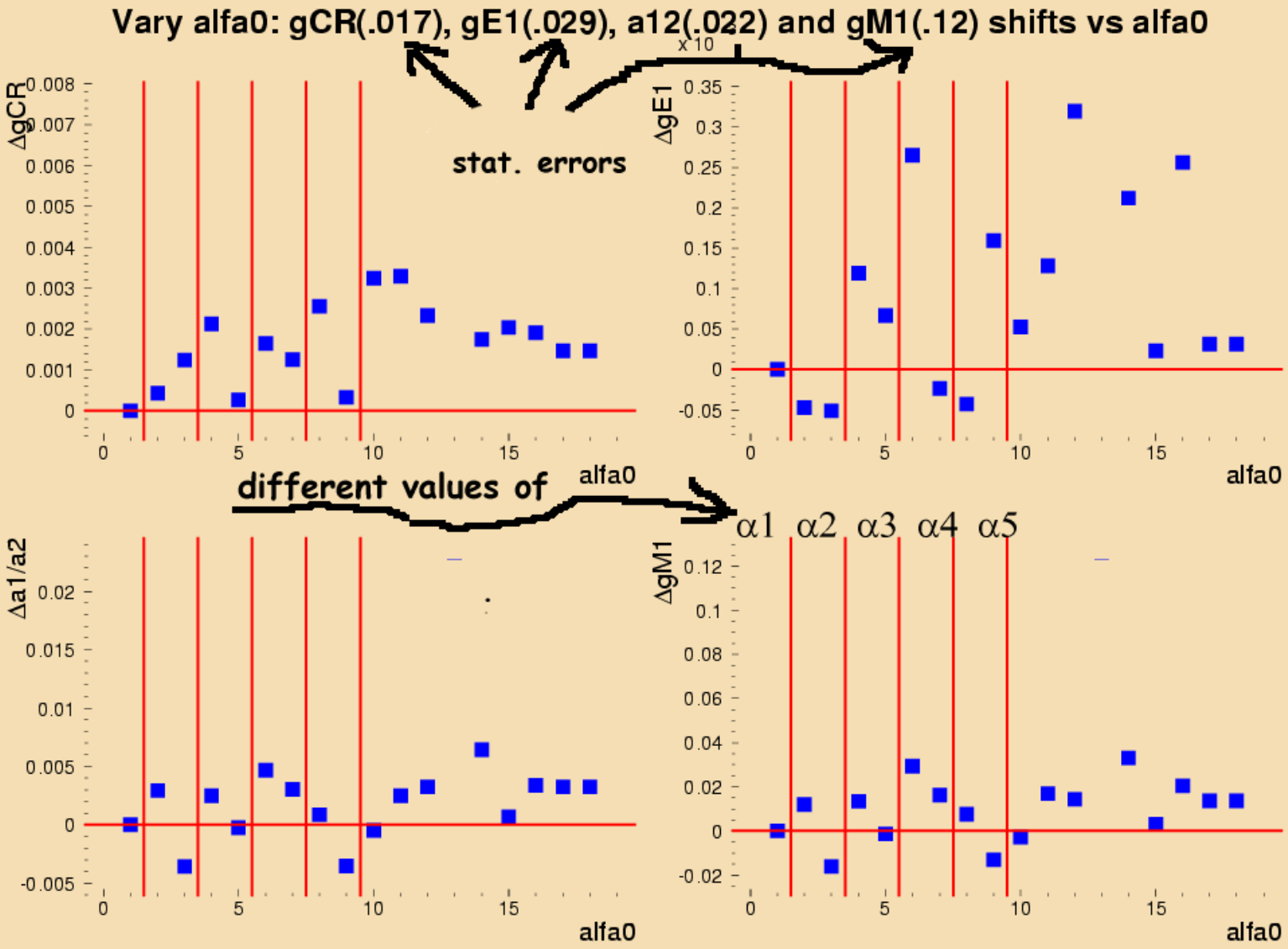
α'_0=(.09,.0,-.81,.8), α_0=(.16,.0,-.73,1.2) re-weighted to α', black: red/blue

4. Vary α_0 to Check Re-weighting: MC re-weighted to MC″



α''_0=(.24,.09,-.65,1.6), α_0=(.16,.0,-.73,1.2) re-weighted to α'', black: red/blue

5. α_0 Variation Shows Shifts at $\approx 1\%$ Level.



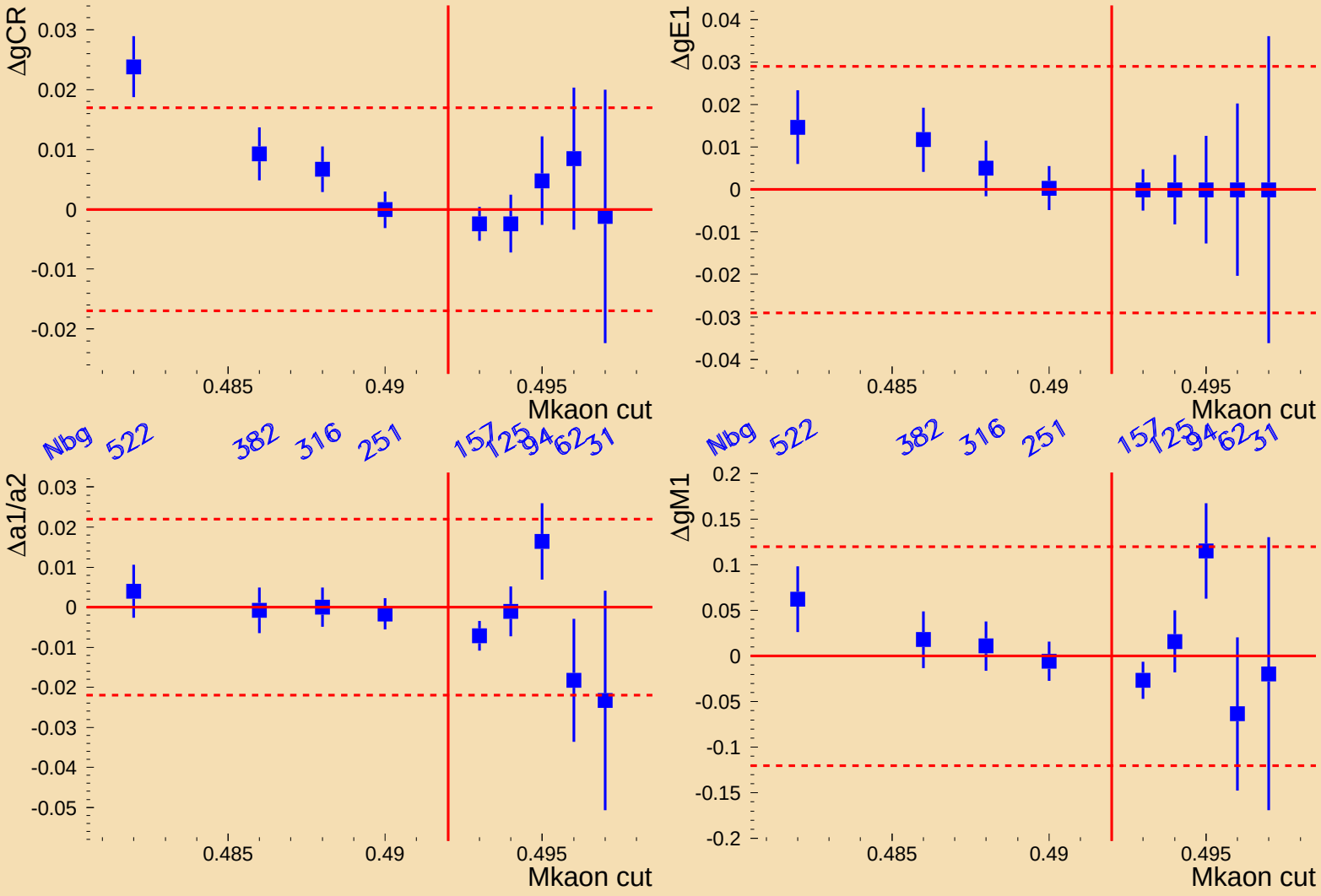
6. Study Systematic Effects with Cut Variation.

- ✓ To estimate the effect of the background we vary 3 cuts: kaon mass, P_T^2 and pp0kine, and assign the systematic uncertainty keeping in mind that the changes are correlated.

- ✓ Then other cuts and assign systematic uncertainty assuming no correlation.

