Update on the Maximum Likelihood Fits in $K_L \rightarrow \pi^+\pi^-e^+e^-$ Analysis

Alexander Golossanov
University of Virginia

May 17, 2003*

KTeV Collaboration Meeting
Fermilab†

*Last compilation May 17, 2003
†In this document (if you have access to the internet) you can click on any text in *purple color* for additional information

1. What Am I Going to Talk About?

- √ Will remind the **fitting procedure and the re-weighting** technique.

- √ Will show that **re-weighting works now**. The problem which we had before **has been fixed**.

- √ Have **re-visited** results from the previous fits:
  1-par fit for g_{CR} ('97 only) and **2-par fit** for g_{M_1} and $\frac{a_1}{a_2}$ (DPF'02).

- √ Will **present** preliminary results from the **new 4 parameter fit**:
  g_{CR}, g_{E_1}, $\frac{a_1}{a_2}$ and g_{M_1}.

2. Description of Our Model

✓ Use **matrix element for $K_L \to \pi^+\pi^-e^+e^-$** obtained by Sehgal et al.
Call this matrix element μ.

✓ The expression for μ is written as a function of **five independent variables**.
This set of variables defines a point in the phase space, call it

$$x = (\phi, cos\theta_{e^+}, cos\theta_{\pi^-}, M_{\pi\pi}, M_{ee})$$

✓ This function has a number of parameters. Divide all the **parameters** in two sets

$$\alpha = \left(\frac{a_1}{a_2}; g_{M_1}; g_{CR}; g_{E1}\right)$$

and β, which defines all other parameters.

✓ Thus, we have the matrix element $\mu = \mu\,(x; \alpha, \beta)$.
It represents **our model** for $K_L \to \pi^+\pi^-e^+e^-$ decay.

3. How to Test Our Model?

- **The goal** is to find the set of parameters α which corresponds to the **best fit between our model and** the KTeV $K_L \to \pi^+\pi^- e^+ e^-$ **data** sample.

- We use the method of **maximum likelihood** to extract the parameters α. In general, the **likelihood** is a function defined as

$$\mathcal{L}(\alpha) = \prod_{i=1}^{N_d} p(x_i;\alpha), \qquad \int p(x;\alpha)\, dx = 1$$

  The idea of the method is to find α, which corresponds to the maximum of the function $\mathcal{L}(\alpha)$.

- Here p is the hypothetical **p.d.f. describing the data** sample. For our model it can be written in terms of μ and acceptance of our detector a.

$$p(x_i;\alpha) = \frac{\mu(x_i;\alpha)\cdot a(x_i;\alpha)}{\int \mu(x;\alpha)\cdot a(x;\alpha)\, dx}$$

4. We use Re-weighting Technique to Calculate $\mathcal{L}(\alpha)$

- The **straightforward way** would be to generate MC around each phase space point x_i for every necessary α and then determine $a(x_i;\alpha)$ required during the minimization of $\mathcal{L}(\alpha)$. We would also have to calculate the normalization integral.

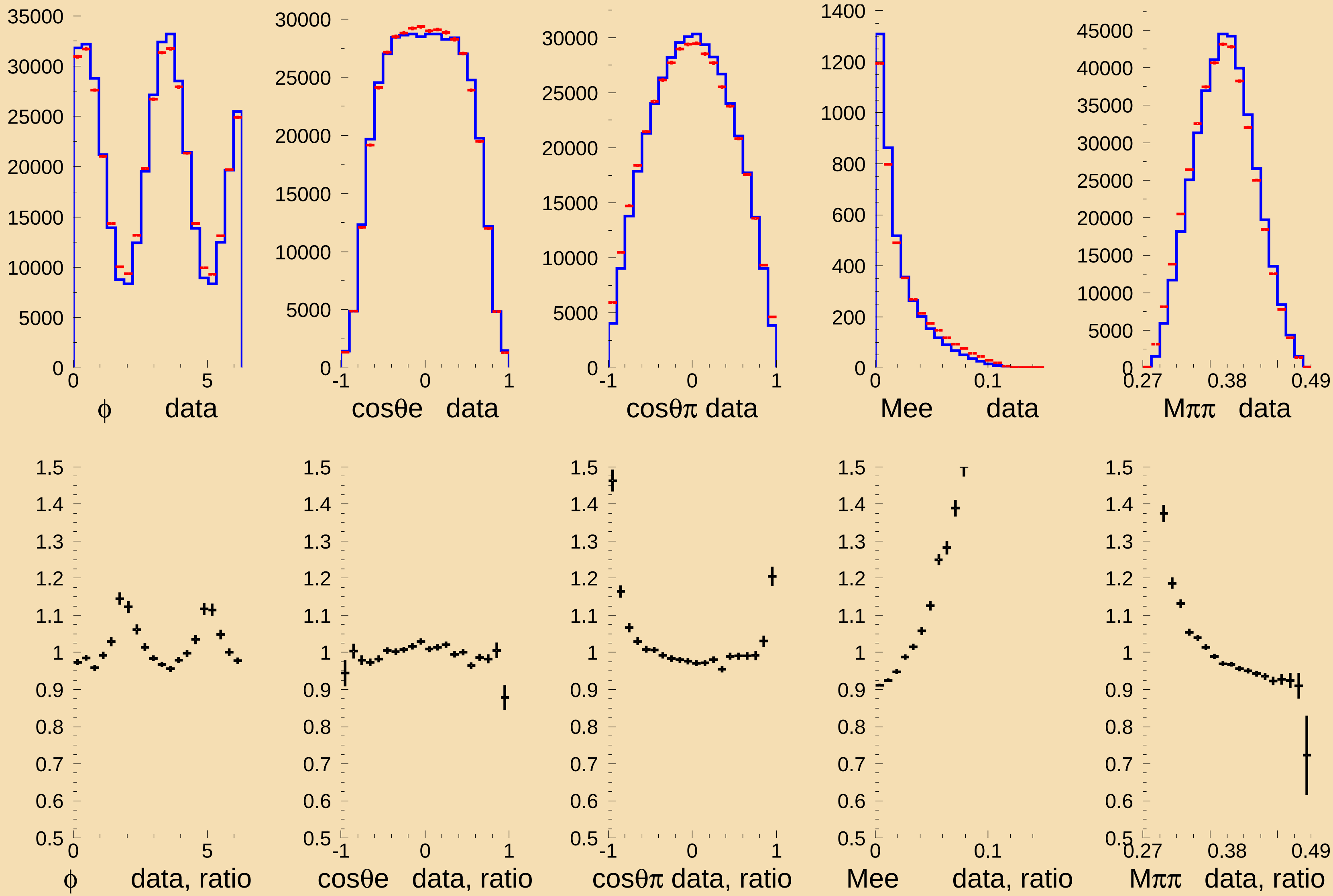
- To avoid doing that we use the **trick: re-weighting technique**. It can be shown that

