Update on Measurements of $K_L \to \pi^+\pi^-e^+e^-$

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KTeV Collaboration Meeting
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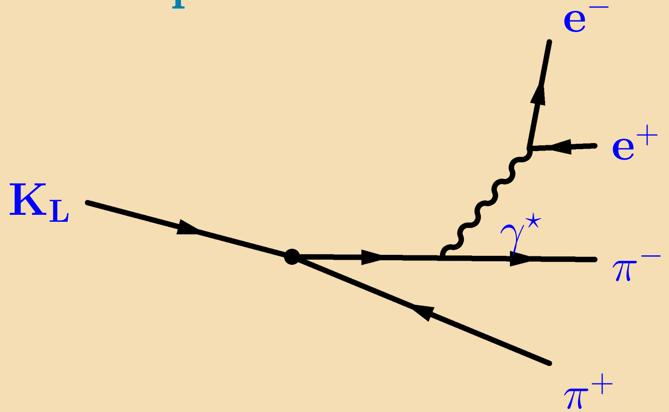
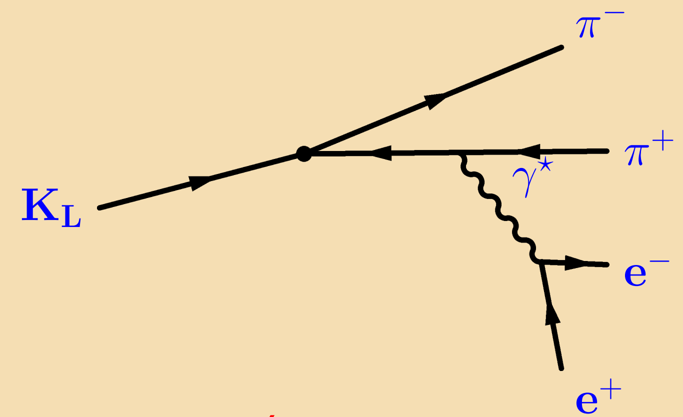
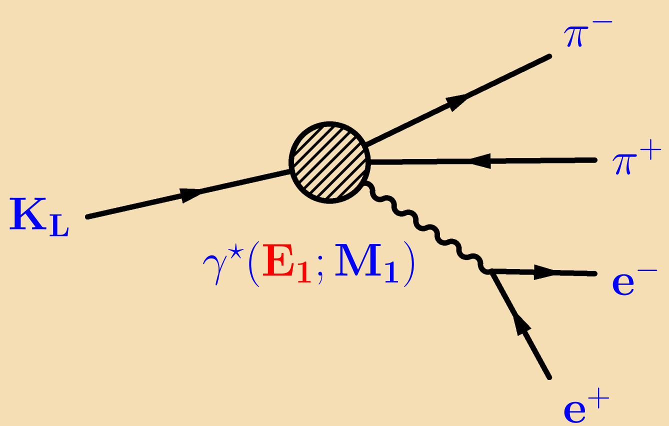
1. Motivation and Current Measurements

1.1. The Amplitudes





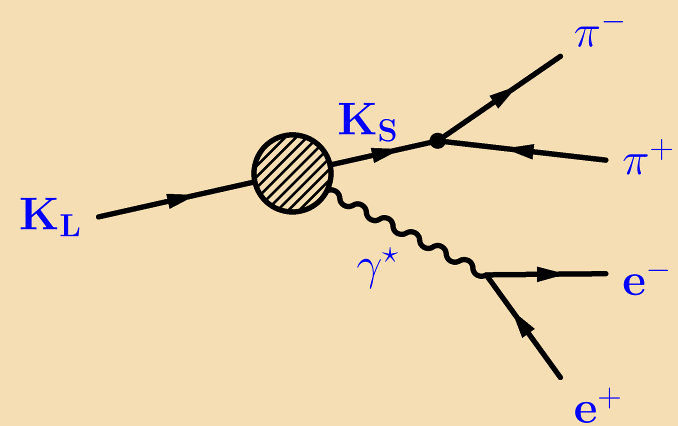
Inner Bremsstrahlung (IB) — ***Indirect*** $\not{C}P$