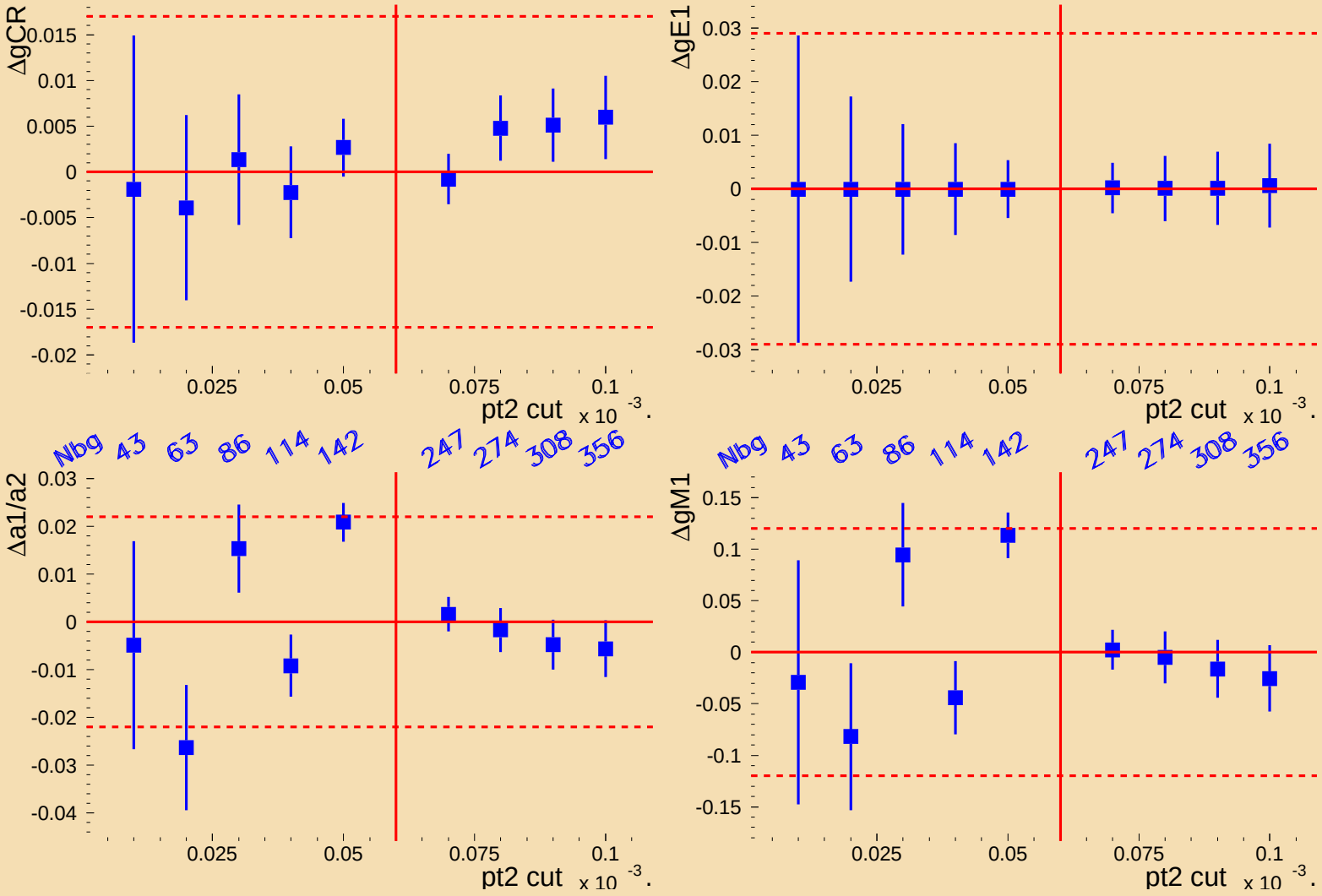
7. Vary Kaon Mass Cut: estimated N_{bg} is written in blue.

Estimated shifts: ΔgCR=0.005, ΔgE1=0.005, Δa12=0, ΔgM1=0



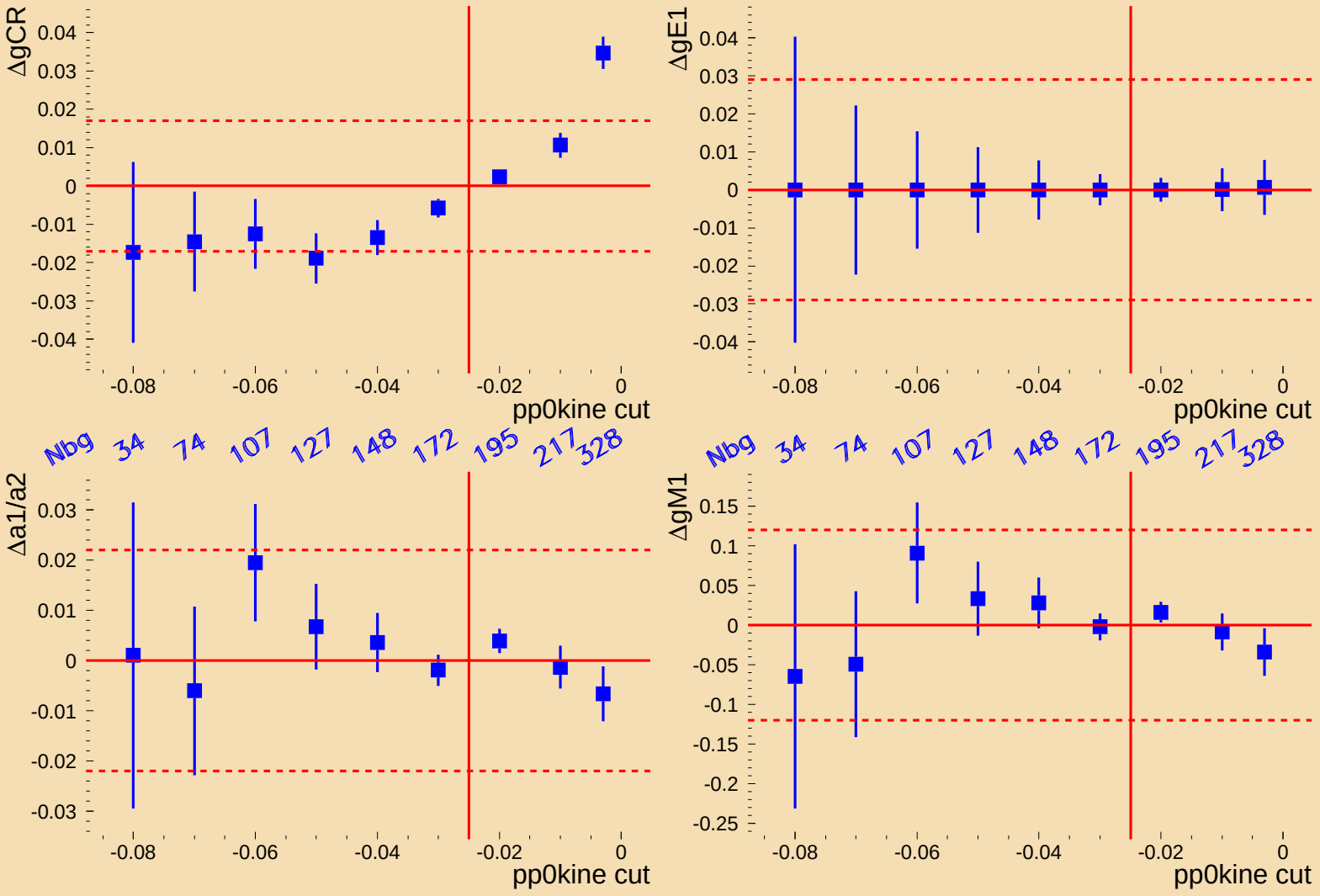
8. Vary P_T^2 Cut: estimated N_{bg} is written in blue.

