Update on Measurements of $K_L \to \pi^+\pi^-e^+e^-$

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Fermilab

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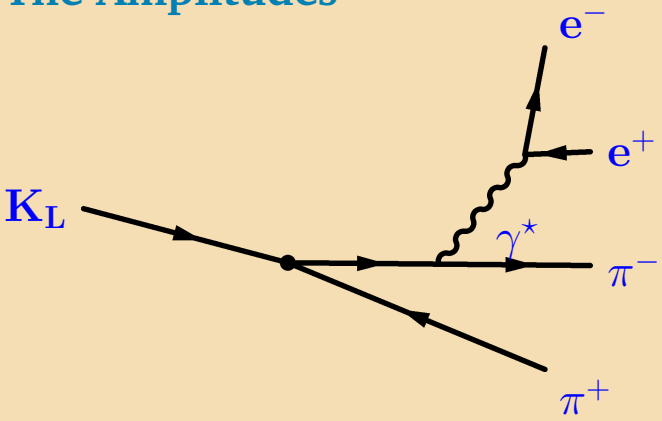
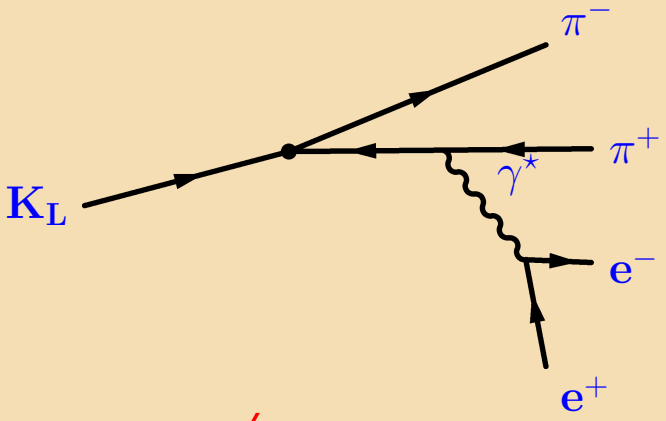
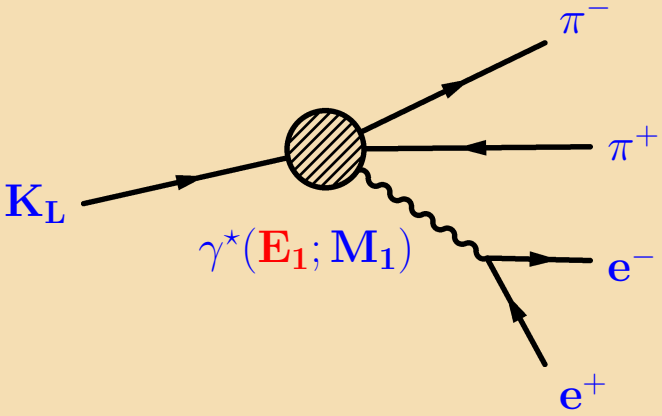
1. Motivation and Current Measurements

1.1. The Amplitudes





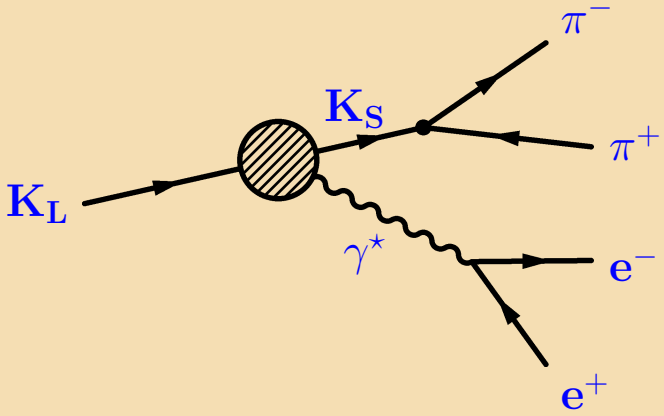
Inner Bremsstrahlung (IB) — *Indirect $\not{C}\not{P}$*



Direct Emission (DE)

E_1 — *Indirect $\not{C}\not{P}$*

M_1 — *CP Conserving*



K^0 Charge Radius (CR)