Direct Emission (DE)

E_1 **—** ***Indirect*** $\not{C}P$

M_1 **—** ***CP Conserving***



K^0 Charge Radius (CR)

CP Conserving

1.2. One of the Form Factors ...

The M_1 Direct Emission contribution:

- $g_{M_1} = ie^{i\delta_1(M_{\pi\pi}) + \Phi_{+-}} \times F\left(\frac{a_1}{a_2}; \tilde{g}_{M_1}\right)$,

  where

$$F = \tilde{\mathbf{g}}_{\mathbf{M_1}} \left[1 + \frac{\mathbf{a1/a2}}{(M_\rho^2 - M_K^2) + 2M_K E_{ee}}\right]$$

Default Parametrization	
Parameter	**Value**
$\lvert\eta_{+-}\rvert$	$(2.285 \pm .019) \times 10^{-3}$
$\mathbf{\Phi}_{+-}$	$43.7° \pm .6°$
$\tilde{\mathbf{g}}_{\mathbf{M_1}}$	$\mathbf{1.35}^{+.20}_{-.17} \pm .04$
$\mathbf{a_1/a_2}$	$-\mathbf{.72} \pm .03 \pm .01$
$\lvert g_{E_1}\rvert < 0.05\ \tilde{g}_{M_1}$	$.038 \pm .038$
$\lvert g_{CR}\rvert$	$.15$

- The new values were obtained using maximum likelihood fit of MC to the Data ('97+'99). These numbers were shown at DPF2002:

$$\frac{\mathbf{a_1}}{\mathbf{a_2}} = -\mathbf{0.75} \pm 0.03(stat) \pm 0.02(syst); \qquad \tilde{\mathbf{g}}_{\mathbf{M_1}} = \mathbf{1.10} \pm 0.10(stat) \pm 0.06(syst)$$

They are in agreement with the earlier published KTeV results:

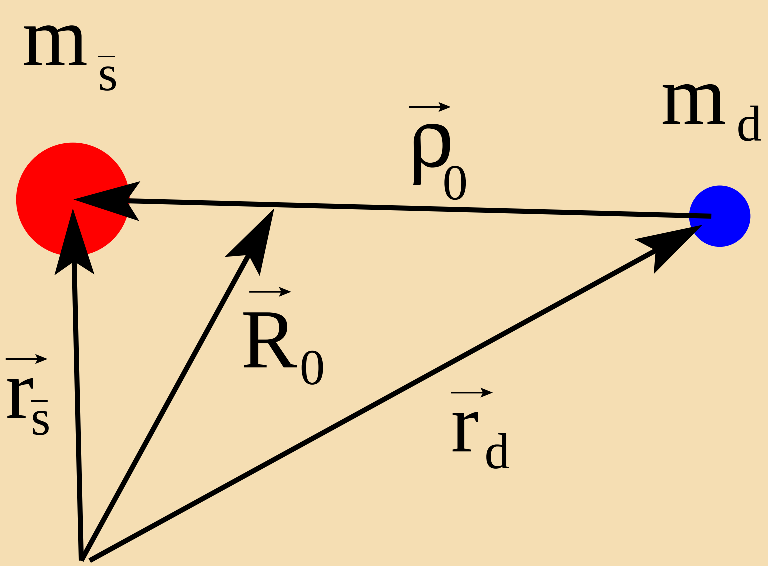
- $\frac{a_1}{a_2}$ = -0.72 $\pm$ 0.03(stat)$\pm$ 0.009(syst) GeV^2/c^2; $\tilde{g}_{M_1} = 1.35^{+0.20}_{-0.17}$(stat)$\pm$ 0.04(syst)

  *(Phys.Rev.Lett. **84**, 408 (2000))* — from '97 $\mathbf{K_L} \to \pi^+\pi^-\mathbf{e^+e^-}$ data.
- $\frac{a_1}{a_2}$ = -0.737$\pm$ 0.026(stat)$\pm$ 0.022(syst) GeV^2/c^2

  *(Phys.Rev.Lett. **86**, 761 (2001))* — from '97 $\mathbf{K_L} \to \pi^+\pi^-\gamma$ data.

