Update on $K_{L,S} \rightarrow \pi^+\pi^-\gamma$

Michael Ronquest
UVA
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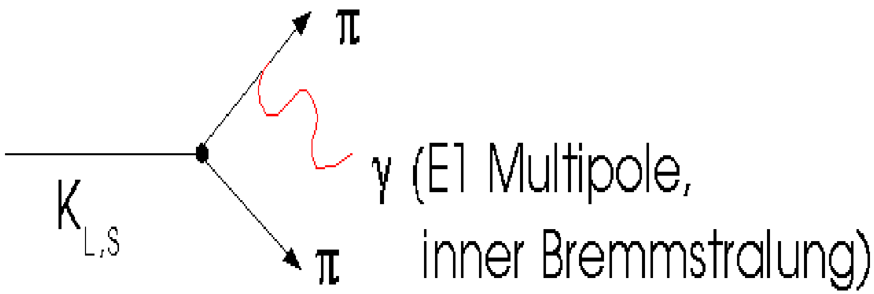
Outline of Today's Talk

- A recent theoretical analysis (Valencia & Tandean, PhysRev D62 (2000) 116007) of K -> $\pi\pi\gamma$ has cleared up some ambiguities about the parameters involved in the measurement of CP violation in this mode. The study also complicates the simulation of these events.
- Today I'll:
  - Review the general facts of this mode
  - Detail the recent developments

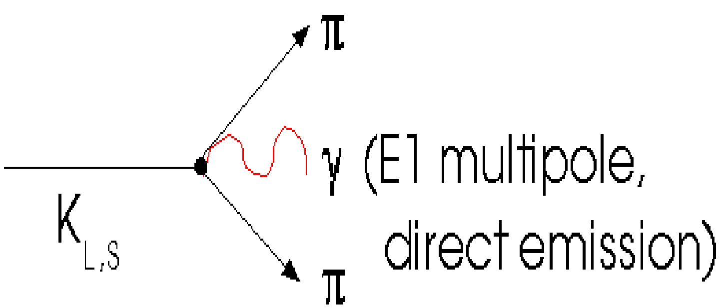
Review

- The main goal of the analysis is to measure the CP violating parameter $\eta_{+-\gamma}$ which characterizes the amount of CP violation in $K \rightarrow \pi^+\pi^-\gamma$ decays.
- This allows for a measurement of direct CP violation in this mode via the calculation of $\varepsilon'_{+-\gamma} = \eta_{+-\gamma} - \eta_{+-}$
- Non- zero $\varepsilon'_{+-\gamma}$ *DOES* indicate direct CP violation in excess of ε' .

Diagrams Contributing To $K_{L,S}$ - > $\pi^+\pi^-\gamma$



CP conserving for K_S
CP violating for K_L



CP conserving for K_S
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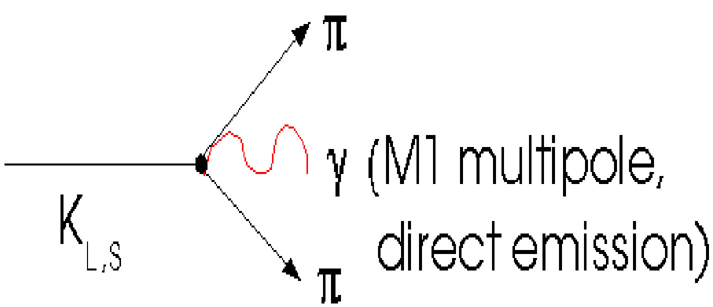

CP violating for K_S
CP conserving for K_L

Measurement Of $\eta_{+-\gamma}$

- The starting point for this discussion of $\eta_{+-\gamma}$ is the equation describing the distribution of the $\pi\pi\gamma$ decay vertex in the regenerator beam as a function of proper time:

$$\frac{dN}{d\tau} \approx \left|\rho^2\right| \Gamma_{K_S \to \pi^+\pi^-\gamma}\, \mathrm{e}^{-\frac{\tau}{\tau_s}} + \Gamma_{K_L \to \pi^+\pi^-\gamma}\, \mathrm{e}^{-\frac{\tau}{\tau_L}}$$

$$+ 2\,Re\left[\rho\,\gamma_{LS}^{*}\,\mathrm{e}^{i\Delta m_\tau}\right] \mathrm{e}^{-\left(\frac{1}{\tau_L}+\frac{1}{\tau_S}\right)\frac{\tau}{2}}$$

- The following definitions are useful:

$$\left|\eta_{+-}\right|^2 = \frac{\Gamma^{IB}_{K_L \to \pi^+ \pi^- \gamma}}{\Gamma^{IB}_{K_S \to \pi^+ \pi^- \gamma}}$$

$$r = \frac{\Gamma^{M1}_{K_L \to \pi^+ \pi^- \gamma}}{\Gamma^{E1}_{K_L \to \pi^+ \pi^- \gamma}}$$