CP Conserving

1.2. History of FF's Measurements

Who and When?	Parameters Measured			
	$\tilde{\mathbf{g}}_{\mathbf{M_1}}$	$\mathbf{a_1/a_2}$, GeV^2/c^2	$\lvert\mathbf{g_{CR}}\rvert$	$\lvert\mathbf{g_{E_1}}\rvert$
before KTeV	$\mathbf{F = 0.76}$		$\mathbf{0.15}$	$\mathbf{0.038}$
KTeV '97, *PRL(2000)*	$\mathbf{1.35}^{+.20}_{-.17} \pm .04$	$-\mathbf{.72} \pm .03 \pm .01$	-	-
KTeV '96, *PRL(2001)*	-	$-\mathbf{.734} \pm .034 \pm .01$	-	-
KTeV '97, BCP4(2001)	-	-	$\mathbf{.100} \pm .018 \pm .013$	-
KTeV '97+'99, DPF2002	$\mathbf{1.10} \pm .10 \pm .06$	$-\mathbf{.75} \pm .03 \pm .02$	-	-
", ", Madison	$\mathbf{1.201}^{+.128}_{-.116}$	$-\mathbf{.730}^{+.024}_{-.027}$	$\mathbf{.188} \pm .013$	-
", ", today	$\mathbf{1.15}^{+.11}_{-.12} \pm \ldots$	$-\mathbf{.73} \pm .02 \pm \ldots$	$\mathbf{.18}^{+.02}_{-.01} \pm \ldots$	$< \mathbf{.025}$

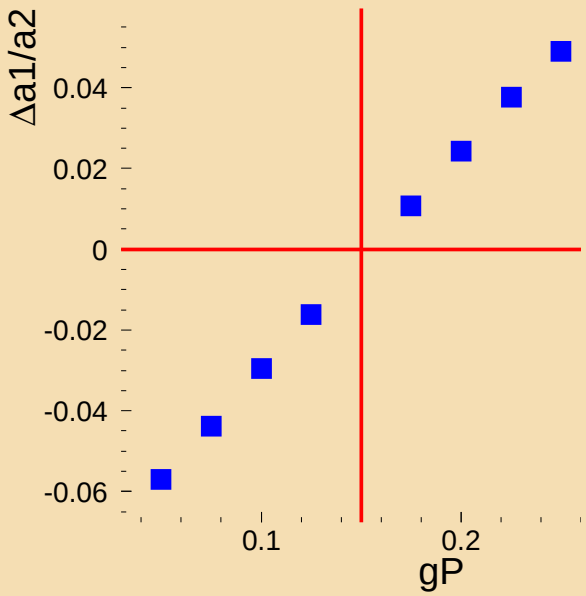
- Inner Bremsstrahlung: $\mathbf{g_{IB}} = |\eta_{+-}| e^{i\delta_0(M_K) + \Phi_{+-}}$
- M_1 Direct Emission: $\mathbf{g_{M_1}} = i e^{i\delta_1(M_{\pi\pi}) + \Phi_{+-}} \times \mathbf{F}\left(\frac{a_1}{a_2}; \tilde{g}_{M_1}\right)$,

  where

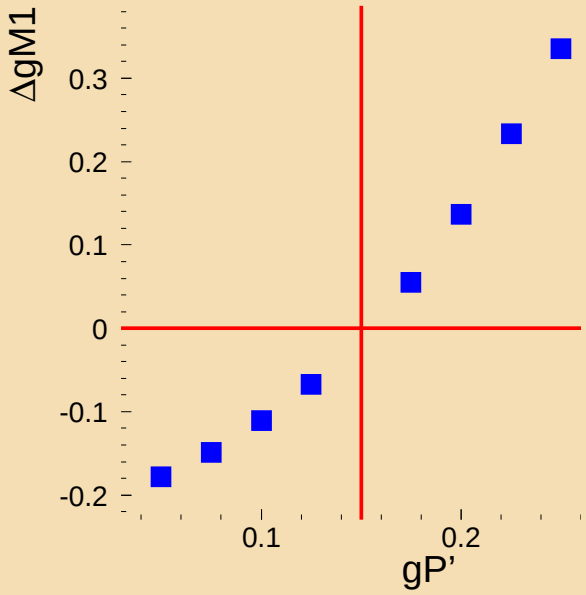
$$\mathbf{F} = \tilde{\mathbf{g}}_{\mathbf{M_1}} \left[1 + \frac{\mathbf{a1/a2}}{(M_\rho^2 - M_K^2) + 2M_K E_{ee}}\right]$$

