Update on Measurements of $K_L \to \pi^+\pi^-e^+e^-$

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KTeV Collaboration Meeting
Fermilab

Contents

1. Introduction

1.1. Expressions for the Amplitudes

- Inner Bremsstrahlung: $\mathbf{g_{IB}} = |\eta_{+-}| e^{i\delta_0(M_K) + \Phi_{+-}}$
- M_1 Direct Emission: $\mathbf{g_{M_1}} = i e^{i\delta_1(M_{\pi\pi}) + \Phi_{+-}} \times \mathbf{F}\left(\frac{a_1}{a_2}; \tilde{g}_{M_1}\right)$,

  where

$$\mathbf{F} = \tilde{\mathbf{g}}_{\mathbf{M_1}} \left[1 + \frac{\mathbf{a1/a2}}{(M_\rho^2 - M_K^2) + 2M_K E_{ee}} \right]$$

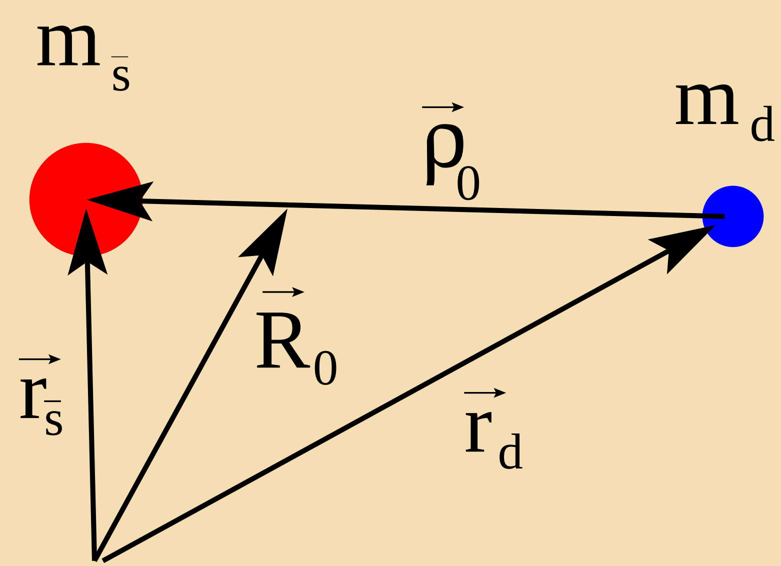
- E_1 Direct Emission: $\mathbf{g_{E_1}} = i|\mathbf{g_{E_1}}| e^{i\delta_1(M_{\pi\pi}) + \Phi_{+-}} \times \mathbf{F}\left(\frac{a_1}{a_2}; \tilde{g}_{M_1}\right)$
- Charge Radius: $\mathbf{g_{CR}} = |\mathbf{g_{CR}}| e^{i\delta_0(M_{\pi\pi})}$,

  where $|\mathbf{g_{CR}}| = -\frac{1}{3}\langle \mathbf{R^2(K^0)} \rangle M_K^2$

1.2. The Measurement of K_L Charge Radius

$\mathbf{K^0}$ consists $\bar{\mathrm{s}}$ and **d** quark, the relative and center-of-mass coordinates are

$$\vec{\rho}_0 \equiv \vec{\mathbf{r}}_{\bar{s}} - \vec{\mathbf{r}}_d; \qquad \vec{\mathbf{R}}_0 \equiv \frac{\mathbf{m_s \vec{r}_{\bar{s}} + m_d \vec{r}_d}}{\mathbf{m_s + m_d}}$$



Heavier strange quark is confined to a smaller radius thus giving $\mathbf{K^0}$ a positively charged core:

$$\left\langle \mathbf{R^2} \right\rangle \equiv \left\langle \sum \mathbf{q_i} (\vec{\mathbf{r}}_i - \vec{\mathbf{R}}_0)^2 \right\rangle = -\frac{1}{3}\frac{\mathbf{m_s - m_d}}{\mathbf{m_s + m_d}} \left\langle \rho_0^2 \right\rangle$$