Estimated shifts: ΔgCR=0, ΔgE1=0, Δa12=0.01, ΔgM1=0.04



9. Vary p0kine Cut: estimated N_{bg} is written in blue.

Estimated shifts: ΔgCR=0.01, ΔgE1=0, Δa12=0, ΔgM1=0

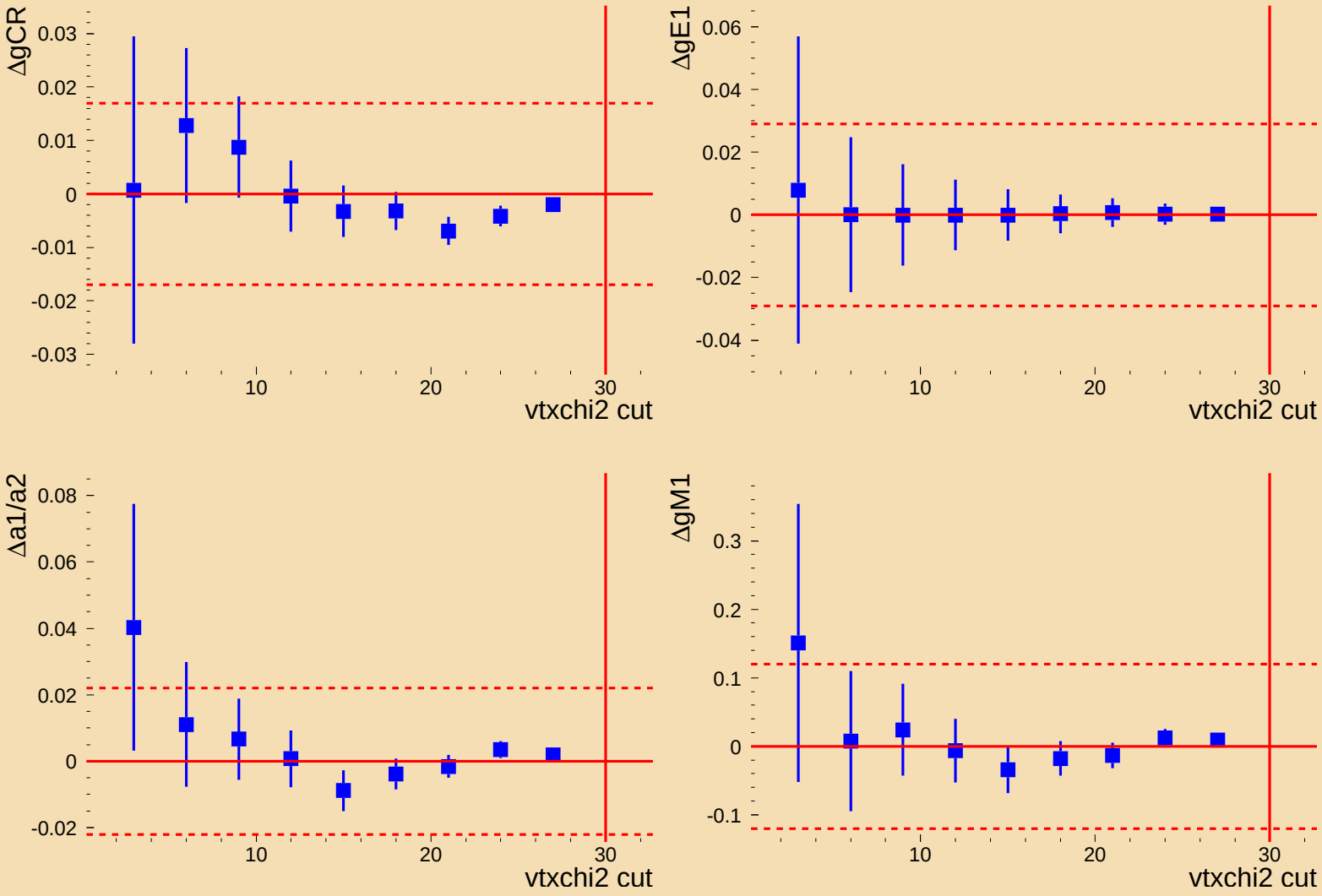


10. Summarize Background Systematics

Cut	Estimated Shift			
	Δg_{CR}	$\Delta \frac{\lvert g_{E_1} \rvert}{\lvert g_{M_1} \rvert}$	$\Delta \frac{a_1}{a_2}$	Δg_{M_1}
Kaon Mass	0.005	0.005	0.0	0.0
P_T^2	0.0	0.0	0.01	0.04
pp0kine	0.01	0.0	0.0	0.0
Assigned Error	0.01	0.005	0.01	0.04
Compare to Stat.	0.017	0.028	0.022	0.12