$$log\mathcal{L}(\alpha) = \sum_{i=1}^{N_d} log\mu(x_i;\alpha) - N_d \cdot log \sum_{j=1}^{N_{mc}} \frac{\mu(x_j;\alpha)}{\mu(x_j;\alpha_0)} + f(x)$$

  where f is function of x only and does not depend on α. We generate **one large MC sample** for a certain choice of $\alpha = \alpha_0$ and then re-weight it to obtain $\mathcal{L}(\alpha)$ for all other values of α.

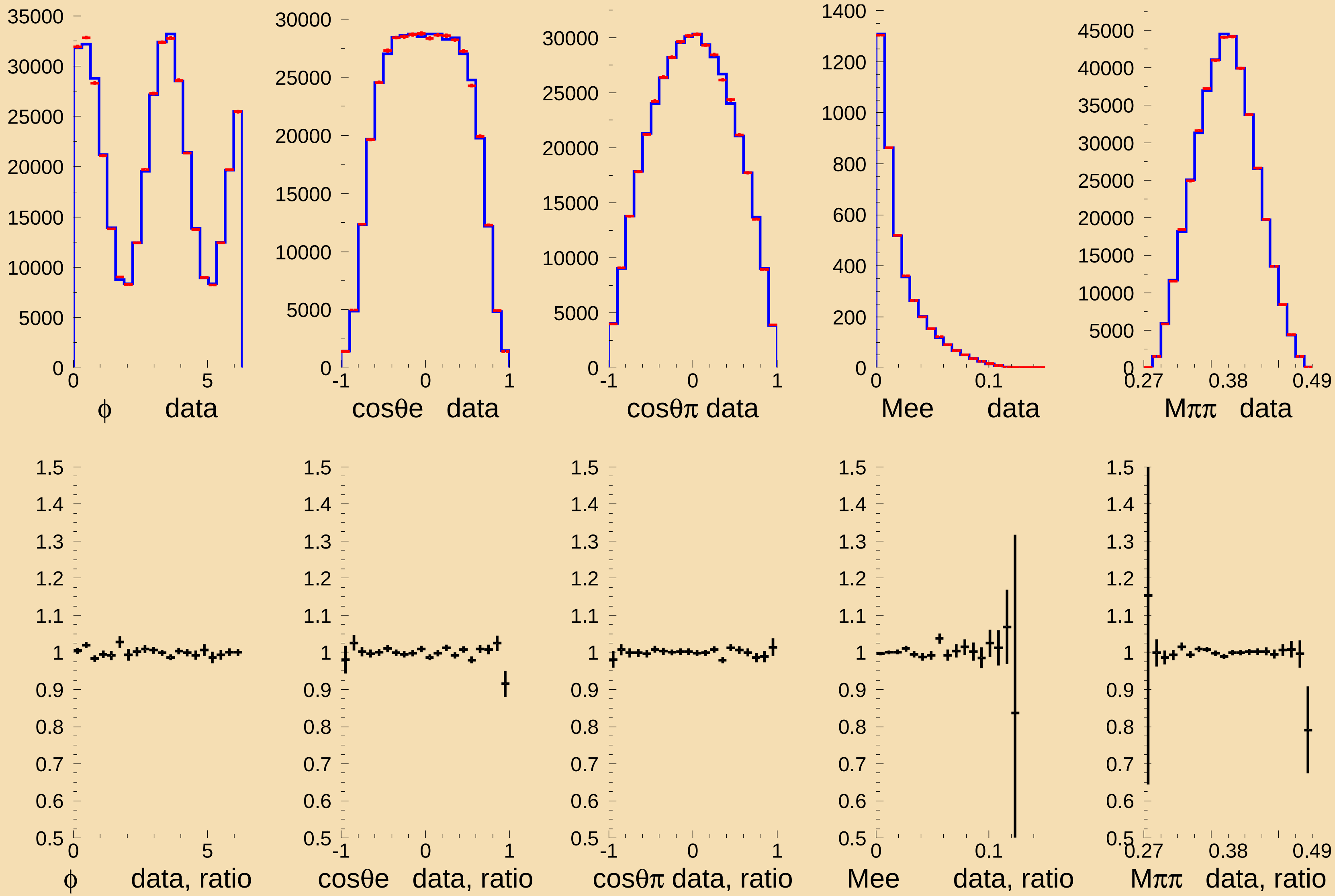
- To calculate μ we use **the routine from KTEVMC**, which was used during the generation.