On the other hand $\langle \mathbf{R^2} \rangle$ is part of the expression for $\mathbf{g_{CR}}$:

$$\mathbf{g_{CR}} = -\frac{1}{3} \left\langle \mathbf{R^2} \left(\mathbf{K^0} \right) \right\rangle \mathbf{M_K^2} e^{i\delta_0(M_{\pi\pi})}; \qquad |\mathbf{g_{CR}}| \equiv -\frac{1}{3} \left\langle \mathbf{R^2} \left(\mathbf{K^0} \right) \right\rangle \mathbf{M_K^2}$$

KTeV Preliminary Measurement ***(Work by Sasha Ledovskoy on '97 Data)***:

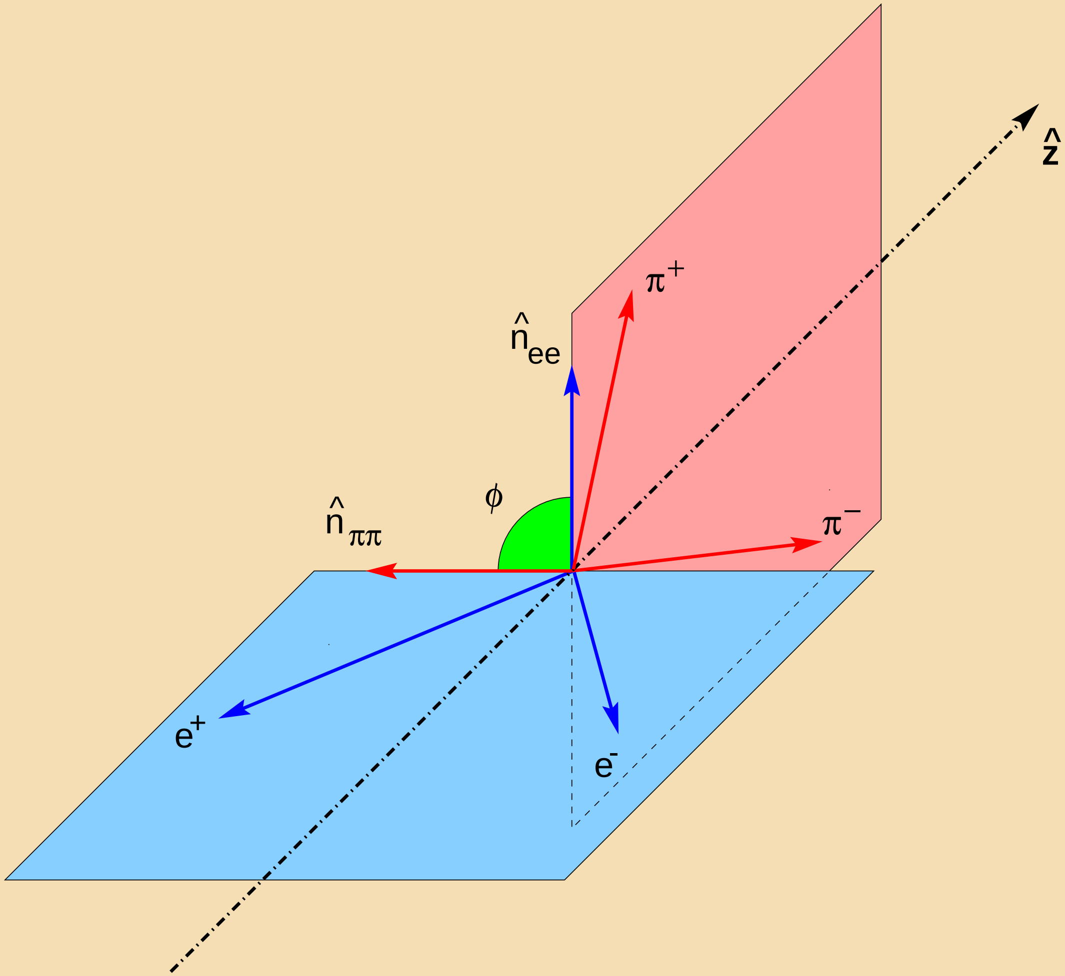
$$|\mathbf{g_{CR}}| = \mathbf{0.100 \pm 0.018 \pm 0.013}; \qquad \left\langle \mathbf{R^2 (K^0)} \right\rangle = \mathbf{-0.047 \pm 0.008 \pm 0.006 [fm^2]}$$

1.3. The CP Violating Asymmetry in the Angular Variable

It can be shown *(Sehgal and Wanninger, Phys.Rev.D.* ***46****, 1035 (1992))* that the polarization of the photon will manifest itself as an asymmetry in the angle ϕ.