1.3. The Measurement of K_L Charge Radius

$\mathbf{K^0}$ consists $\bar{s}$ and $\mathbf{d}$ quark, the relative and center-of-mass coordinates are



$$\vec{\rho}_0 \equiv \vec{r}_{\bar{s}} - \vec{r}_d; \qquad \vec{R}_0 \equiv \frac{m_s \vec{r}_{\bar{s}} + m_d \vec{r}_d}{m_s + m_d}$$

Heavier strange quark is confined to a smaller radius thus giving $\mathbf{K^0}$ a positively charged core:

$$\langle R^2 \rangle \equiv \left\langle \sum q_i (\vec{r}_i - \vec{R}_0)^2 \right\rangle = -\frac{1}{3}\frac{m_s - m_d}{m_s + m_d} \langle \rho_0^2 \rangle$$

On the other hand $\langle R^2 \rangle$ is part of the expression for g_{CR}:

$$g_{CR} = -\frac{1}{3} \langle R^2 \left(K^0 \right) \rangle M_K^2 e^{i\delta_0(M_{\pi\pi})}; \qquad |g_{CR}| \equiv -\frac{1}{3} \langle R^2 \left(K^0 \right) \rangle M_K^2$$

KTeV Preliminary Measurement ***(Work by Sasha Ledovskoy on '97 Data)***:

$$|g_{CR}| = 0.100 \pm 0.018 \pm 0.013; \qquad \langle R^2(K^0) \rangle = -0.047 \pm 0.008 \pm 0.006 [\mathrm{fm}^2]$$