- E_1 Direct Emission: $\mathbf{g_{E_1}} = i|\mathbf{g_{E_1}}| e^{i\delta_1(M_{\pi\pi}) + \Phi_{+-}} \times \mathbf{F}\left(\frac{a_1}{a_2}; \tilde{g}_{M_1}\right)$
- Charge Radius: $\mathbf{g_{CR}} = |\mathbf{g_{CR}}| e^{i\delta_0(M_{\pi\pi})}$, where $|\mathbf{g_{CR}}| = -\frac{1}{3}\langle \mathbf{R^2(K^0)} \rangle M_K^2$

1.3. The Uncertainties of the Two-Parameter Fit



Source	Uncertainty on the Parameter		
	$\Delta a_1/a_2$	Δg_{M_1}	$\Delta\mathcal{A}$
Background	.012	.04	.004
Variation of Cuts	.012	.024	.0028
Resolution	.003	.008	.007
Limited MC	.01	.03	.002
Δg_{E_1}	.005	.001	.004
$\Delta\eta_{+-}$	.002	.0003	.003
$\Delta\Phi_{+-}$	.0002	.00015	.0015
Combined	**.021**	**.056**	**.010**



Note that this systematic errors do not include the effect of g_{CR} which was fixed at the value of .15 for this analysis.

The Result was shown at DPF2002 ...

$$\frac{a_1}{a_2} = -0.75 \pm 0.03(\text{stat}) \pm 0.02(\text{syst})$$

$$\tilde{g}_{M_1} = 1.10 \pm 0.10(\text{stat}) \pm 0.06(\text{syst})$$

2. The Four-Parameter Fit

2.1. The Method

- Use Maximum Likelihood Method to estimate the parameters. The logarithm of the likelihood function can be written in terms of the relative weights of the event (data and Monte Carlo)

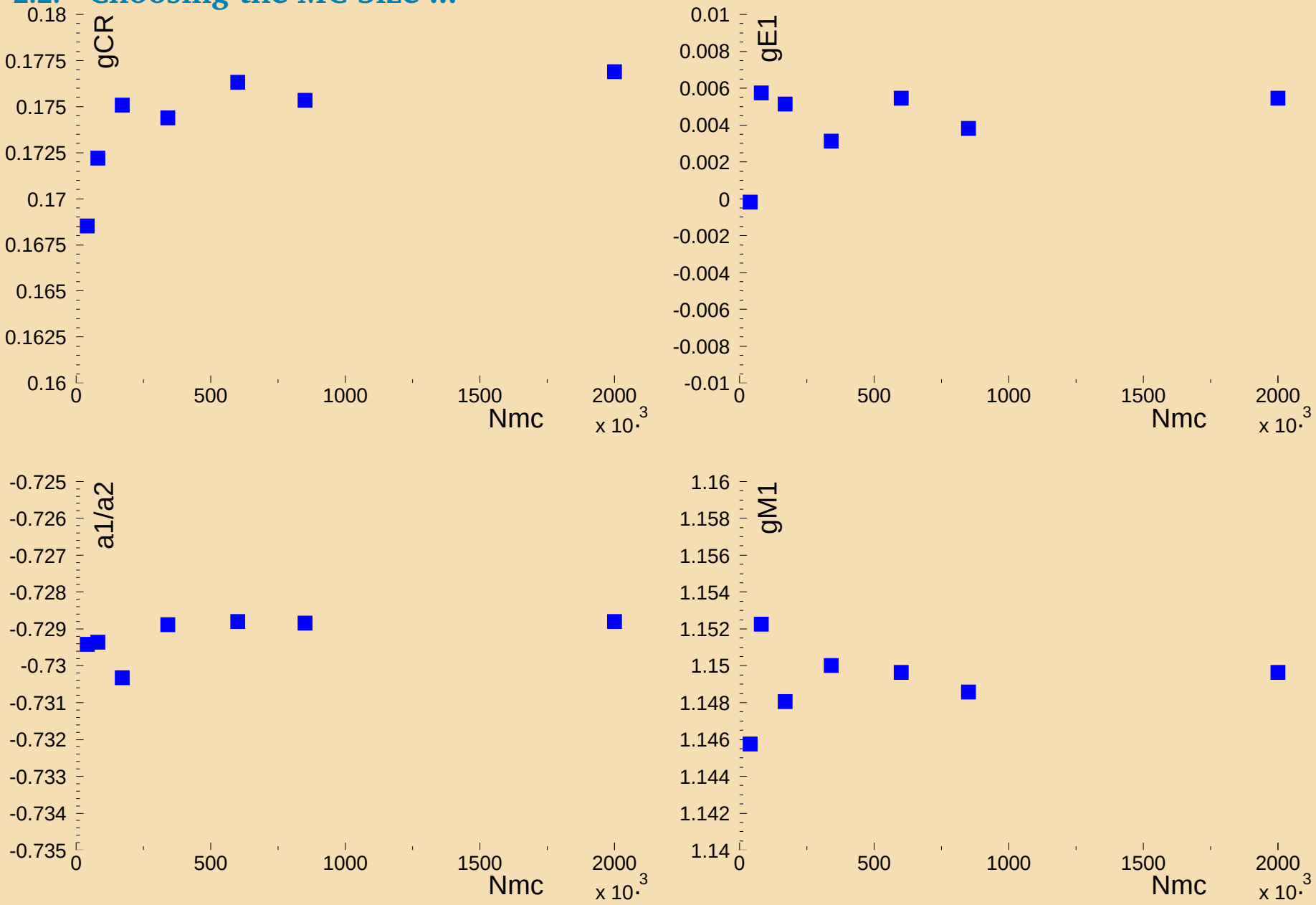
$$\log\mathcal{L}\left(\vec{\alpha}\right) = \left[\sum_{i=1}^{N_d} \log w_i\left(\vec{\alpha}, \vec{x}\right)\right] - N_d \log \sum_{j=1}^{N_{mc}} \frac{w_i(\vec{\alpha}, \vec{x})}{w_i(\vec{\alpha_0}, \vec{x})}$$