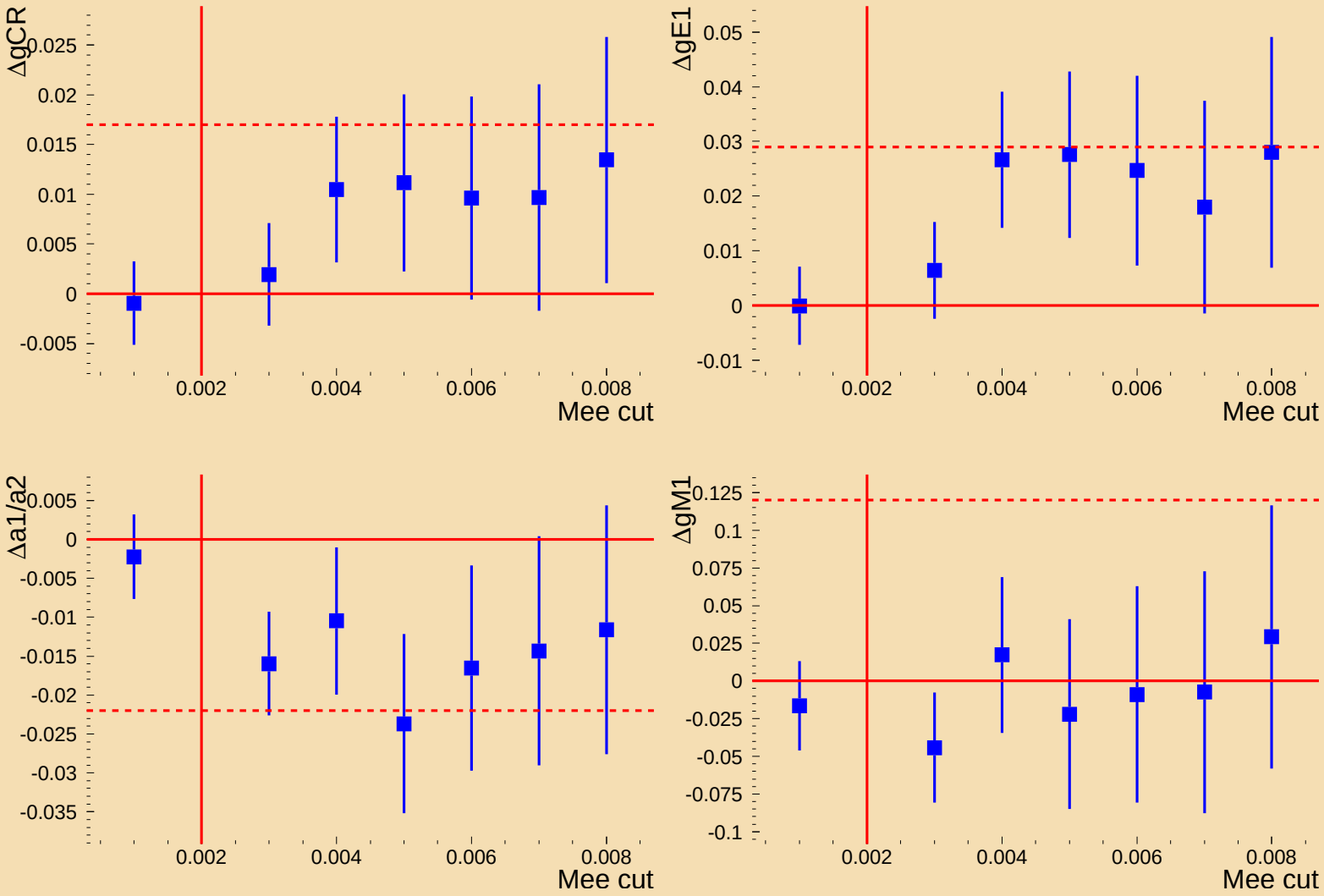
11. Vary χ^2_{vtx} Cut

Estimated shifts: ΔgCR=0.002, ΔgE1=0, Δa12=0, ΔgM1=0



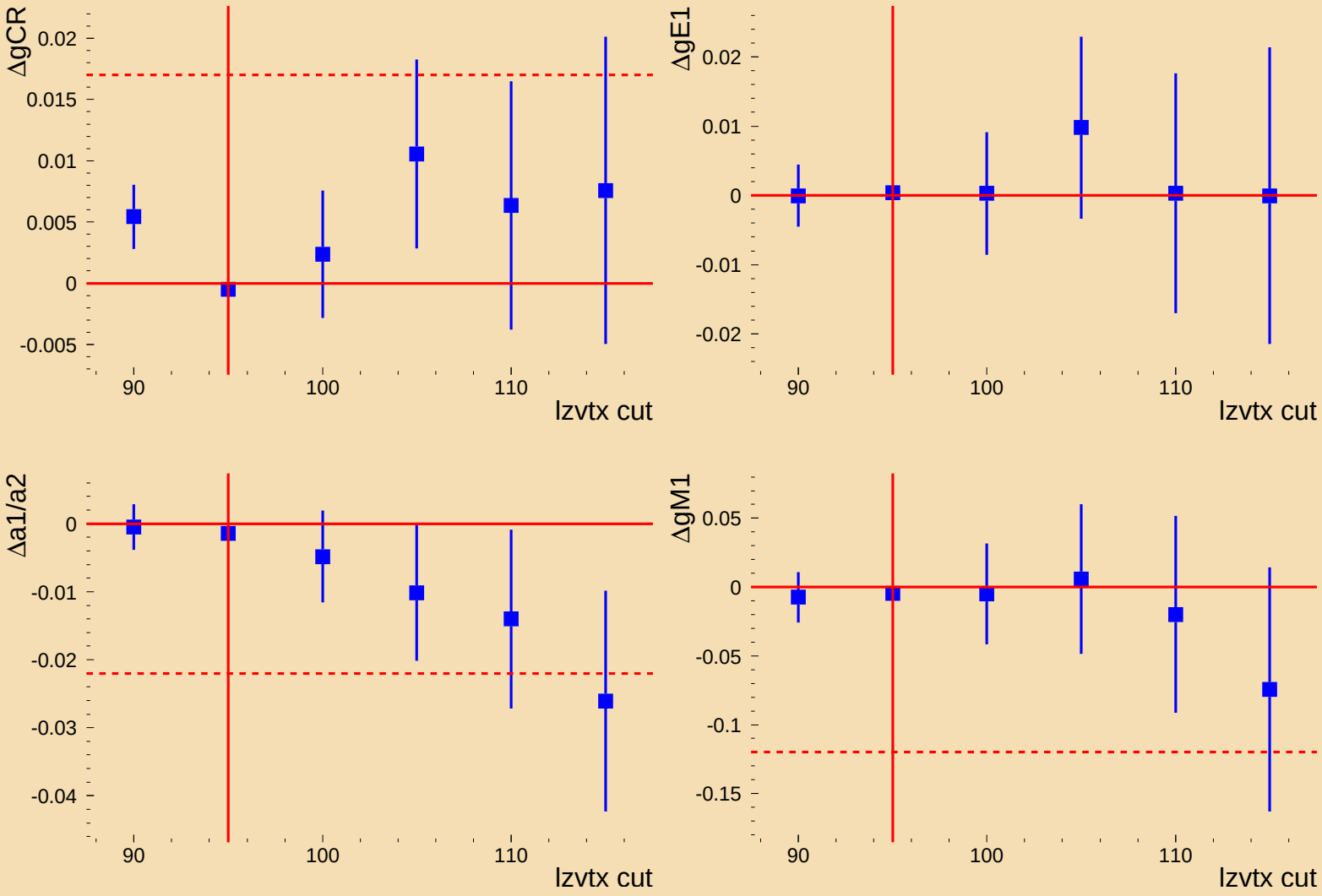
12. Vary M_{ee} Cut

Estimated shifts: ΔgCR=0.005, ΔgE1=0.01, Δa12=0.01, ΔgM1=0



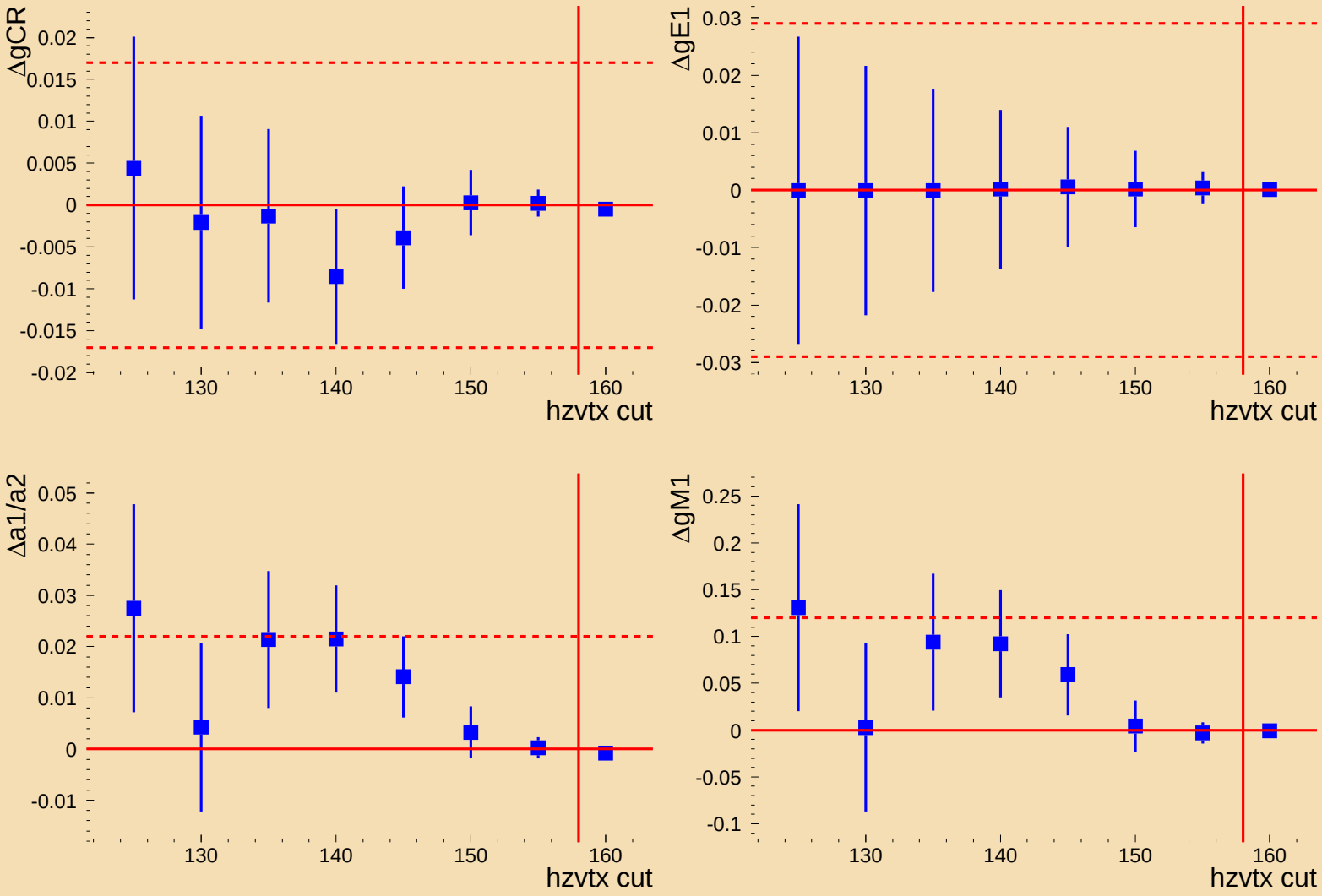
13. Vary Lower Edge of Z_{vtx} Cut