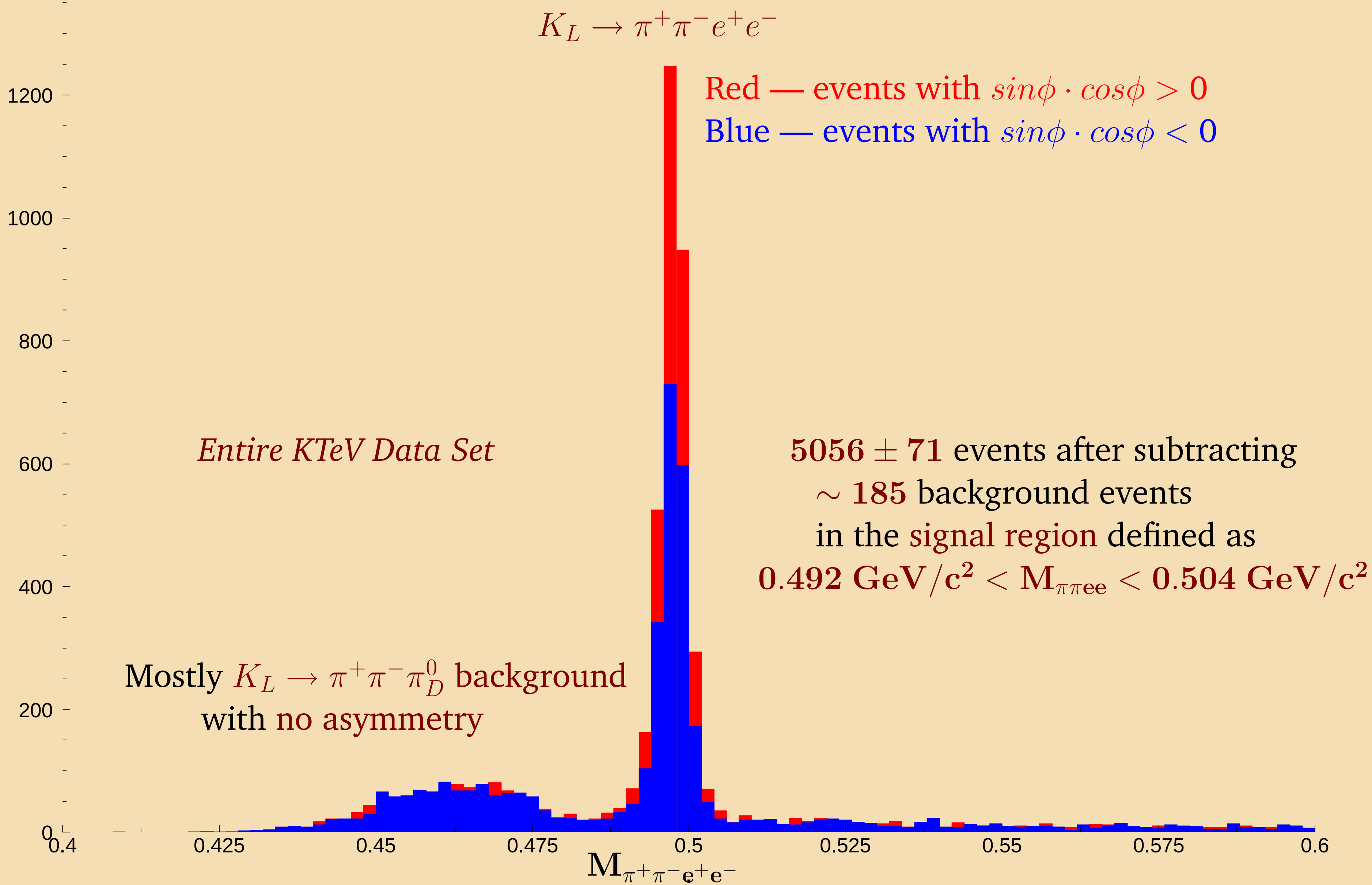
2. The Data

2.1. Summary of Main Cuts

- Event has 4 tracks
- Particle ID: electrons if $0.95 < \frac{E}{P} < 1.05$ and pions if $\frac{E}{P} < 0.9$ or $\frac{E}{P} > 1.1$
- $P^2_{\pi^0} < -0.025\ \mathrm{GeV}^2/c^2$
- $M_{ee} > 0.002\ \mathrm{GeV}/c^2$
- $95\mathrm{m} < Z_{vtx} < 158\mathrm{m}$
- $P_t^2 < 6 \times 10^{-5}\ \mathrm{GeV}^2/c^2$
- $E_{\pi\pi ee} < 200\ \mathrm{GeV}$
- $0.492\ \mathrm{GeV}/c^2 < M_{\pi\pi ee} < 0.504\ \mathrm{GeV}/c^2$

The residual background under the mass peak was estimated by a fitting procedure.

2.2. The Final Event Sample



3. The Measurements

3.1. Maximum Likelihood Fit

Use Maximum Likelihood Method to estimate the parameters. The logarithm of the likelihood function can be written in terms of the relative weights of the event (data and Monte Carlo)

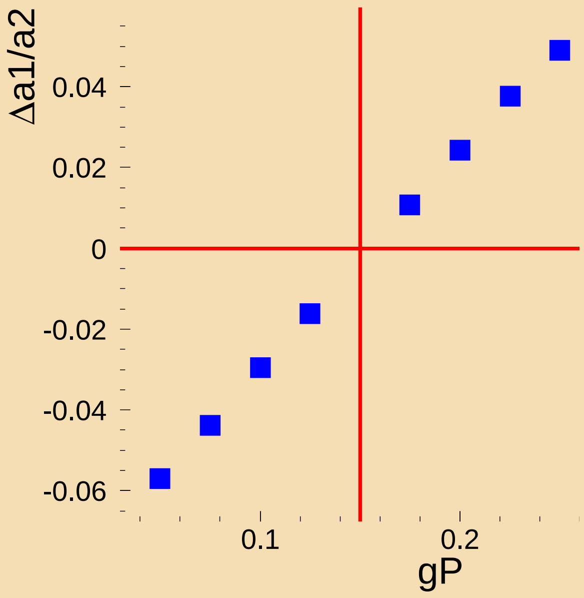
$$\mathbf{log}\mathcal{L}\left(\vec{\alpha}\right) = \left[\sum_{\mathbf{i=1}}^{\mathbf{N_d}} \mathbf{log} \mathbf{w_i}\left(\vec{\alpha}, \vec{\mathbf{x}}\right)\right] - \mathbf{N_d} \mathbf{log} \sum_{\mathbf{j=1}}^{\mathbf{N_{mc}}} \frac{\mathbf{w_i}(\vec{\alpha}, \vec{\mathbf{x}})}{\mathbf{w_i}(\vec{\alpha_0}, \vec{\mathbf{x}})}$$

where $\vec{\mathbf{x}}$ is the vector of measured variables and $\vec{\alpha}$ is the vector of parameters to be estimated, i.e.