5. Check The Re-weighting: MC and MC′ with $\alpha \neq \alpha'$



blue: $g_{CR}= .03$, red: $g_{CR}= .29$, black: red/blue

6. Re-weighting Works Now: MC re-weighted to MC′



blue: g_{CR} = .03, red: g_{CR} = .29, re-weighted to g_{CR} = .03, black: red/blue

7. But it Didn't Work Before.

√ The assumption had been made, when calculating the weights for the re-weighting technique, that the variables used in the KTEVMC matrix element subroutine were the "Sehgal" variables (which we use for all plots in order to be consistent in our definitions with Sehgal's previously published papers).

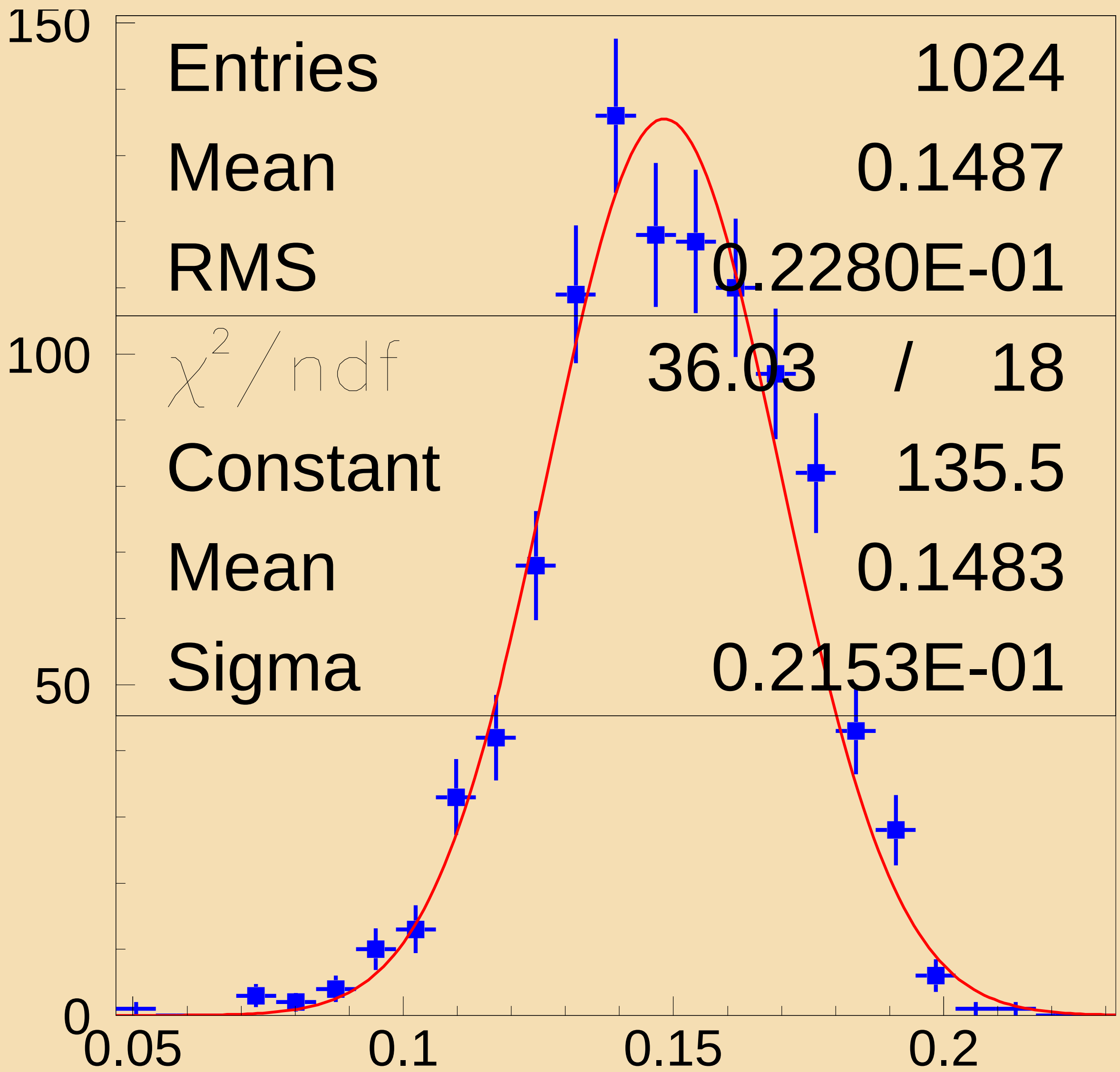
√ However, the KTEVMC defines all θ-angles w.r.t. $\pi\pi$ direction (Z direction). Sehgal defines θ_e similar to KTEVMC, w.r.t. $\pi\pi$ direction, but he defines θ_π w.r.t. ee direction (-Z direction). So we have