where $\vec{x}$ is the vector of measured variables and $\vec{\alpha}$ is the vector of parameters to be estimated, i.e.

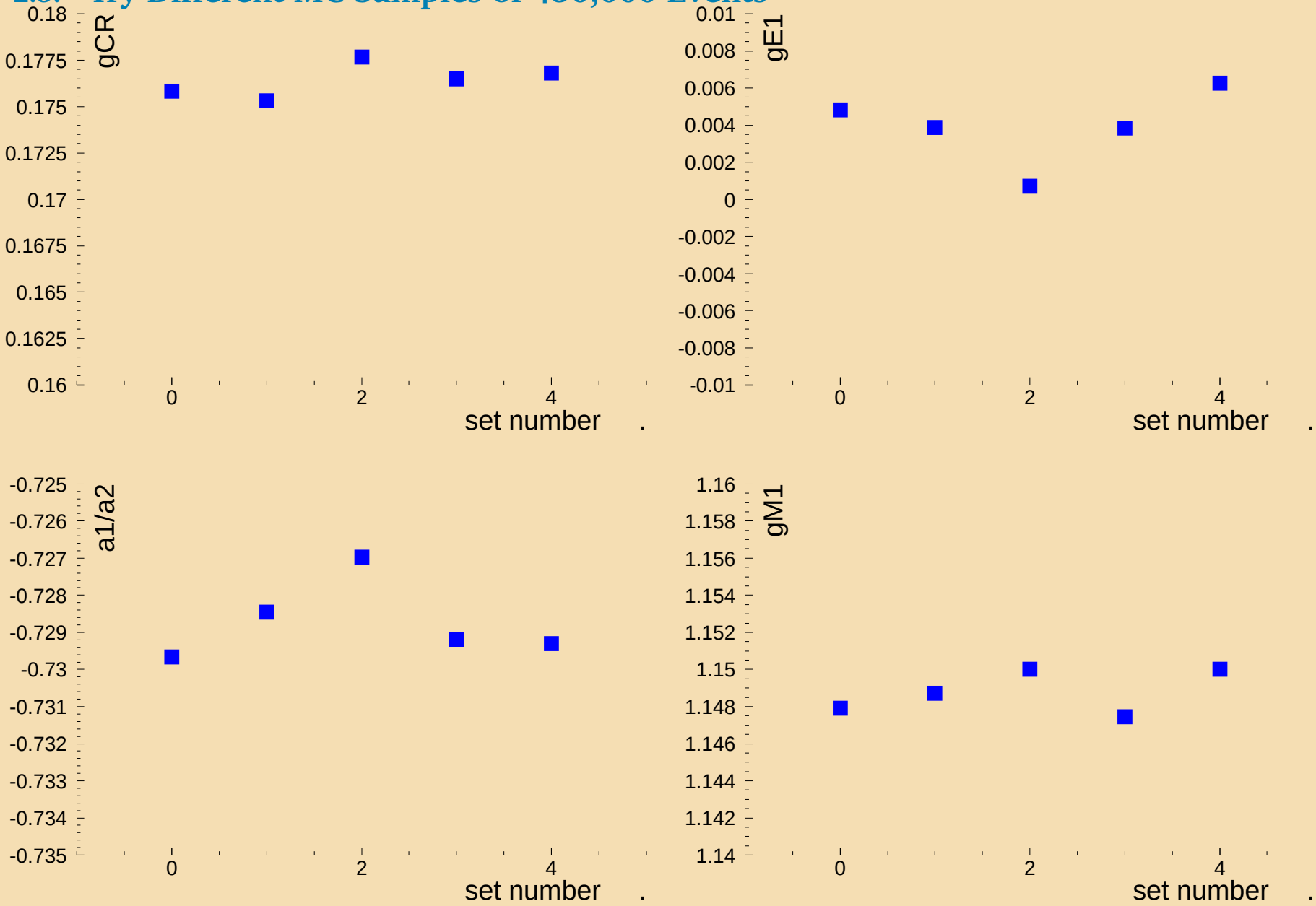
$$\vec{\alpha} = \left(\frac{a_1}{a_2}; g_{M_1}; g_{CR}; g_{E1}\right); \qquad \vec{x} = (\phi, \theta_{e^+}, \theta_{\pi^-}, M_{\pi\pi}, M_{ee})$$