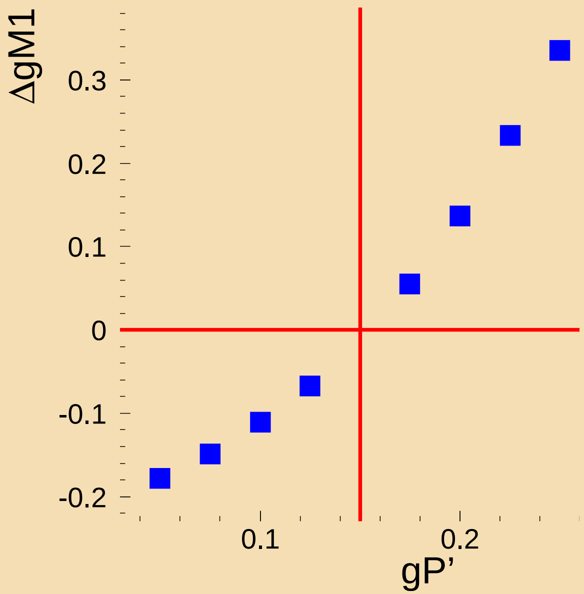
$$\vec{\alpha} = \left(\frac{\mathbf{a_1}}{\mathbf{a_2}}; \mathbf{g_{M_1}}\right) \text{ or } \vec{\alpha} = \left(\frac{\mathbf{a_1}}{\mathbf{a_2}}; \mathbf{g_{M_1}}; \mathbf{g_{CR}}\right); \qquad \vec{\mathbf{x}} = (\phi, \theta_{\mathbf{e^+}}, \theta_{\pi^-}, \mathbf{M_{\pi\pi}}, \mathbf{M_{ee}})$$

A large sample of Monte Carlo events was generated with nominal values of the parameters $\vec{\alpha_0}$ and then each event was re-weighted for any other set of floating parameters.

3.2. The Uncertainties of the Two-Parameter Fit



Source	Uncertainty on the Parameter		
	$\Delta a_1/a_2$	Δg_{M_1}	$\Delta \mathcal{A}$
Background	.012	.04	.004
Variation of Cuts	.012	.024	.0028
Resolution	.003	.008	.007
Limited MC	.01	.03	.002
Δg_{E_1}	.005	.001	.004
$\Delta \eta_{+-}$	.002	.0003	.003
$\Delta \Phi_{+-}$	.0002	.00015	.0015
Combined	**.021**	**.056**	**.010**



Note that this systematic errors do not include the effect of g_{CR} which was fixed at the value of .15 for this analysis.

The Result was shown at DPF2002 ...