$$\theta_e^{KTEVMC} = \pi - \theta_e^{Sehgal}$$

√ **To make the re-weighting work one needs to make the following transformation** before calculating the matrix element using the KTEVMC subroutine:

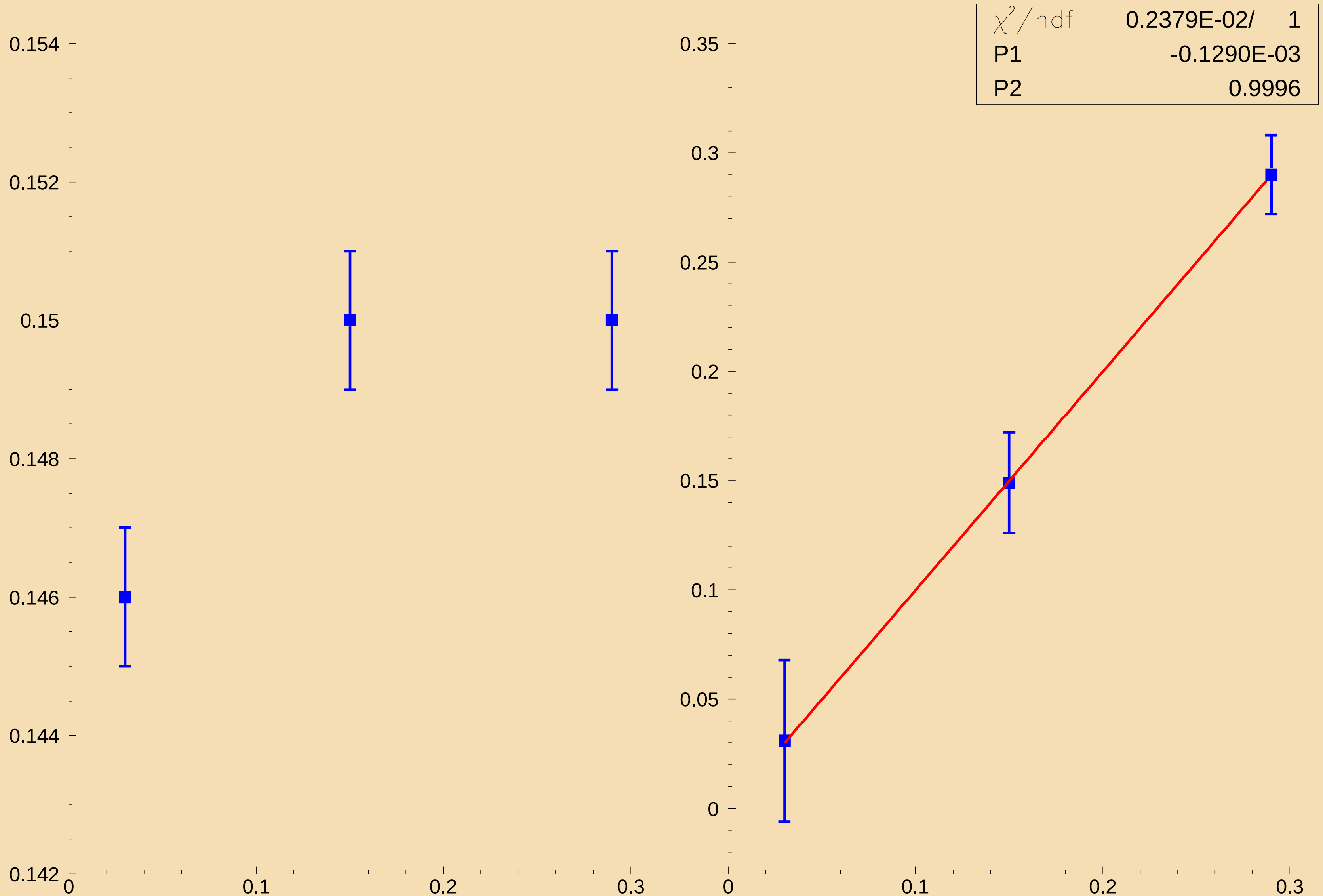
$$cos\theta_e' = -cos\theta_e \text{ and } \phi' = \phi \pm \pi$$