Estimated shifts: ΔgCR=0, ΔgE1=0, Δa12=0.01, ΔgM1=0



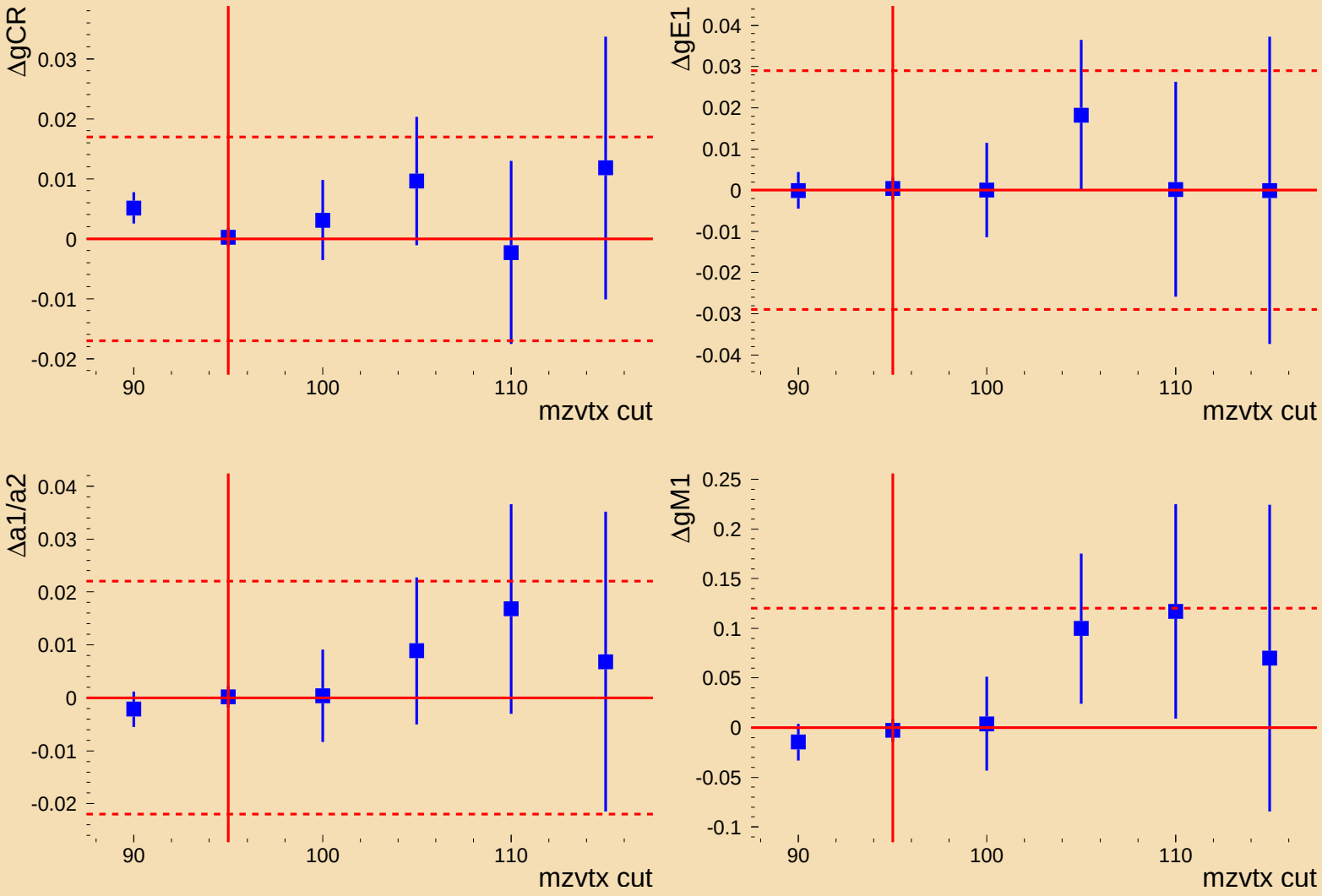
14. Vary Higher Edge of Z_{vtx} Cut

Estimated shifts: ΔgCR=0, ΔgE1=0, Δa12=0.01, ΔgM1=0.02



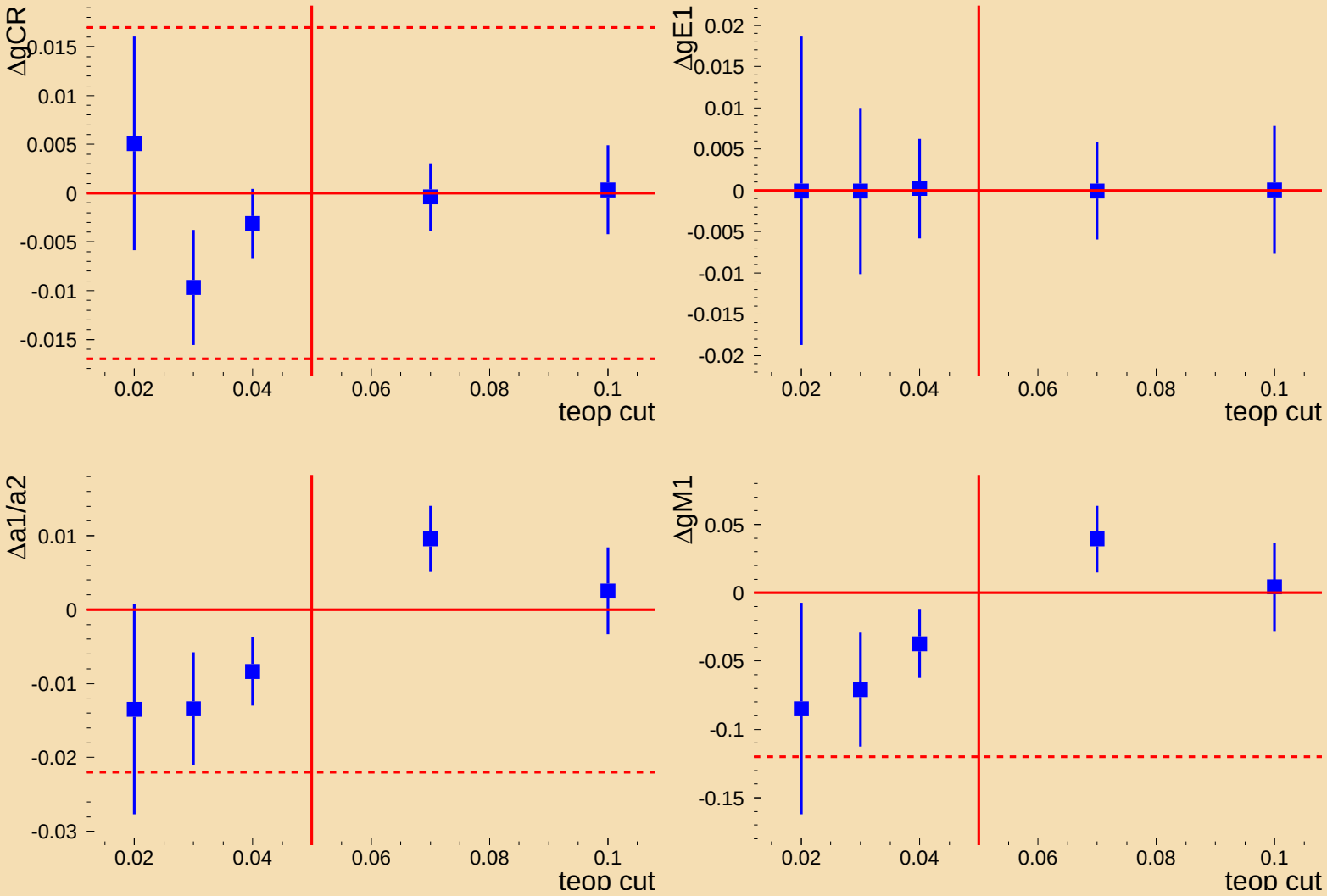
15. Vary Both Ends of Z_{vtx} Cut

Estimated shifts: ΔgCR=0, ΔgE1=0, Δa12=0, ΔgM1=0