$$\frac{a_1}{a_2} = -0.75 \pm 0.03(\text{stat}) \pm 0.02(\text{syst})$$

$$\tilde{g}_{M_1} = 1.10 \pm 0.10(\text{stat}) \pm 0.06(\text{syst})$$

3.3. The Three-Parameter Fits.

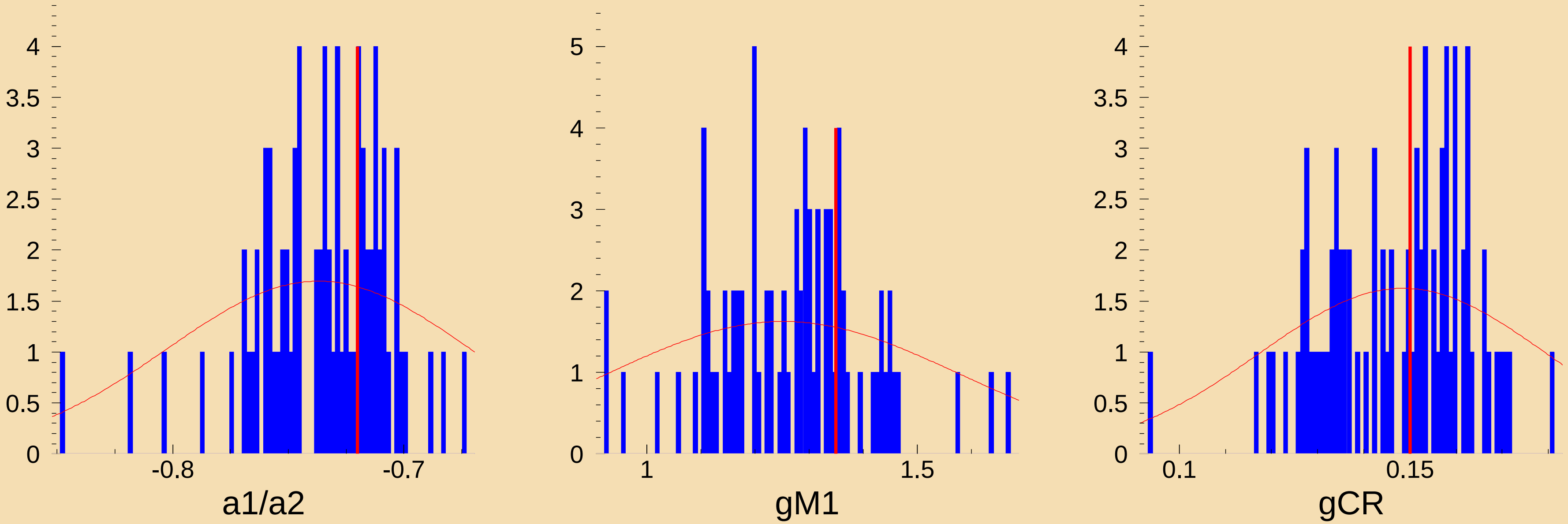
$$\log \mathcal{L}\left(\vec{\alpha}\right); \qquad \vec{\alpha} = \left(\frac{\mathbf{a_1}}{\mathbf{a_2}}; \mathbf{g_{M_1}}; \mathbf{g_{CR}}\right)$$

Three different techniques to extract the parameters and their errors (fastest to slowest):

1. Use CERN *Minuit* package with estimation of errors by MINOS utility. Minimum labor and maximum information, but doesn't necessarily work!
2. Custom Fitter (code from Sasha Ledovskoy and "Numerical Recipes"), which uses *Powell Algorithm* to minimize a function. This is quick and robust, but doesn't give errors on parameters.

   After the minimum has been found, estimate the errors by scanning the function versus all arguments in the vicinity of the maximum. This is very reliable and illustrative, but CPU and labor intensive.
3. Use either one of the fitters on real data to find the parameters. And then generate multiple MC samples and fit this "fake data" to estimate the errors directly. Always works, but takes a long time.