8. Repeat 1-par Fit for g_{CR} ('97 only): Test on Fake Data



$g_{CRin} = .15$ (blue: distribution of g_{CRout}, red: it's fit to a Gaussian)

9. 1-par Fit for g_{CR} : Test For Biases (Fake Data)



g_{CRout} versus g_{CR0} and g_{CRin}

10. 1-par Fit for g_{CR} : Check Systematics

Source	Uncertainty on the Parameter Δg_{CR}
Background	?
Variation of Cuts	?
Limited MC	.001
$\Delta\eta_{+-}$	.001
$\Delta\Phi_{+-}$	.0004
syst1	.001
compare to stat	.022

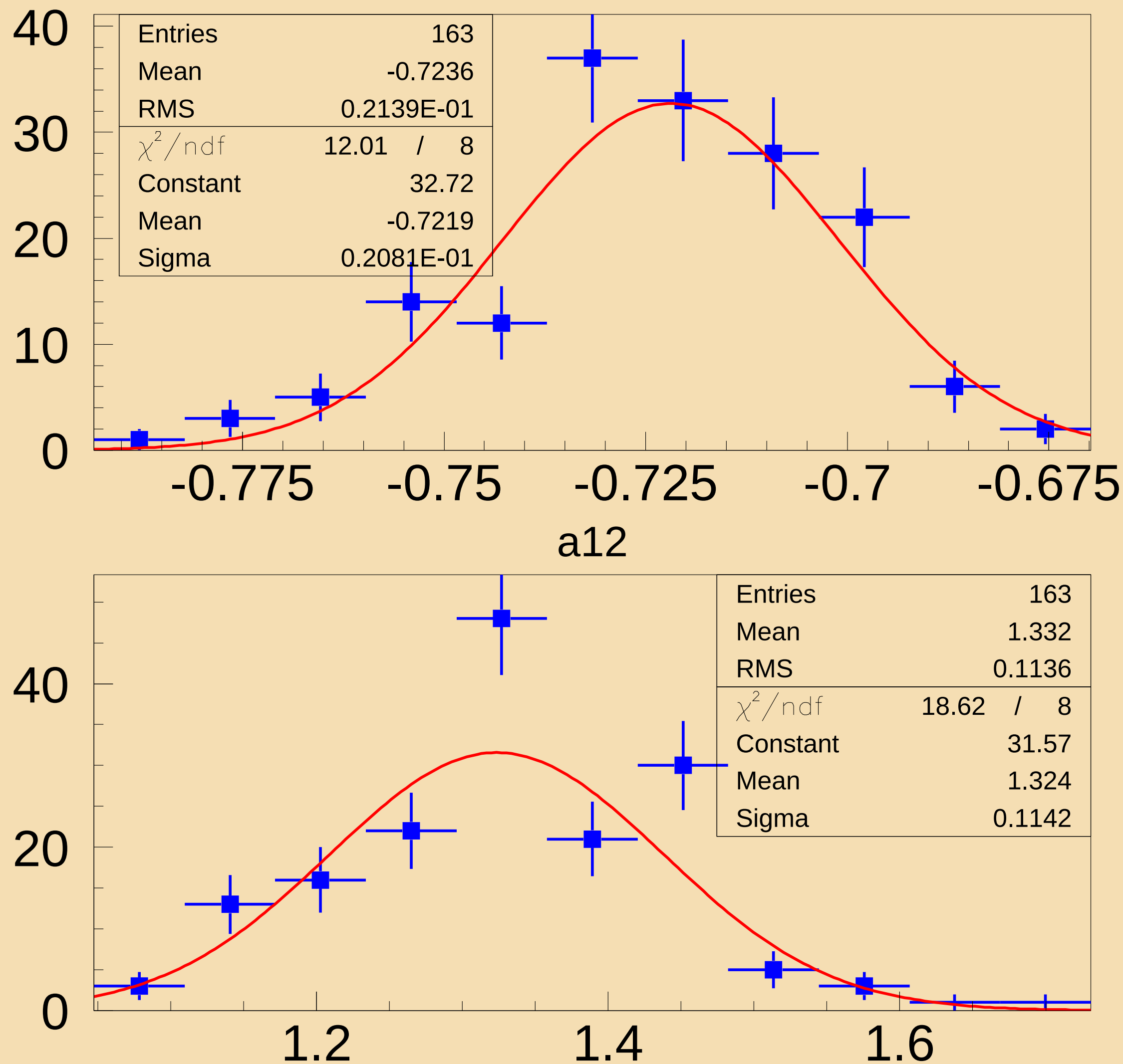
√ Note that **syst1** errors do not include cut variation, background and g_{CR0} effect. This still remains to be done, call it **syst2**.