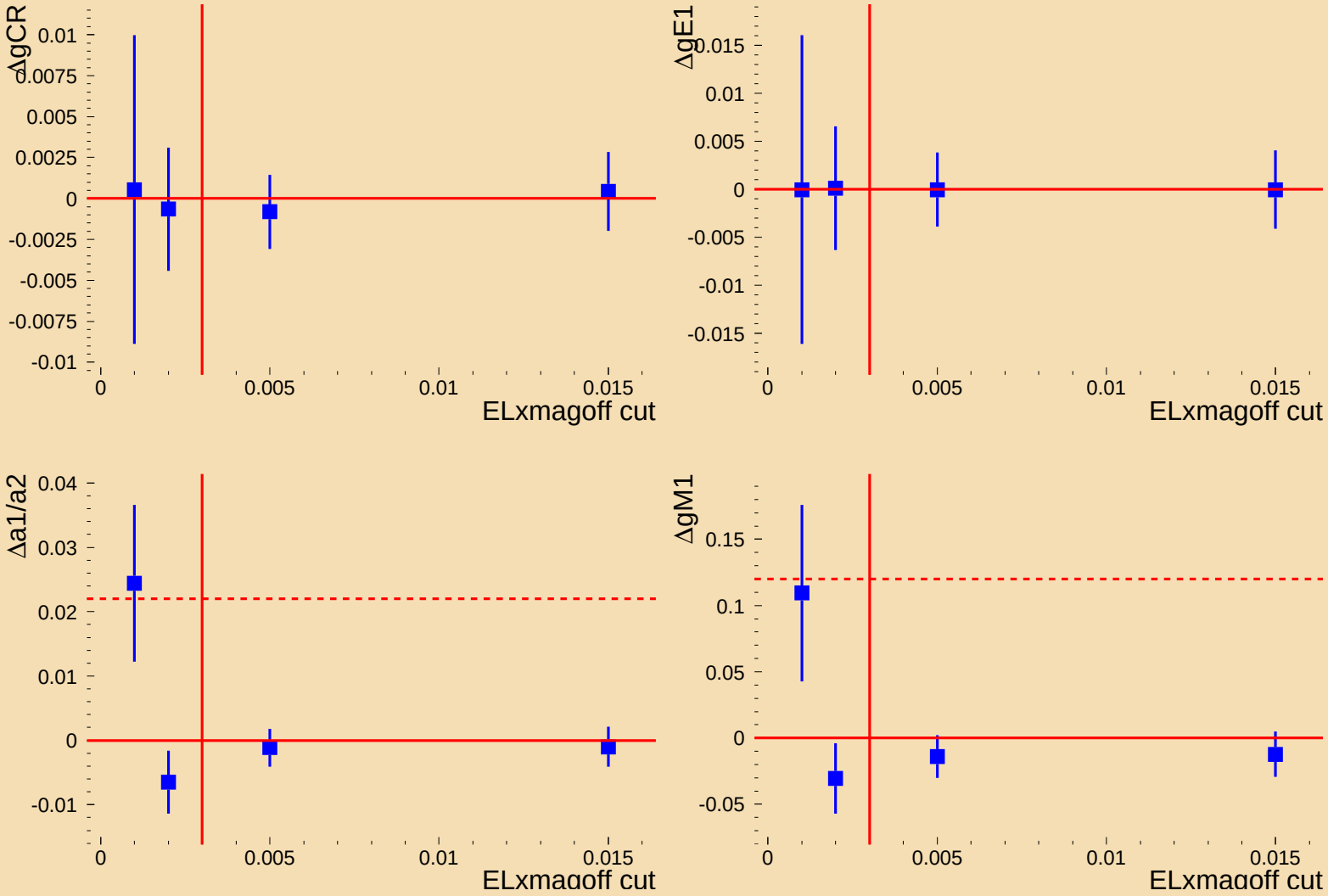
16. Vary E/P Cut

Estimated shifts: ΔgCR=0, ΔgE1=0, Δa12=0.005, ΔgM1=0.02



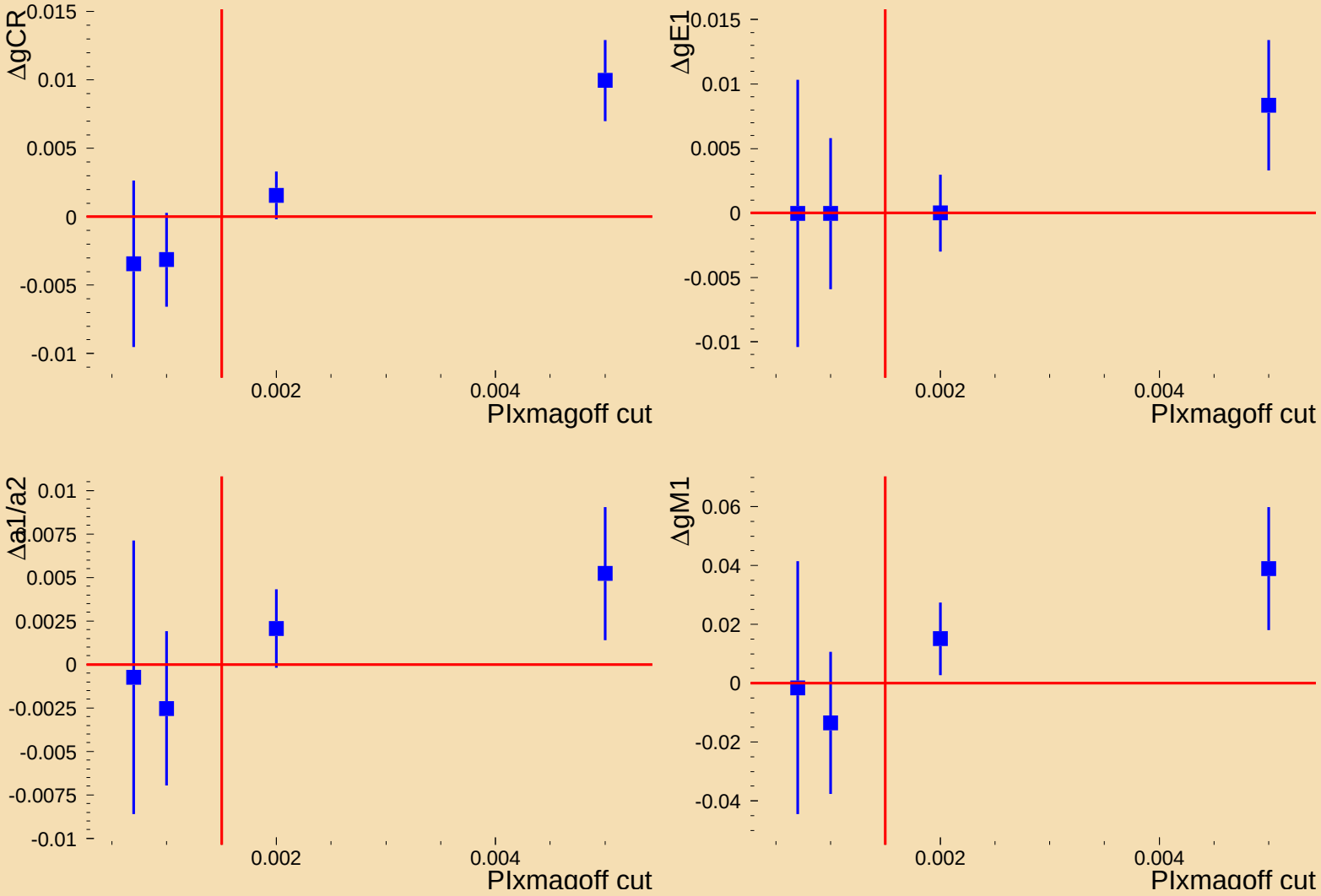
17. Vary Cut on the Magnet Offset of Electron

Estimated shifts: ΔgCR=0, ΔgE1=0, Δa12=0, ΔgM1=0



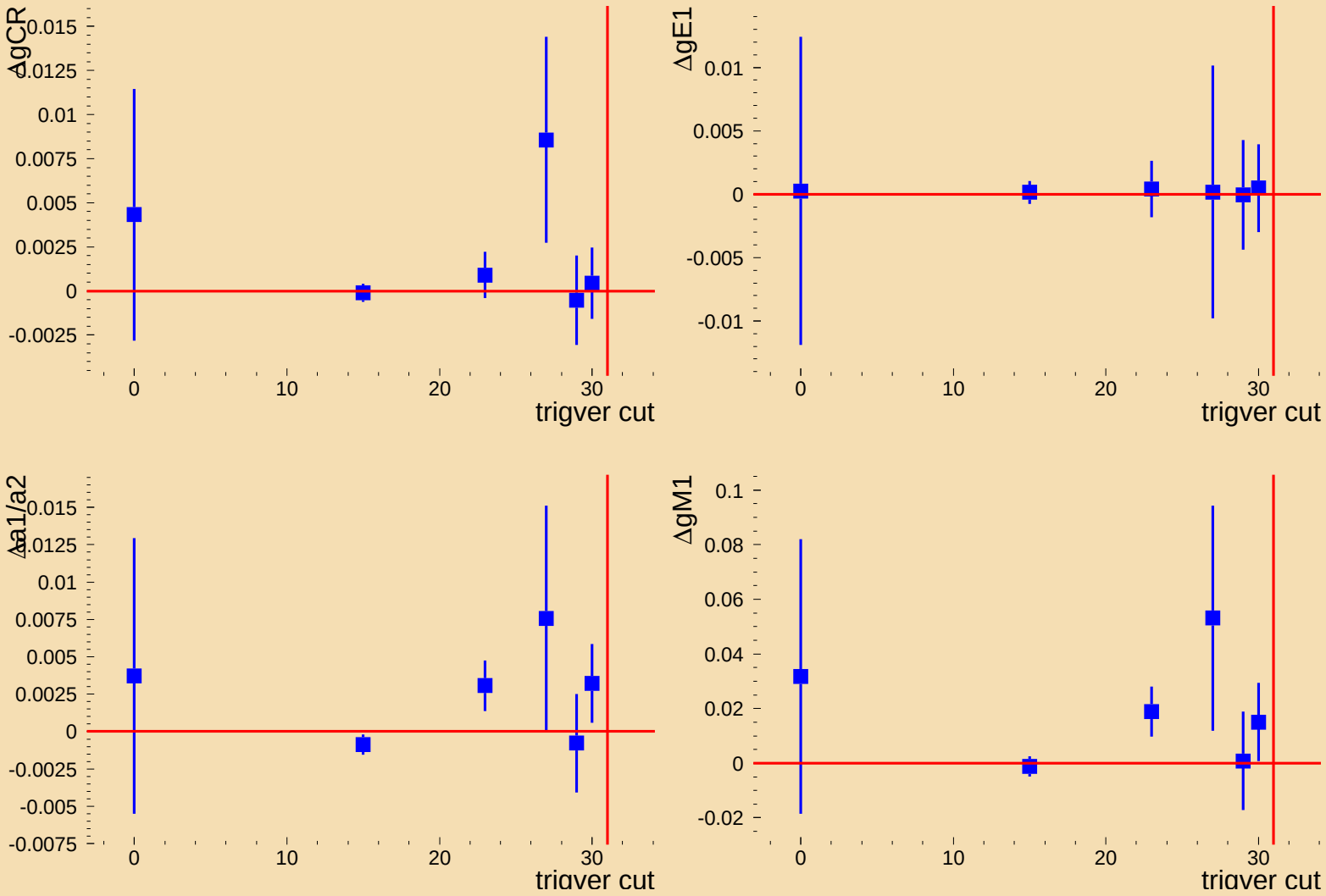
18. Vary Cut on the Magnet Offset of Pion