- Generate one Monte Carlo sample for a set of values of the parameters $\vec{\alpha_0}$ and then re-weighted each event for any other set of floating parameters.
- Use Custom Fitter (code from Sasha Ledovskoy and "Numerical Recipes"), which uses *Powell Algorithm* to minimize the $\log\mathcal{L}\left(\vec{\alpha}\right)$

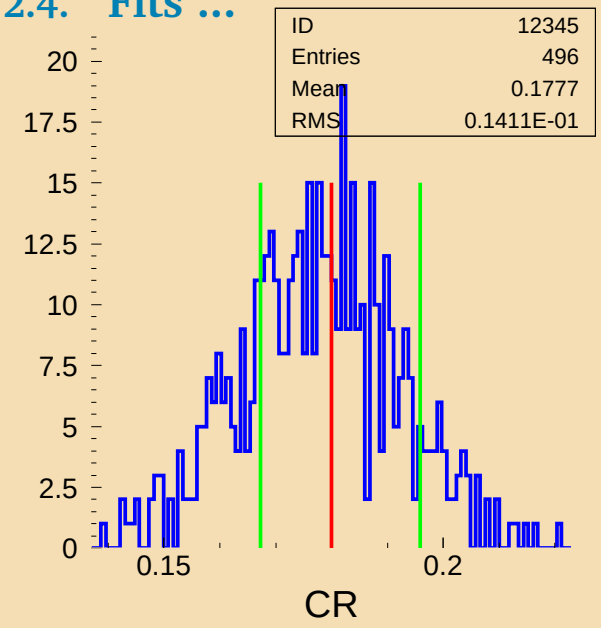
2.2. Choosing the MC Size ...