11. The 1-par Fit for g_{CR}: Result

$$\mathbf{g_{CR} = 0.172 \pm 0.022(stat) \pm 0.001(syst1) \pm \ldots (syst2)}$$

('97 only, $\boldsymbol{g_{E_1}}$ = .038, $\boldsymbol{g_{M_1}}$ = 1.35, $\boldsymbol{\frac{a_1}{a_2}}$ = -.72)

12. Repeat 2-par Fit for g_{M_1} and $\frac{a_1}{a_2}$ ('97 + '99, DPF'02)



$g_{M_1 in} = 1.35$, $\frac{a_1}{a_2}{}_{in} = -.72$ (blue: distributions for fake data, red: their fit to a Gaussian)

13. 2-par Fit for g_{M_1} and $\frac{a_1}{a_2}$: Check Systematics

Source	Uncertainty on the Parameter	
	Δg_{M_1}	$\Delta \frac{a_1}{a_2}$
Background	?	?
Variation of Cuts	?	?
Limited MC	.01	.002
$\Delta \eta_{+-}$	.005	.002
$\Delta \Phi_{+-}$	.003	.001
syst1	.01	.003
stat	.11	.021

√ Note that **syst1** errors do not include cut variation, background and g_{CR0} effect. This still remains to be done, call it **syst2**.

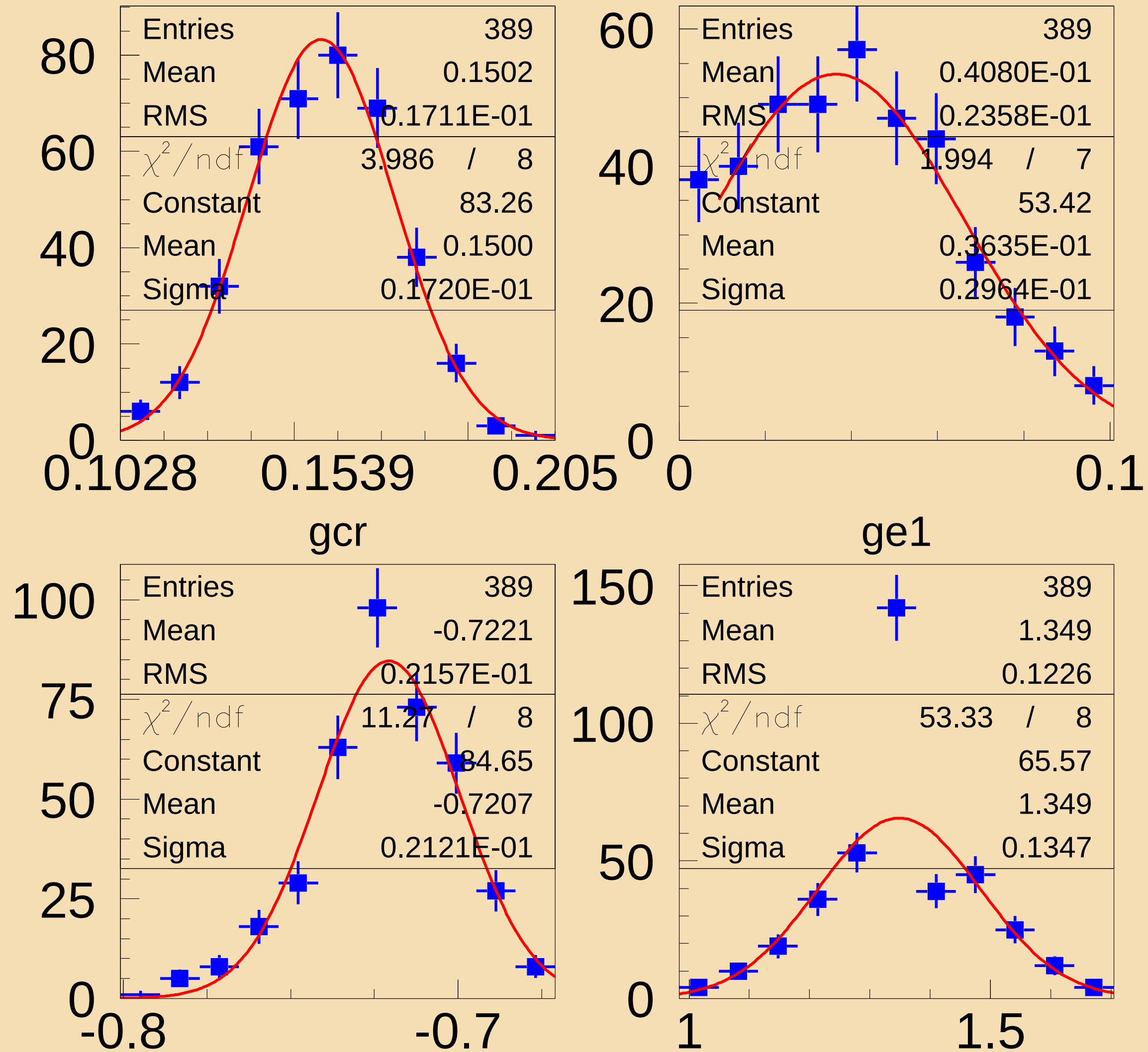
14. The 2-par Fit for g_{M_1} and $\frac{a_1}{a_2}$: Result

$$\mathbf{g_{M1} = 1.16 \pm 0.11(stat) \pm 0.01(syst1) \pm \ldots (syst2)}$$
$$\mathbf{\frac{a_1}{a_2} = -0.737 \pm 0.021(stat) \pm 0.003(syst1) \pm \ldots (syst2)}$$