3.4. The Method Verification.



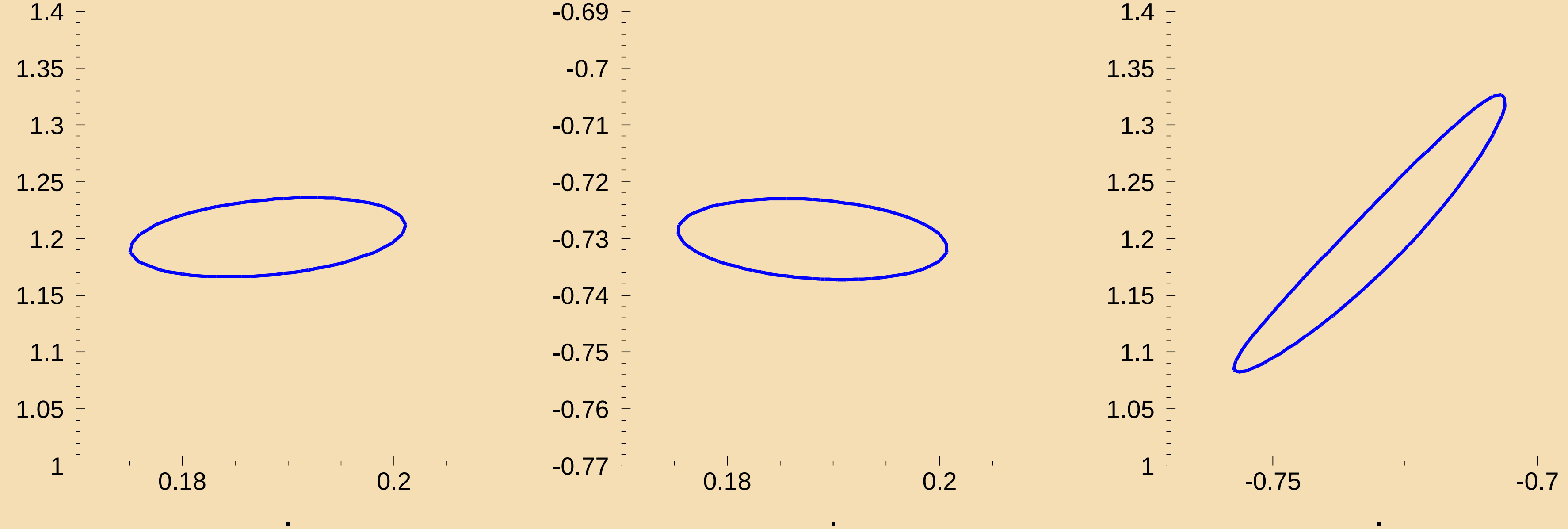
- Generated 64 MC samples of "fake data" for the KTeV experiment, using (vertical lines)

$$\frac{\mathbf{a_1}}{\mathbf{a_2}} = .72 \qquad g_{M1} = 1.35 \qquad g_{CR} = .15$$

- Made the three-parameter fit for each sample. The means ($\pm\sigma$) of the Gaussian fit (red curves) to the distributions (histograms) are:

$$\frac{\mathbf{a_1}}{\mathbf{a_2}} = .737 \pm .066 \qquad g_{M1} = 1.25 \pm .16 \qquad g_{CR} = .148 \pm .031$$

3.5. The Result From a Three-Parameter Fit



The three-parameter fit with the real data:

- Find the minimum with the Customized Fitter.
- Scan the Likelihood function. Inspect the $1-\sigma$ contours in two-parameter projections and estimate errors on parameters from the corresponding projections.
- The new result is (statistical errors only):

$$\frac{\mathbf{a_1}}{\mathbf{a_2}} = .730^{+.024}_{-.027} \qquad \mathbf{g_{M1}} = 1.201^{+.128}_{-.116} \qquad \mathbf{g_{CR}} = .188 \pm .013$$

Minuit with MINOS error estimation gives **the same result!**.

4. Conclusions and Plans

- Now we have $\mathbf{5056 \pm 71\ K_L \rightarrow \pi^+\pi^-e^+e^-}$ events.
- Reported New Preliminary Measurements at DPF2002
  - CP Violating Asymmetry in the ϕ angular variable.
    - ⋆ $\mathcal{A} = (\mathbf{13.3 \pm 1.4(stat) \pm 1.0(syst)})\,\%$
  - Vector Form Factor Parameters
    - ⋆ $\frac{\mathbf{a_1}}{\mathbf{a_2}} = \mathbf{-0.75 \pm 0.03(stat) \pm 0.02(syst)}$
    - ⋆ $\mathbf{\tilde{g}_{M_1} = 1.10 \pm 0.10(stat) \pm 0.06(syst)}$
- Made a three-parameter fit to include $\mathbf{g_{CR}}$ (which will be the measurement of K^0 Charge Radius).

$$\frac{\mathbf{a_1}}{\mathbf{a_2}} = \mathbf{.730^{+.024}_{-.027}} \qquad \mathbf{g_{M1} = 1.201^{+.128}_{-.116}} \qquad \mathbf{g_{CR} = .188 \pm .013}$$

- Plans and Future Prospectives:
  - Add parameter $\mathbf{g_{E_1}}$ (to search for CP Violating E_1 direct emission) and try four-parameter fit.
  - Measure new value for the Branching Ratio
  - Finalize, write up and publish ...