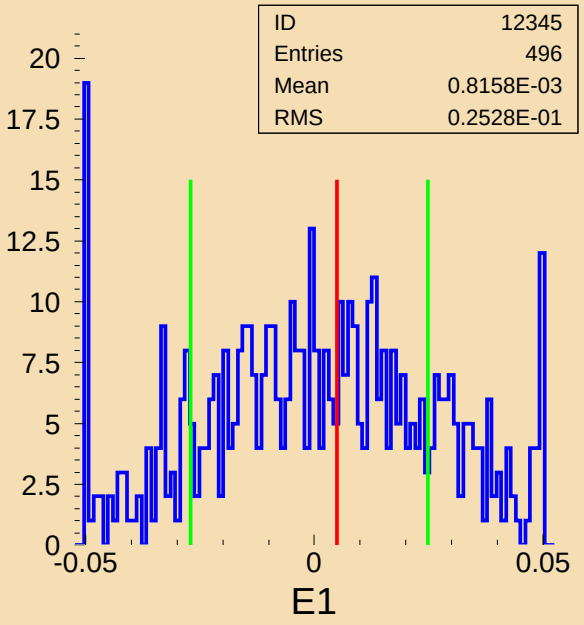


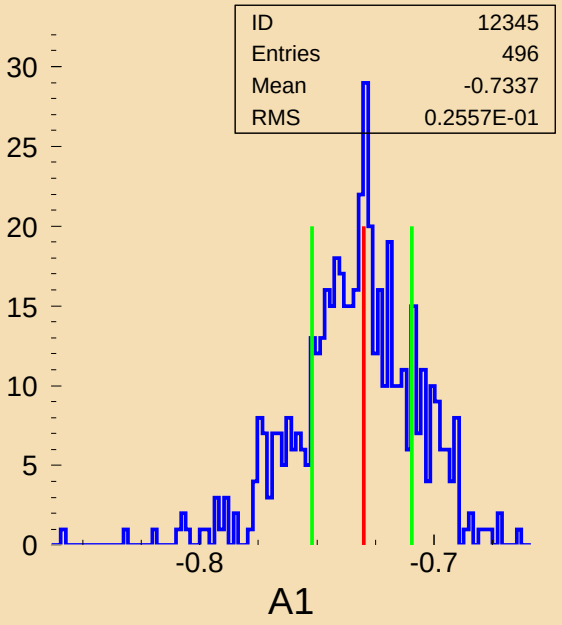
2.3. Try Different MC Samples of 430,000 Events

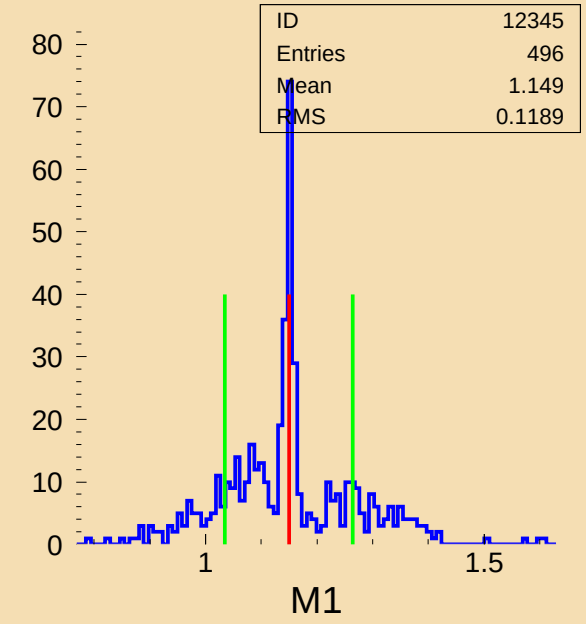


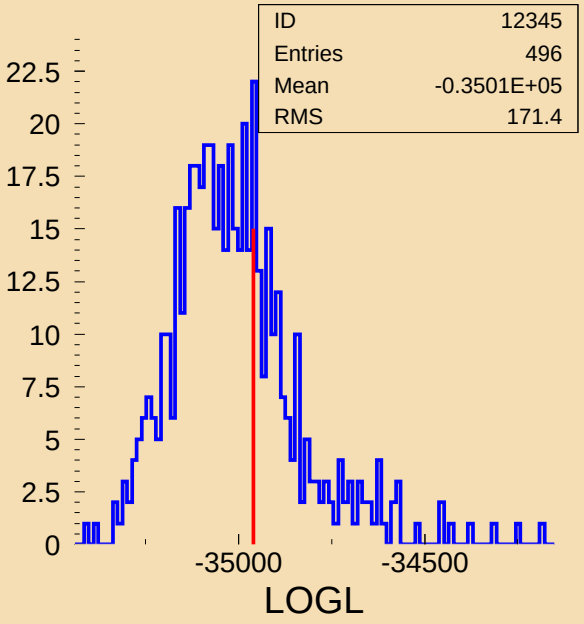
2.4. Fits ...











Done 496 + 1 Fits

blue - “Fake” Data (MC)
red - Real Data
green - Errors

3. Conclusions and Plans

- With the final sample of $\mathbf{5056 \pm 71}$ $\mathbf{K_L \to \pi^+\pi^-e^+e^-}$ events made a four-parameter fit, including $\mathbf{g_{CR}}$ (which is be the measurement of K^0 Charge Radius) and $\mathbf{g_{E1}}$.

$$\frac{\mathbf{a_1}}{\mathbf{a_2}} = \mathbf{.73 \pm .02 \pm \ldots} \qquad \mathbf{g_{M1} = 1.15^{+.11}_{-.12} \pm \ldots} \qquad \mathbf{g_{CR} = .18^{+.02}_{-.01} \pm \ldots} \qquad \mathbf{g_{E1} < .025}$$

- Plans and Future Prospectives:
  - Attempt to improve the values for the phase shitfs.
  - Estimate systematic errors on all parameters.
  - Measure new value for the Branching Ratio and Asymmetry.
  - Finalize, write up and publish ...