(’97 + ’99, g_{E_1}= .038, g_{CR}= .15)

$$\mathbf{g_{M1} = 1.10 \pm 0.10(stat) \pm 0.06(syst)[DPF'02]}$$
$$\mathbf{\frac{a_1}{a_2} = -0.75 \pm 0.03(stat) \pm 0.02(syst)[DPF'02]}$$

15. Illustration of Statistical Uncertainty for 4-par Fit



$$g_{CRin} = .15,\ g_{E_1 in} = .038,\ g_{M_1 in} = 1.35,\ \frac{a_1}{a_2}in = -.72$$

(blue: distributions for fake data, red: their fit to a Gaussian)

16. Check Systematics for 4 Parameter Fit

Source	Uncertainty on the Parameter			
	Δg_{CR}	$\Delta \frac{\lvert g_{E_1}\rvert}{\lvert g_{M_1}\rvert}$	Δg_{M_1}	$\Delta \frac{a_1}{a_2}$
Background	?	?	?	?
Variation of Cuts	?	?	?	?
Limited MC	.001	.001	.01	.002
$\Delta\eta_{+-}$	.002	.0002	.01	.0001
$\Delta\Phi_{+-}$	.0005	.0005	.005	.0006
syst1	.002	.001	.02	.002
stat	.017	.028	.12	.022

√ Note that **syst1** errors do not include cut variation, background and g_{CR0} effect. This still remains to be done, call it **syst2**.

17. Summary of the Fit Results

- Repeated 1 parameter fit for g_{CR} ('97 only, g_{E_1}= .038, g_{M_1}= 1.35, $\frac{a_1}{a_2}$= -.72)

$$\mathbf{g_{CR} = 0.172 \pm 0.022(stat) \pm 0.001(syst1) \pm \ldots (syst2)}$$

- Repeated 2 parameter fit for g_{M_1} and $\frac{a_1}{a_2}$ ('97 + '99, g_{E_1}= .038, g_{CR}= .15)

$$\mathbf{g_{M1} = 1.16 \pm 0.11(stat) \pm 0.01(syst1) \pm \ldots (syst2)}$$

$$\mathbf{\frac{a_1}{a_2} = -0.737 \pm 0.021(stat) \pm 0.003(syst1) \pm \ldots (syst2)}$$

$$\mathbf{g_{M1} = 1.10 \pm 0.10(stat) \pm 0.06(syst)[DPF'02]}$$

$$\mathbf{\frac{a_1}{a_2} = -0.75 \pm 0.03(stat) \pm 0.02(syst)[DPF'02]}$$

- New results of the 4 parameter fit ('97 +'99)

$$\mathbf{g_{CR} = 0.161 \pm 0.017(stat) \pm .002(syst1) \pm \ldots (syst2)}$$

$$\mathbf{\frac{|g_{E1}|}{|g_{M1}|} < 0.029 + \ldots (syst2)}$$

$$\mathbf{g_{M1} = 1.16 \pm 0.12(stat) \pm 0.02(syst1) \pm \ldots (syst2)}$$

$$\mathbf{\frac{a_1}{a_2} = -0.729 \pm 0.022(stat) \pm 0.002(syst1) \pm \ldots (syst2)}$$

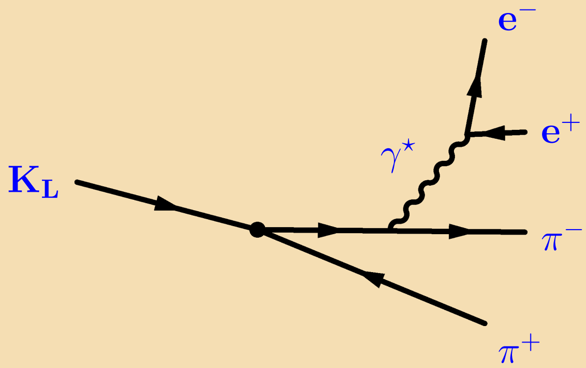
18. Conclusions and Plans

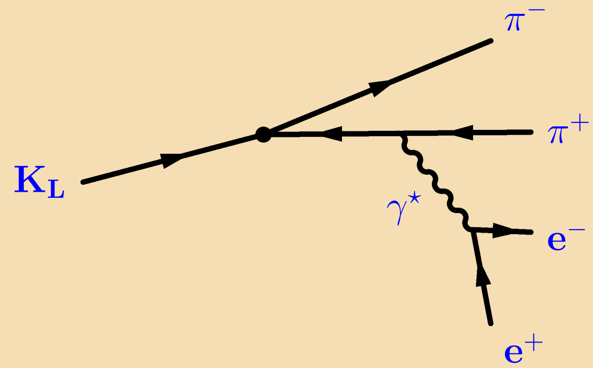
- √ Re-weighting now really works!
- √ New Values for FF's are measured from the 4 parameter fit.
- √ Analysis of the systematic uncertainty needs to be completed.
- √ The next step is to measure the asymmetry, branching ratio and the CPT phase.