Both $\hat{n}_{ee}$ and $\hat{n}_{\pi\pi}$ are axial vectors while $\hat{z}$ is polar vector and therefore $\sin\phi\cos\phi$ is odd under CP and T transformations. The asymmetry is large and theory predicts the value of $\sim 14\%$. It can be defined as follows:

$$\mathcal{A}(\phi) \equiv \frac{\mathbf{N}_{\sin\phi\cos\phi>\mathbf{0}} - \mathbf{N}_{\sin\phi\cos\phi<\mathbf{0}}}{\mathbf{N}_{\sin\phi\cos\phi>\mathbf{0}} + \mathbf{N}_{\sin\phi\cos\phi<\mathbf{0}}}$$



1.4. History of $K_L \to \pi^+\pi^- e^+e^-$ Measurements

Large **When?**	**Measured Values**					
	$\tilde{\mathbf{g}}_{\mathbf{M_1}}$	$\mathbf{a_1/a_2}$, GeV^2/c^2	$\lvert\mathbf{g_{CR}}\rvert$	$\lvert\mathbf{g_{E_1}}\rvert$	$\mathcal{A}$, %	$\mathcal{BR}$, $\times 10^{-7}$
Before KTeV	$\mathbf{F = 0.76}$		$\mathbf{0.15}$	$\mathbf{0.038}$	-	-
one day, *PRL(1996)*	-	-	-	-	-	$\mathbf{3.2 \pm .6}$
Winter, ICHEP98	-	-	-	-	-	$\mathbf{3.32 \pm .14}$
'97, EPS HEP99	-	-	-	-	-	$\mathbf{3.63 \pm .11}$
'97, *PRL(2000)*	$\mathbf{1.35 \pm .20}$	$\mathbf{-.72 \pm .03}$	-	-	$\mathbf{13.6 \pm 2.5}$	-
'96, *PRL(2001)*	-	$\mathbf{-.734 \pm .034}$	-	-	-	-
'97, BCP4(2001)	-	-	$\mathbf{.100 \pm .018}$	-	-	-
'97+'99, DPF2002	$\mathbf{1.10 \pm .10}$	$\mathbf{-.75 \pm .03}$	-	-	$\mathbf{13.3 \pm 1.4}$	-
", ", Madison	$\mathbf{1.20 \pm .13}$	$\mathbf{-.73 \pm .03}$	$\mathbf{.19 \pm .01}$	-	-	-
", ", Sept 2002	$\mathbf{1.15 \pm .12}$	$\mathbf{-.73 \pm .02}$	$\mathbf{.18 \pm .02}$	$\mathbf{< .03}$	-	-
", ", today	$\mathbf{1.14 \pm .12}$	$\mathbf{-.73 \pm .02}$	$\mathbf{.20 \pm .01}$	$\mathbf{.09 \pm .03}$	$\mathbf{14.1 \pm 1.4}$	-

2. $\pi\pi$ S- and P-wave Phase Shifts

2.1. New Functions ($s \equiv M^2_{\pi\pi}$)

$$\tan\delta^I_\ell = \sqrt{1 - \frac{4M^2_\pi}{s}}\, q^{2\ell}\left\{A^I_\ell + B^I_\ell q^2 + C^I_\ell q^4 + D^I_\ell q^6\right\}\left(\frac{4M^2_\pi - s^I_\ell}{s - s^I_\ell}\right), \tag{17.1}$$

with $s = 4(M^2_\pi + q^2)$. The numerical values of the coefficients are:

$$\begin{array}{lll} A^0_0 = 0.220, & A^1_1 = 0.379 \times 10^{-1}, & A^2_0 = -0.444 \times 10^{-1}, \\ B^0_0 = 0.268, & B^1_1 = 0.140 \times 10^{-4}, & B^2_0 = -0.857 \times 10^{-1}, \\ C^0_0 = -0.139 \times 10^{-1}, & C^1_1 = -0.673 \times 10^{-4}, & C^2_0 = -0.221 \times 10^{-2}, \\ D^0_0 = -0.139 \times 10^{-2}, & D^1_1 = 0.163 \times 10^{-7}, & D^2_0 = -0.129 \times 10^{-3}, \end{array} \tag{17.2}$$