Estimated shifts: ΔgCR=0, ΔgE1=0, Δa12=0, ΔgM1=0



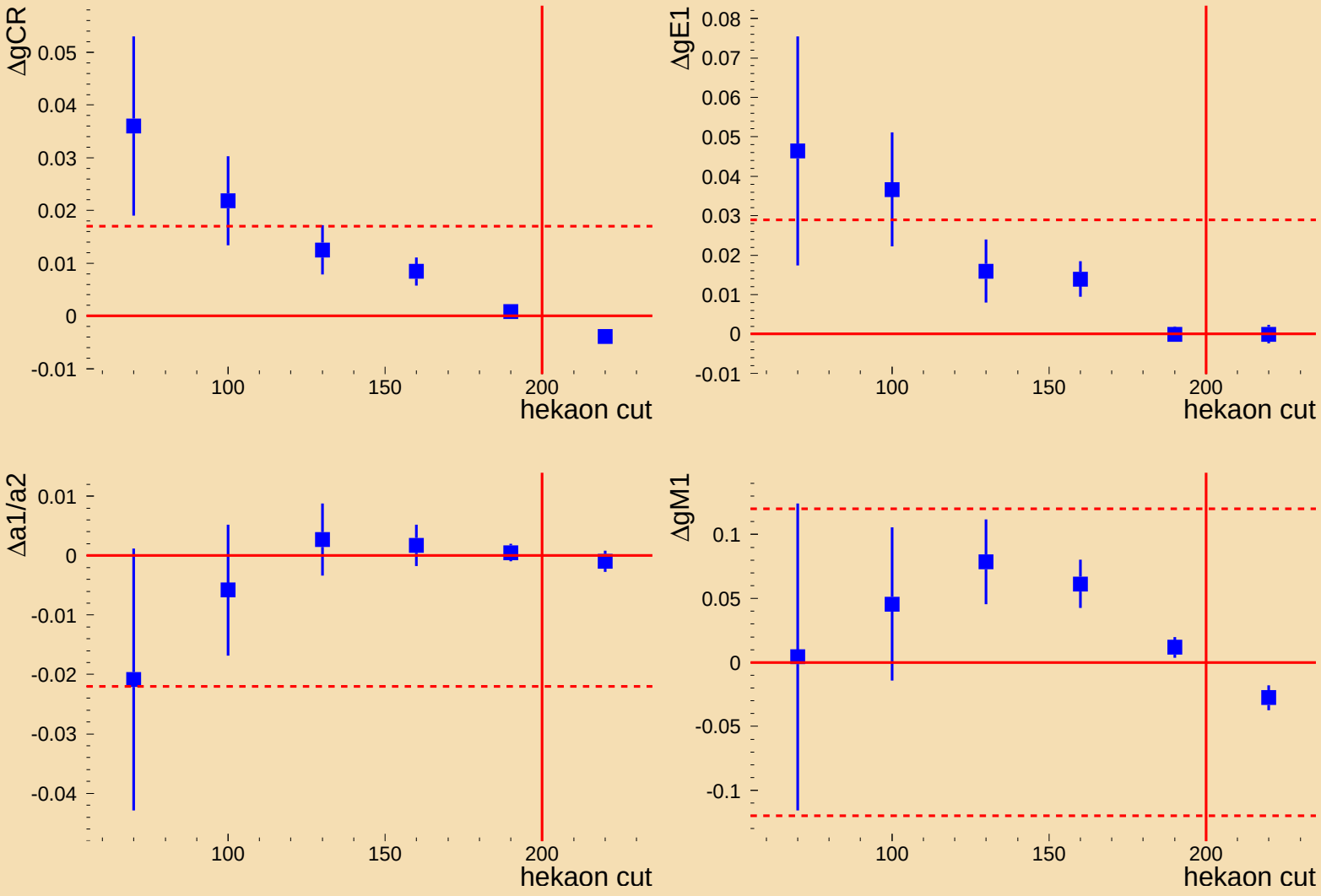
19. Vary Trigger Verification Requirements

Estimated shifts: ΔgCR=0, ΔgE1=0, Δa12=0, ΔgM1=0



20. Vary Kaon Energy Cut

Estimated shifts: ΔgCR=0.02, ΔgE1=0.015, Δa12=0, ΔgM1=0.03



21. Summarize Cut Variation Systematics

Cut	Estimated Shift			
	Δg_{CR}	$\Delta \frac{\lvert g_{E_1} \rvert}{\lvert g_{M_1} \rvert}$	$\Delta \frac{a_1}{a_2}$	Δg_{M_1}
χ^2_{vtx}	0.002	0.0	0.0	0.0
M_{ee}	0.005	0.01	0.01	0.0
Z_{vtx}	0.0	0.0	0.01	0.02
E/P	0.0	0.0	0.005	0.02
E_{kaon}	0.02	0.015	0.0	0.03
Assigned Error	0.021	0.018	0.015	0.041
Compare to Stat.	0.017	0.028	0.022	0.12

22. Summarize All Systematics

Source	Uncertainty on the Parameter			
	Δg_{CR}	$\Delta \frac{\lvert g_{E_1}\rvert}{\lvert g_{M_1}\rvert}$	$\Delta \frac{a_1}{a_2}$	Δg_{M_1}
Background	0.01	0.005	0.01	0.04
Variation of Cuts	0.021	0.018	0.015	0.041
Variation of α_0	0.001	0.001	0.005	0.02
Limited MC	0.001	0.001	0.002	0.01
$\Delta\eta_{+-}$	0.002	0.0002	0.0001	0.01
$\Delta\Phi_{+-}$	0.0005	0.0005	0.0006	0.005
Assigned Error	0.023	0.019	0.019	0.06
Compare to Stat.	0.017	0.028	0.022	0.12

23. Summary of the Fit Results

√ **Results** of the 4 parameter fit for the total dataset

$$g_{CR} = 0.161 \pm 0.017(\text{stat}) \pm 0.023(\text{syst})$$

$$\frac{|g_{E1}|}{|g_{M1}|} < 0.028(\text{stat}) \pm 0.19(\text{syst})$$

$$\frac{a_1}{a_2} = -0.729 \pm 0.022(\text{stat}) \pm 0.019(\text{syst})$$

$$g_{M1} = 1.16 \pm 0.12(\text{stat}) \pm 0.06(\text{syst})$$

√ **Covariance** matrix (*in the above order*):

$$V = \begin{pmatrix} 2.821 & 1.633 & 0.8246 & 7.761 \\ 1.633 & 5.760 & 0.2548 & 10.25 \\ 0.8246 & 0.2548 & 4.452 & 23.47 \\ 7.761 & 10.25 & 23.47 & 153.7 \end{pmatrix} \cdot 10^{-4} \pm \begin{pmatrix} 0.42 & 0.14 & 0.0015 & 0.0070 \\ 0.14 & 10.1 & 1.93 & 122.4 \\ 0.0015 & 1.93 & 3.11 & 120.2 \\ 0.0070 & 122.4 & 120.2 & 5462.15 \end{pmatrix} \cdot 10^{-8}$$

√ **Correlation** matrix:

$$\rho = \begin{pmatrix} 1.00 & 0.41 & 0.23 & 0.37 \\ 0.41 & 1.00 & 0.050 & 0.34 \\ 0.23 & 0.050 & 1.00 & 0.90 \\ 0.37 & 0.34 & 0.90 & 1.00 \end{pmatrix}$$