19. Reference Slides

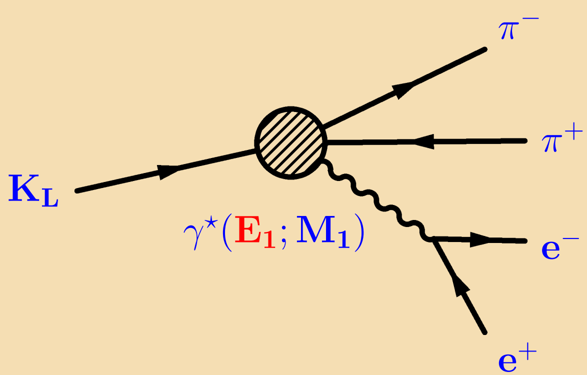
√ On the **next pages** there are some reference slides.

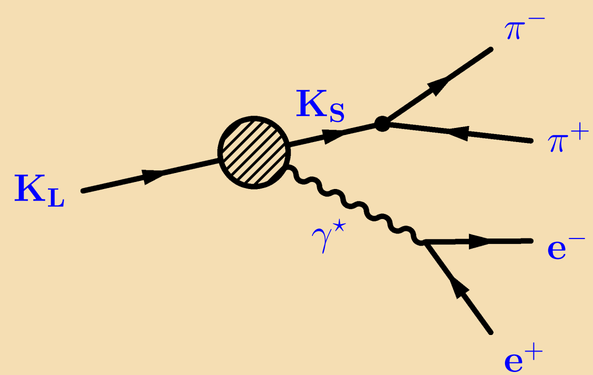
20. Contributions to the Decay $K_L \to \pi^+\pi^-e^+e^-$





Inner Bremsstrahlung (IB) — *Indirect $\not{CP}$*





Direct Emission (DE)
$\mathrm{E_1}$ — ***Indirect $\not{CP}$***
$\mathrm{M_1}$ — ***CP Conserving***

$\mathrm{K^0}$ Charge Radius (CR)
CP Conserving

21. Expressions for the Form Factors

- **Inner Bremsstrahlung**: $g_{IB} = |\eta_{+-}| e^{i(\delta_0(M_K) + \Phi_{+-})}$
- M_1 **Direct Emission**: $g_{M_1} = i e^{i\delta_1(M_{\pi\pi})} \times |g_{M_1}|$,

  where

$$|g_{M_1}| \equiv \tilde{g}_{M_1} \left[1 + \frac{a1/a2}{(M_\rho^2 - M_K^2) + 2 M_K E_{ee}} \right]$$

- E_1 **Direct Emission**: $g_{E_1} = \frac{|g_{E_1}|}{|g_{M_1}|} e^{i(\delta_1(M_{\pi\pi}) + \Phi_{+-})} \times |g_{M_1}|$
- **Charge Radius**: $g_{CR} = |g_{CR}| e^{i\delta_0(M_{\pi\pi})}$,

  where $|g_{CR}| = -\frac{1}{3} \langle R^2(K^0) \rangle M_K^2$

22. History of $K_L \to \pi^+\pi^-e^+e^-$ Measurements

When?	Measured Values					
	$\tilde{g}_{M_1}$	a_1/a_2, GeV^2/c^2	$\|g_{CR}\|$	$\|g_{E_1}\|$	$\mathcal{A}$, %	$\mathcal{BR}$, $\times 10^{-7}$
Before KTeV	F = 0.76		0.15	0.038	-	-
one day, *PRL(1996)*	-	-	-	-	-	$3.2 \pm .6$
Winter, ICHEP98	-	-	-	-	-	$3.32 \pm .14$
'97, EPS HEP99	-	-	-	-	-	$3.63 \pm .11$
'97, *PRL(2000)*	$1.35 \pm .20$	$-.72 \pm .03$	-	-	13.6 ± 2.5	-
'96, *PRL(2001)*	-	$-.734 \pm .034$	-	-	-	-
'97, BCP4(2001)	-	-	$.100 \pm .018$	-	-	-
'97+'99, DPF2002	$1.10 \pm .10$	$-.75 \pm .03$	-	-	13.3 ± 1.4	-
", ", Madison	$1.20 \pm .13$	$-.73 \pm .03$	$.19 \pm .01$	-	-	-
", ", Sept 2002	$1.15 \pm .12$	$-.73 \pm .02$	$.18 \pm .02$	$< .03$	-	-
", ", Jan 2003	$1.14 \pm .12$	$-.73 \pm .02$	$.20 \pm .01$	$.09 \pm .03$	14.1 ± 1.4	-
", ", March 2003	$1.27 \pm .12$	$-.71 \pm .02$	$.25 \pm .01$	$.14 \pm .03$	13.9 ± 1.4	$3.67 \pm .07$
", ", today	$1.16 \pm .12$	$-.73 \pm .02$	$.16 \pm .02$	$< .03$	-	-