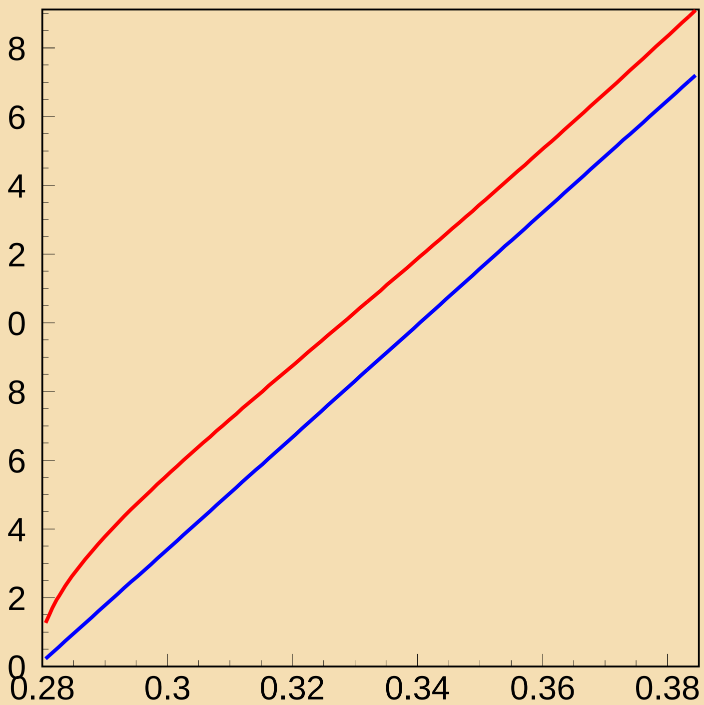
in units of M_π. In particular, the constants A^I_ℓ represent the scattering lengths of the three partial waves under consideration, while the B^I_ℓ are related to the effective ranges.

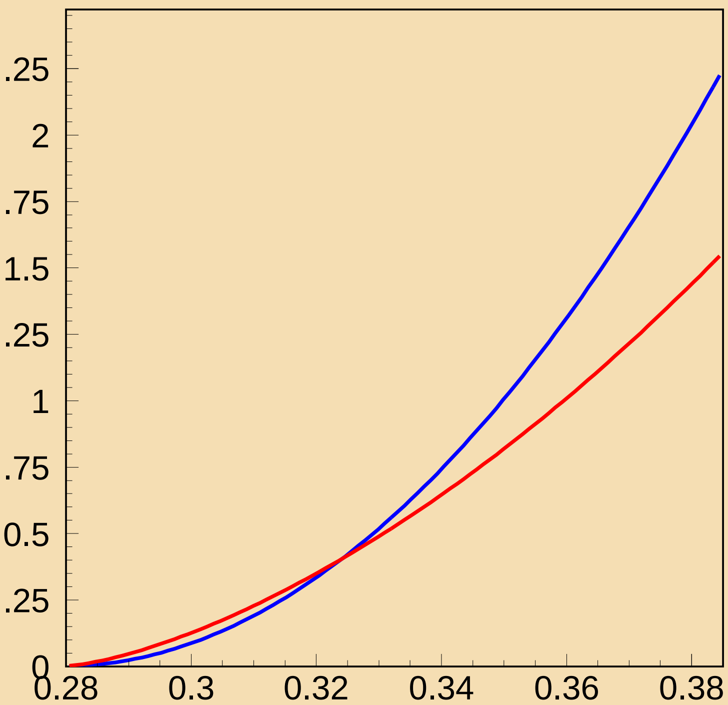
The parameters s^I_ℓ specify the value of s where $\delta^I_\ell(s)$ passes through 90°:

$$s^0_0 = 36.77 M^2_\pi, \qquad s^1_1 = 30.72 M^2_\pi, \qquad s^2_0 = -21.62 M^2_\pi. \tag{17.3}$$