Amplitudes

$$\gamma_{LS} = \int d[PS]\left(\left[E_{IB}(K_L) + E_{DE}(K_L)\right]\left[E^*_{IB}(K_S) + E^*_{DE}(K_S)\right] + M(K_L)\,M^*(K_S)\right)$$

where

$$d[PS] = d\cos\theta\, dE^*_\gamma \frac{\left(\beta\ E^*_\gamma\right)^3 \sin\theta}{32\pi^3 m_K^3}\left(1 - \frac{2E^*_\gamma}{M_K}\right)$$

and

$$\beta = \sqrt{1 - \frac{4m_\pi^2}{m_K^2 - 2E_\gamma m_K}}$$

- It is also necessary to note that:

$$\eta_{+-\gamma} \neq \frac{E_{IB}(K_L) + E_{DE}(K_L)}{E_{IB}(K_S) + E_{DE}(K_S)}$$

- However, one can define:

$$\widetilde{\eta_{+-\gamma}} = \frac{E_{IB}(K_L) + E_{DE}(K_L)}{E_{IB}(K_S) + E_{DE}(K_S)}$$

which is *NOT* the same parameter that appears in the vertex distribution.

- Using the following matrix elements, we can write down theoretical predictions for the previous parameters in terms of the 'a's and 'g's.

$$E_{IB}(K_S) = \left(\frac{M_K}{E_\gamma^2}\right) \frac{\sqrt{2}\, R\, e\, A_0 e^{i\delta_0}}{1-\beta^2\cos^2(\theta)}$$

$$E_{IB}(K_L) = \left(\frac{M_K}{E_\gamma^2}\right) \frac{\sqrt{2}\, R\, e\, A_0 \eta_{+-}\, e^{i\delta_0}}{1-\beta^2\cos^2(\theta)}$$

$$M(K_L) = i\left|g_{M1}\right|\left(\frac{a_1}{m_\rho^2 - m_K^2 + 2E_\gamma M_K} + a_2\right)e^{i\delta_1}$$

$$E_{DE}(K_S) = \frac{\left|g_{E1(i)}\right|}{\left|\epsilon\right|} e^{i\delta_1}$$

$$E_{DE}(K_L) = \left|g_{E1(i)}\right| e^{i(\delta_1+\phi_\epsilon)} + i\left|g_{E1(d)}\right| e^{i\delta_1}$$

same as g_{E1} for $K_L \rightarrow \pi\pi\gamma$ and $\pi\pi ee$ (indirectly CP violating)

new parameter (directly CP violating)

- The most interesting parameter to calculate is $\tilde{\eta}_{+-\gamma}$:

$$\widetilde{\eta_{+-\gamma}} = \frac{\left(\frac{M_K}{E_\gamma^2}\right)\frac{\sqrt{2}\,R\;e\;A_0\,\eta_{+-}\;e^{i\delta_0}}{1-\beta^2\cos^2(\theta)} + |g_{E1(i)}|\,e^{i(\delta_1+\phi_\epsilon)} + i\,|g_{E1(d)}|\,e^{i\delta_1}}{\left(\frac{M_K}{E_\gamma^2}\right)\frac{\sqrt{2}\,R\;e\;A_0\,e^{i\delta_0}}{1-\beta^2\cos^2(\theta)} + \frac{|g_{E1(i)}|}{|\epsilon|}\,e^{i\delta_1}}$$

$$\approx \eta_{+-} + \frac{|g_{E1(i)}|\,e^{i(\delta_1+\phi_\epsilon)} + i\,|g_{E1(d)}|\,e^{i\delta_1} - \eta_{+-}\frac{|g_{E1(i)}|}{|\epsilon|}\,e^{i\delta_1}}{\sqrt{2}\,R\;e\;A_0\,M_K}\,E_\gamma^2\left(1-\beta^2\cos^2(\theta)\right)e^{-i\delta_0}$$

$$\approx \eta_{+-} + \frac{|g_{E1(d)}|}{\sqrt{2}\,R\;e\;A_0\,M_K}\,E_\gamma^2\left(1-\beta^2\cos^2(\theta)\right)e^{i\left(\delta_1-\delta_0+\frac{\pi}{2}\right)}$$

indirect CP violation terms are eliminated

- Following Valencia, we can then define a new direct CP violation parameter

$$\hat{\epsilon} = \frac{g_{E1(d)} M_K}{4\sqrt{2}\, Re\, A_0}$$

This leads to

$$\tilde{\epsilon}'_{+-\gamma} = \widetilde{\eta_{+-\gamma}} - \eta_{+-} = \hat{\epsilon}\, e^{i\left(\delta_1 - \delta_0 + \frac{\pi}{2}\right)} \left(2\frac{E_\gamma}{M_K}\right)^2 \left(1 - \beta^2 \cos^2(\theta)\right)$$

which is *not* constant throughout phase space. Moving on to the observable, we can define

$$\epsilon'_{+-\gamma} = \frac{1}{\Gamma_{K_S \to \pi^+\pi^-\gamma}} \int d[PS]\, \tilde{\epsilon}'_{+-\gamma} \left| E_{IB}(K_S) + E_{DE}(K_S) \right|^2$$