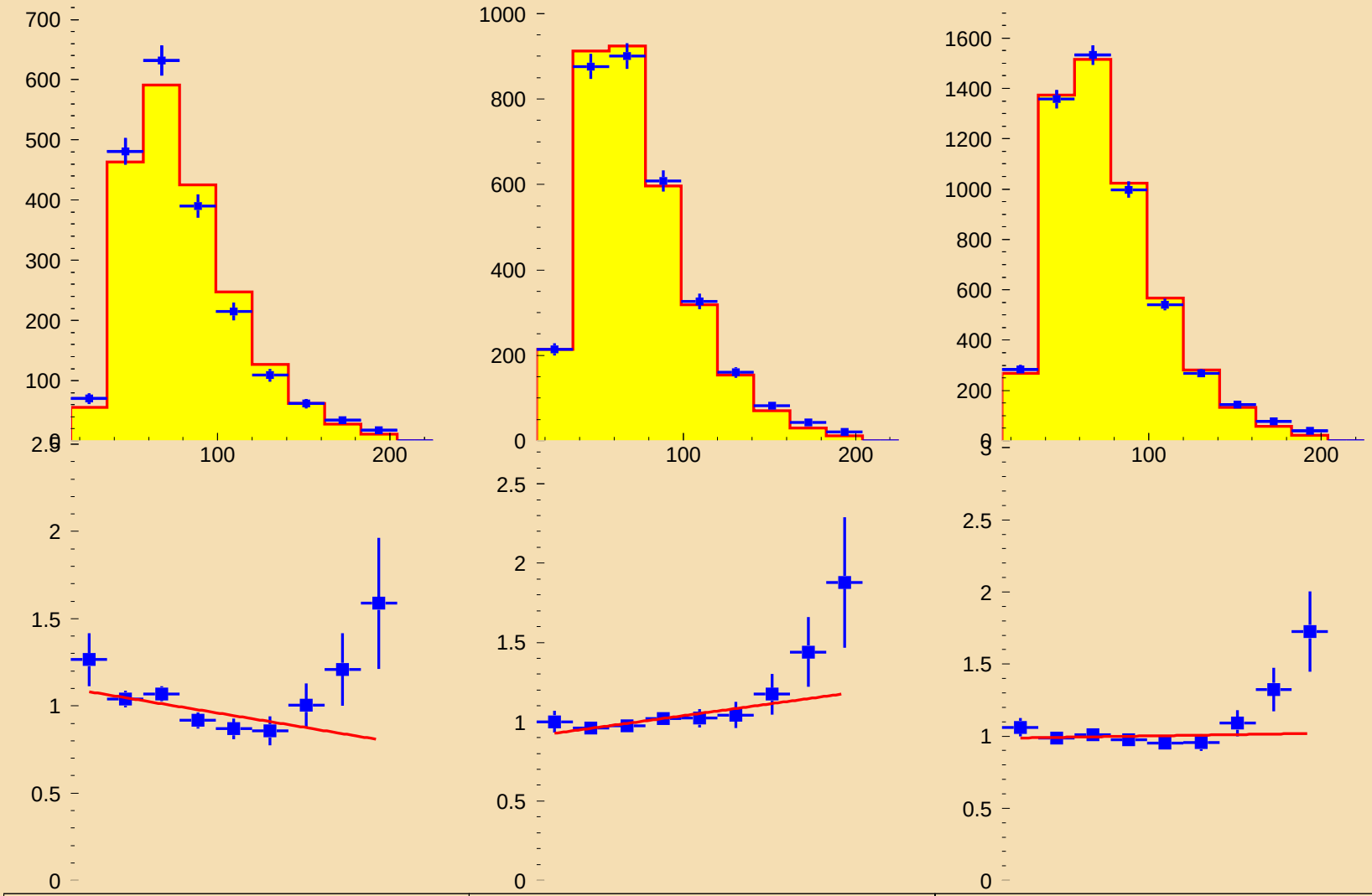
24. Conclusions and Plans

- ✓ Re-weighting doesn't depend on the initial values of the parameters for MC.
- ✓ The Values for FF's are measured from the 4 parameter fit and uncertainties are estimated.
- ✓ The systematic uncertainty due to E_{kaon} cut requires further study.
- ✓ Measure the asymmetry, branching ratio and the CPT phase.

25. Reference Slides

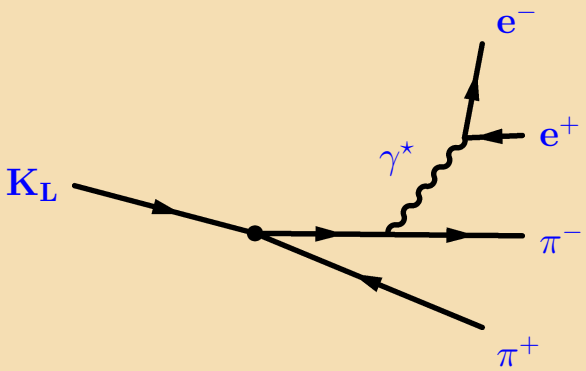
√ On the **next pages** there are some reference slides.

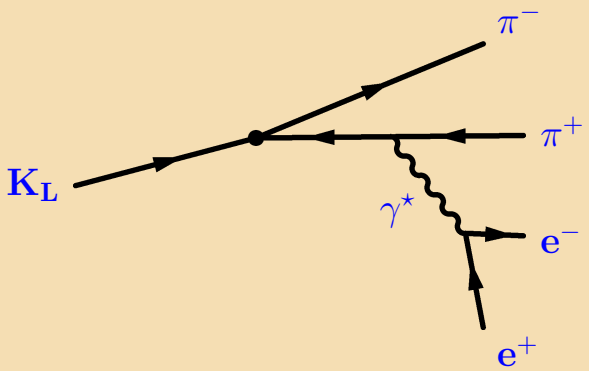
26. Data/MC Comparison for E_{kaon}: '97, '99 and total.



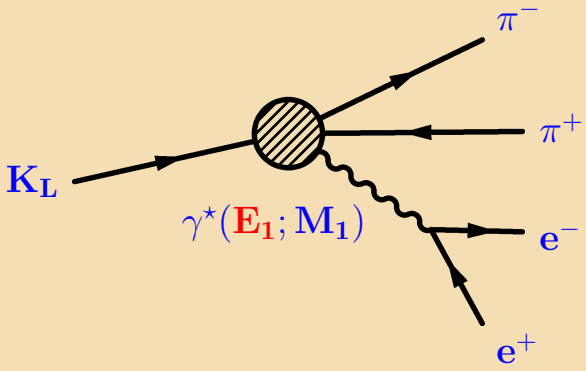
χ^2/ndf	15.57 / 7			χ^2/ndf	6.816 / 7		χ^2/ndf	16.02 / 7
P1	1.123 ±	0.6202E-01		P1	0.8895 ±	0.4585E-01	P1	0.9830 ± 0.3667E-01
P2	-0.1635E-02 ±	0.7107E-03		P2	0.1485E-02 ±	0.5906E-03	P2	0.1834E-03 ± 0.4498E-03

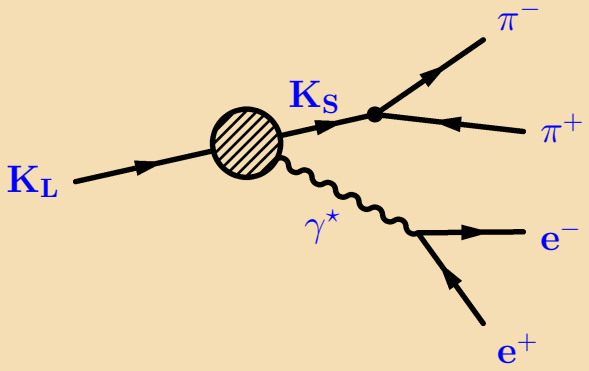
27. Contributions to the Decay $K_L \to \pi^+\pi^-e^+e^-$





Inner Bremsstrahlung (IB) — *Indirect $\not{C}\not{P}$*





Direct Emission (DE)
E_1 — ***Indirect $\not{C}\not{P}$***
M_1 — ***CP Conserving***

K^0 Charge Radius (CR)
CP Conserving

28. Summary of the Fit Results From the Previous Meeting

- Repeated 1 parameter fit for g_{CR} ('97 only, g_{E_1} = .038, g_{M_1} = 1.35, $\frac{a_1}{a_2}$ = -.72)

$$g_{CR} = 0.172 \pm 0.022(\text{stat}) \pm 0.001(\text{syst1}) \pm \ldots (\text{syst2})$$

- Repeated 2 parameter fit for g_{M_1} and $\frac{a_1}{a_2}$ ('97 + '99, g_{E_1} = .038, g_{CR} = .15)

$$g_{M1} = 1.16 \pm 0.11(\text{stat}) \pm 0.01(\text{syst1}) \pm \ldots (\text{syst2})$$

$$\frac{a_1}{a_2} = -0.737 \pm 0.021(\text{stat}) \pm 0.003(\text{syst1}) \pm \ldots (\text{syst2})$$

$$g_{M1} = 1.10 \pm 0.10(\text{stat}) \pm 0.06(\text{syst})[\text{DPF}'02]$$

$$\frac{a_1}{a_2} = -0.75 \pm 0.03(\text{stat}) \pm 0.02(\text{syst})[\text{DPF}'02]$$

- New results of the 4 parameter fit ('97 +'99)

$$g_{CR} = 0.161 \pm 0.017(\text{stat}) \pm .002(\text{syst1}) \pm \ldots (\text{syst2})$$

$$\frac{|g_{E1}|}{|g_{M1}|} < 0.029 + \ldots (\text{syst2})$$

$$g_{M1} = 1.16 \pm 0.12(\text{stat}) \pm 0.02(\text{syst1}) \pm \ldots (\text{syst2})$$

$$\frac{a_1}{a_2} = -0.729 \pm 0.022(\text{stat}) \pm 0.002(\text{syst1}) \pm \ldots (\text{syst2})$$

29. Expressions for the Form Factors

- ✓ **Inner Bremsstrahlung**: $g_{IB} = |\eta_{+-}| e^{i(\delta_0(M_K) + \Phi_{+-})}$
- ✓ **M_1 Direct Emission**: $g_{M_1} = i e^{i\delta_1(M_{\pi\pi})} \times |g_{M_1}|$,

  where

$$|g_{M_1}| \equiv \tilde{g}_{M_1} \left[1 + \frac{a1/a2}{(M_\rho^2 - M_K^2) + 2M_K E_{ee}} \right]$$

- ✓ **E_1 Direct Emission**: $g_{E_1} = \frac{|g_{E_1}|}{|g_{M_1}|} e^{i(\delta_1(M_{\pi\pi}) + \Phi_{+-})} \times |g_{M_1}|$
- ✓ **Charge Radius**: $g_{CR} = |g_{CR}| e^{i\delta_0(M_{\pi\pi})}$,

  where $|g_{CR}| = -\frac{1}{3}\langle R^2(K^0)\rangle M_K^2$