G. Colangelo et al. Nuclear Physics B 603, 125-179, 2001

2.2. Compare Old Shifts Versus New Ones

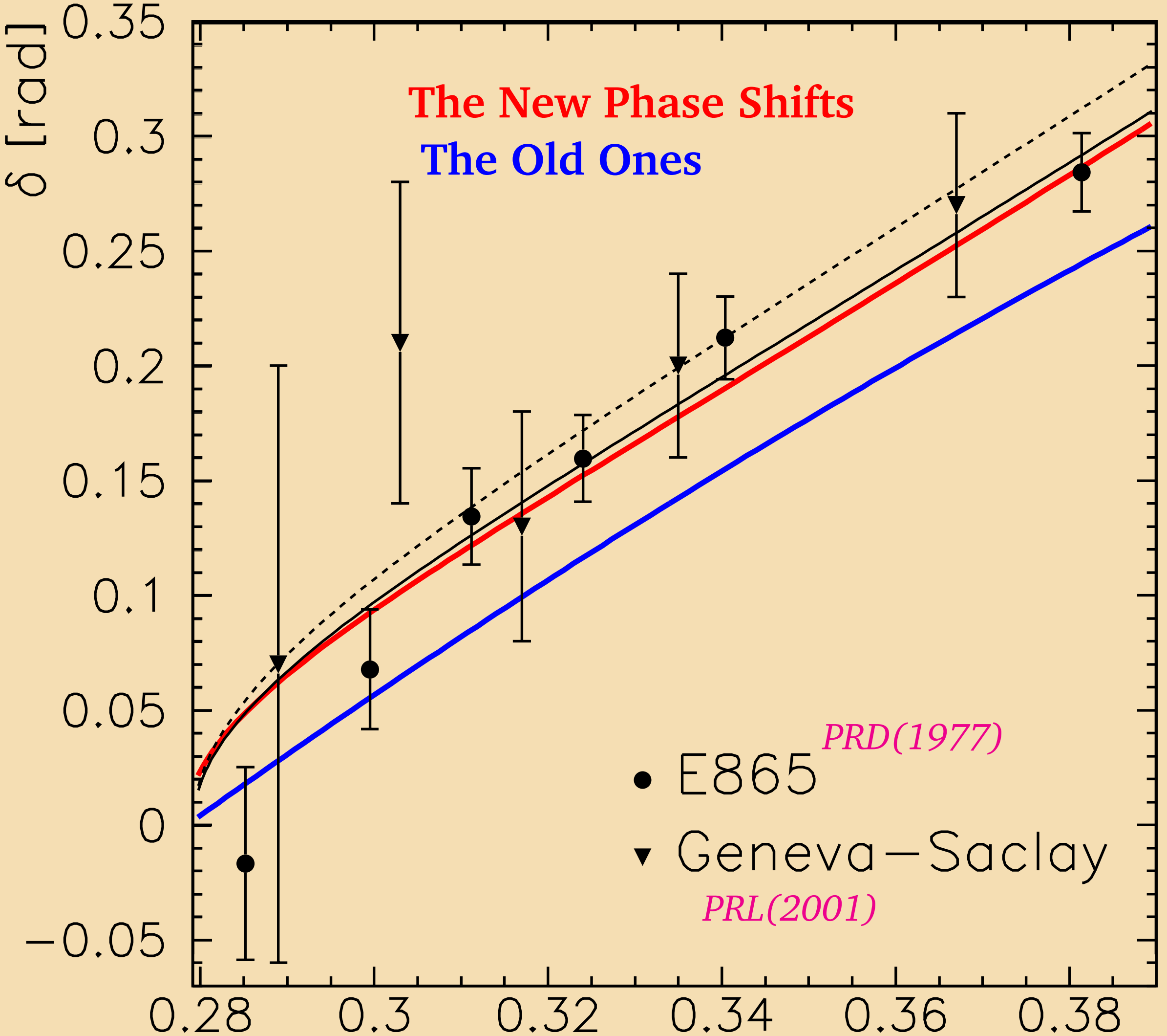




The **old functions** are very simple:

$$\boldsymbol{\delta_0^0 = 2.85(\mathrm{M}_{\pi\pi} - 2\mathrm{M}_{\pi}) \times \frac{180}{\pi}} \quad \text{and} \quad \boldsymbol{\delta_1^1 = 3.5(\mathrm{M}_{\pi\pi} - 2\mathrm{M}_{\pi})^2 \times \frac{180}{\pi}}$$