- Returning to the vertex distribution, neglecting M1 emission from the K_S and using the definitions made so far,

$$\gamma_{LS} = \int d[PS]\left(\widetilde{\eta_{+-\gamma}}\left[E_{IB}(K_S)+E_{DE}(K_S)\right]\left[E^*_{IB}(K_S)+E^*_{DE}(K_S)\right]\right)$$

$$= \int d[PS]\left(\left(\eta_{+-}+\tilde{\epsilon}'_{+-\gamma}\right)\left[E_{IB}(K_S)+E_{DE}(K_S)\right]\left[E^*_{IB}(K_S)+E^*_{DE}(K_S)\right]\right)$$

$$= \left(\eta_{+-}+\epsilon'_{+-\gamma}\right)\Gamma_{K_S\to\pi^+\pi^-\gamma}$$

In a similar fashion,

$$\Gamma_{K_L\to\pi^+\pi^-\gamma} = \int d[PS]\left(\left|E_{IB}(K_L)+E_{DE}(K_L)\right|^2+\left|M(K_L)\right|^2\right)$$

$$= \int d[PS]\left((1+r)\left|E_{IB}(K_L)+E_{DE}(K_L)\right|^2\right)$$

$$= \int d[PS]\left((1+r)\left|\widetilde{\eta_{+-\gamma}}\right|^2\left|E_{IB}(K_S)+E_{DE}(K_S)\right|^2\right)$$

$$\approx \int d[PS]\left((1+r)\left(\left|\eta_{+-}\right|^2+2\,Re\left(\eta^*_{+-}\,\tilde{\epsilon}'_{+-\gamma}\right)\right)\left|E_{IB}(K_S)+E_{DE}(K_S)\right|^2\right)$$

$$\approx (1+r)\left(\left|\eta_{+-}\right|^2+2\,Re\left(\eta^*_{+-}\,\epsilon'_{+-\gamma}\right)\right)\Gamma_{K_S\to\pi^+\pi^-\gamma}$$

- Putting everything together, and dropping a common factor of the partial width of K_S-> $\pi^+\pi^-\gamma$, we get :

$$\frac{dN}{d\tau} \propto |\rho|^2 e^{-\frac{\tau}{\tau_s}} + \left[|\eta_{+-}|^2 + 2\,Re\left(\eta^*_{+-}\,\epsilon'_{+-\gamma}\right)\right](1+r)\,e^{-\frac{\tau}{\tau_L}}$$

$$+2|\eta_{+-} + \epsilon'_{+-\gamma}||\rho|\cos(\Delta m\tau + \phi_\rho - \phi_\eta)\,e^{-(\frac{1}{\tau_L}+\frac{1}{\tau_S})\frac{\tau}{2}}$$

Phase of $\eta_{+-\gamma}$

- Comparing to the traditional form shown below, it is clear that $\eta_{+-\gamma} = \eta_{+-} + \varepsilon'_{+-\gamma}$ as expected.

$$\frac{dN}{d\tau} \propto |\rho|^2 e^{-\frac{\tau}{\tau_s}} + |\eta_{+-\gamma}|^2 (1+r)\,e^{-\frac{\tau}{\tau_L}}$$

$$+2|\eta_{+-\gamma}||\rho|\cos(\Delta m\tau + \phi_\rho - \phi_\eta)\,e^{-(\frac{1}{\tau_L}+\frac{1}{\tau_S})\frac{\tau}{2}}$$

- In light of this, it is now clear that the parameter appearing in the vertex distribution is:

Direct and Indirect CP violation in K-> $\pi^+\pi^-$

$$\eta_{+-\gamma} = \eta_{+-} + \frac{\hat{\epsilon}}{\Gamma_{K_S\to\pi^+\pi^-\gamma}} \int d[PS] e^{i\left(\delta_1-\delta_0+\frac{\pi}{2}\right)} \left(2\frac{E_\gamma}{M_K}\right)^2 \left(1-\beta^2\cos^2(\theta)\right) \left|E_{IB}(K_S)+E_{DE}(K_S)\right|^2$$

Direct CP Violation in K_L -> $\pi^+ \pi^- \gamma$ via E1 Direct Emission

Ramifications

- The value of $\eta_{+-\gamma}$ depends on the area of phase space studied. A fit of dN/dτ for low energy gammas in the kaon rest frame would yield a different value of $\eta_{+-\gamma}$ than higher energy photons.
- This will clearly effect event generation for the MC, and re- weighting techniques. A way must be found to incorporate these new findings into the analysis.

Next Steps

- Devise a MC event generator that takes the kinematic dependence of $\eta_{+-\gamma}$ (and r!) into account, and/or develop a re-weighting scheme that does the same.
- See if the new technique gives better results than the default MC for this mode.