2.3. The Comparison to Data



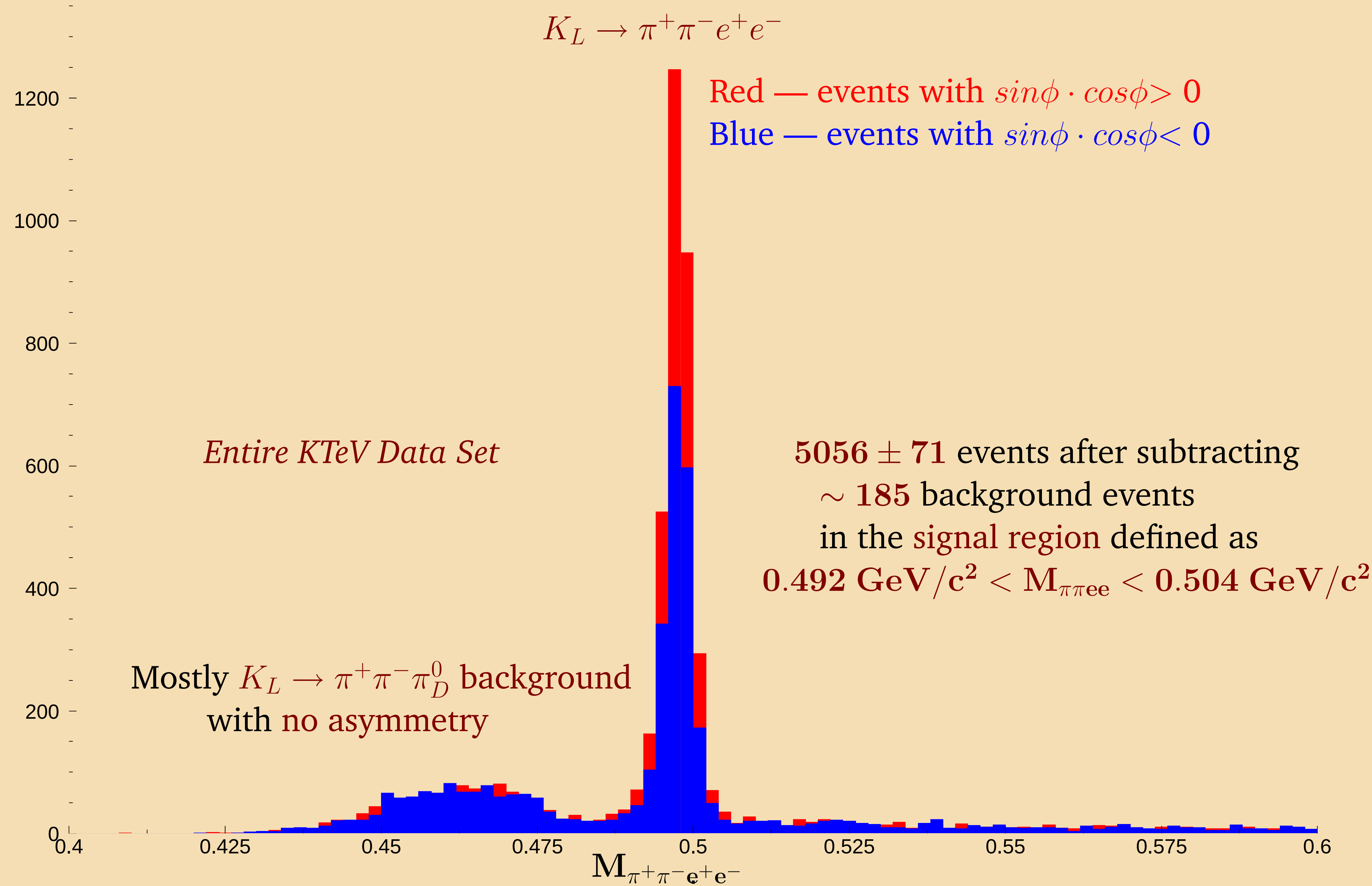
3. The Data

3.1. Summary of Main Cuts

- Event has 4 tracks
- Particle ID: electrons if $0.95 < \frac{E}{P} < 1.05$ and pions if $\frac{E}{P} < 0.9$ or $\frac{E}{P} > 1.1$
- $P^2_{\pi^0} < -0.025 \text{ GeV}^2/c^2$
- $M_{ee} > 0.002 \text{ GeV}/c^2$
- $95m < Z_{vtx} < 158m$
- $P^2_t < 6 \times 10^{-5} \text{ GeV}^2/c^2$
- $E_{\pi\pi ee} < 200 \text{ GeV}$
- $0.492 \text{ GeV}/c^2 < M_{\pi\pi ee} < 0.504 \text{ GeV}/c^2$

The residual background under the mass peak was estimated by a fitting procedure.

3.2. The Final Event Sample



4. The Four-Parameter Fit

4.1. The Method

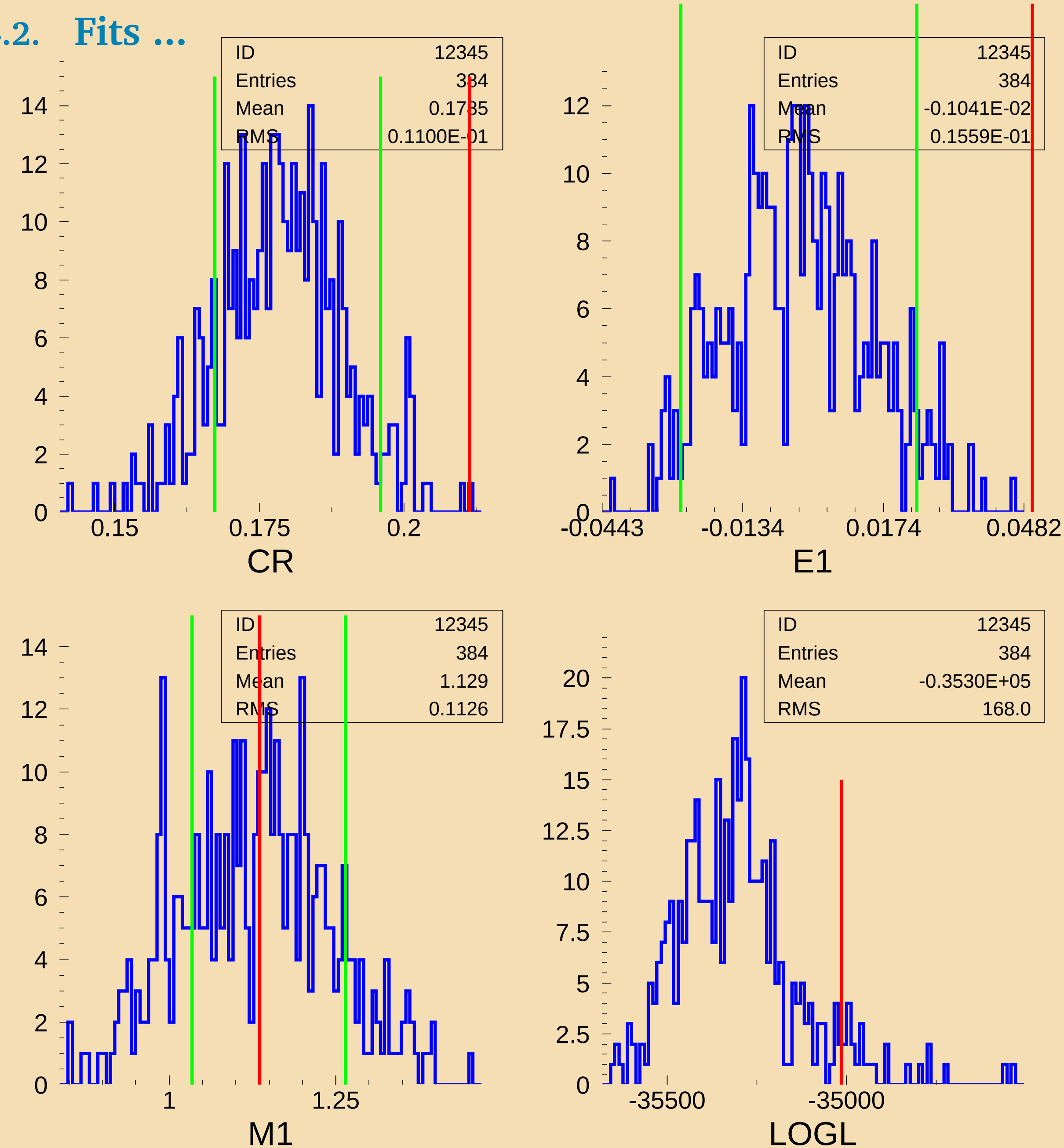
- Use Maximum Likelihood Method to estimate the parameters. The logarithm of the likelihood function can be written in terms of the relative weights of the event (data and Monte Carlo)

$$\mathbf{log}\mathcal{L}\left(\vec{\alpha}\right) = \left[\sum_{\mathbf{i=1}}^{\mathbf{N_d}} \mathbf{log w_i}\left(\vec{\alpha}, \vec{\mathbf{x}}\right)\right] - \mathbf{N_d log}\sum_{\mathbf{j=1}}^{\mathbf{N_{mc}}} \frac{\mathbf{w_i}(\vec{\alpha}, \vec{\mathbf{x}})}{\mathbf{w_i}(\vec{\alpha_0}, \vec{\mathbf{x}})}$$

  where $\vec{\mathbf{x}}$ is the vector of measured variables and $\vec{\alpha}$ is the vector of parameters to be estimated, i.e.

$$\vec{\alpha} = \left(\frac{\mathbf{a_1}}{\mathbf{a_2}}; \mathbf{g_{M_1}}; \mathbf{g_{CR}}; \mathbf{g_{E1}}\right); \qquad \vec{\mathbf{x}} = \left(\phi, \theta_{\mathbf{e}^+}, \theta_{\pi^-}, \mathbf{M}_{\pi\pi}, \mathbf{M_{ee}}\right)$$

- Generate one Monte Carlo sample for a set of values of the parameters $\vec{\alpha_0}$ and then re-weighted each event for any other set of floating parameters.
- Use Custom Fitter (code from Sasha Ledovskoy and "Numerical Recipes"), which uses *Powell Algorithm* to minimize the $\mathbf{log}\mathcal{L}\left(\vec{\alpha}\right)$



Done 384 + 1 Fits

blue - “Fake” Data (MC)

red - Real Data

green - Errors

5. Conclusions and Plans

- With the final sample of $\mathbf{5056 \pm 71\ K_L \to \pi^+\pi^- e^+ e^-}$ events made a four-parameter fit, including $\mathbf{g_{CR}}$ (which is the measurement of K^0 Charge Radius) and $\mathbf{g_{E1}}$.

$$\frac{\mathbf{a_1}}{\mathbf{a_2}} = \mathbf{-0.73 \pm 0.02 \pm \ldots} \qquad \mathbf{g_{M1} = 1.14 \pm 0.12 \pm \ldots}$$

$$\mathbf{g_{CR} = 0.20 \pm 0.02 \pm \ldots} \qquad \mathbf{g_{E1} = 0.09 \pm 0.03 \pm \ldots}$$

$$\mathcal{A} = (\mathbf{14.1 \pm 1.4(stat) \pm \ldots})\,\%$$

$$\langle \mathbf{R^2(K^0)} \rangle \approx \mathbf{-0.094 \pm 0.01 \pm \ldots fm^2}$$

- Plans and Future Prospectives:
  - Estimate systematic errors on all parameters.
  - Branching Ratio, then finalize, write up and publish ...