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Testing the CKM Model with Kaon Experiments

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Kaon decay experiments had considerable impact in motivating and validating the model of Kobayashi and Maskawa. We review these along with ones performing precision studies of CKM matrix elements and searching for physics outside the Standard Model.

Subject Index: 151, 152, 155, 156, 157

§1. Introduction

Kaons have played a central role in the development of the theory of quark mixing. Most importantly, perhaps, was the 1964 observation of CP violation, which presented the problem that motivated Kobayashi and Maskawa to consider a model with 3 generations of quarks. That model, now referred to as the Cabibbo-Kobayashi-Maskawa (CKM) model, accommodated CP violation in a natural way with a single CP violating quantity. Unfortunately, for many years, there was only a single CP violating observable, the parameter ϵ, which describes the CP asymmetry in K^0-$\overline{K}^0$ mixing, providing no real test of the model. For the next 30 years, the search for direct CP violation (also predicted by the CKM model), the decay of the CP odd component of the K_L to the CP-even $\pi\pi$ final state, motivated many of the experiments done in the kaon system.

In this paper, we will review the role of neutral kaon experiments in motivating and testing the Kobayashi and Maskawa model. The discussion begins with a brief review of the features of neutral kaon system that have made it a unique laboratory for testing discrete symmetries in weak interactions. The following sections will describe studies of CP violation in neutral kaons including the search for direct CP violation. We will also review past and future efforts to use kaon decays to make precise measurements of CKM matrix elements.

§2. Neutral kaons and CP violation

The neutral kaon was first observed by Rochester and Butler using a cloud chamber in 1947.[1)] Figure 1 shows a photograph of the first detected "V" particle.[2)] We now know that this image shows the decay of a neutral kaon to $\pi^+\pi^-$, but we cannot tell whether it was a K^0 ($\overline{s}d$) or $\overline{K}^0$ ($s\overline{d}$). This ambiguity is the key to

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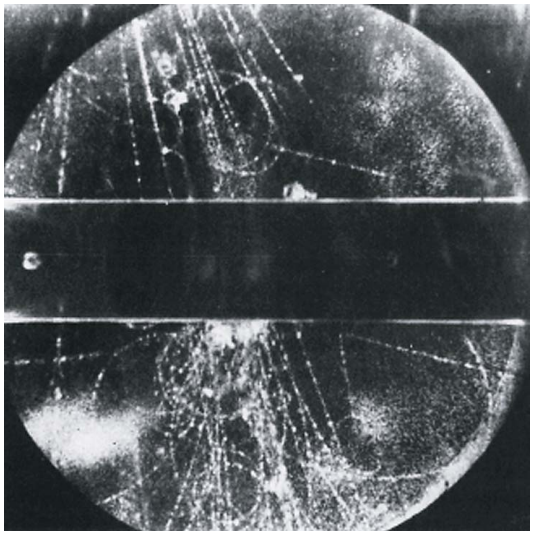

Fig. 1. The first observed "V" particle. Rochester and Butler identified the inverted V in the lower right quadrant as two oppositely-charged particles coming from the decay of a previously unknown neutral particle.

kaon's unique properties. Since the K^0 and $\overline{K}^0$ have common decay modes, they can mix into each other through second-order weak processes that changes strangeness by 2. The physical states that decay will therefore be mixtures of the K^0 and $\overline{K}^0$. Assuming CP invariance, these states are

$$\begin{aligned} |K_{even}\rangle &\sim |K^0\rangle + |\overline{K}^0\rangle, \\ |K_{odd}\rangle &\sim |K^0\rangle - |\overline{K}^0\rangle, \end{aligned} \tag{2·1}$$

where $CP|K_{even}\rangle = +|K_{even}\rangle$ and $CP|K_{odd}\rangle = -|K_{odd}\rangle$.

The CP-even state almost always decays to pairs of pions. Since the CP-odd state cannot decay to the two-pion final state, it should have a longer lifetime than the CP-even state. Gell-Mann and Pais[3)] predicted the existence of the long-lived neutral kaon, which had not yet been observed, based on this argument.[*)] Lederman and collaborators[4)] quickly searched for and observed the predicted particle with a lifetime 580 times longer than that of the short-lived neutral kaon.

The large lifetime difference between the long and short-lived neutral kaons makes it easy to produce a beam of the long-lived, CP-odd state: an experiment may be placed far from a target so that all of the short-lived state has decayed away. Such an experiment was performed by Christenson, Cronin, Fitch, and Turlay in 1963 to investigate an anomalous result on regeneration of K_{even} mesons. They also planned to test CP invariance by obtaining a better limit on $K_{odd} \to \pi^+\pi^-$. To their surprise, they observed about 45 decays of the long-lived neutral kaon to $\pi^+\pi^-$, establishing the existence of CP violation.[5)]

[*)] Their argument assumed C invariance rather than CP invariance.

Already in 1967, connections between CP violation and the early Universe were made, most notably in Sakharov's famous paper on the generation of the baryon asymmetry. Aside from Sakharov's paper, CP violation was essentially unconnected to the rest of physics, somewhat akin to the relationship of gravity to the rest of particle physics. It was important and prompted many ideas about further studies of kaon decays, but it was such a small effect that it was hard to understand if it had any broad implications for particle physics. The model of Kobayashi and Maskawa, discussed in the following section, provided a mechanism to integrate CP violation naturally into the quark sector.

§3. Interpretations of $K_L \to \pi\pi$ decays

3.1. *Mechanism of CP violation in neutral kaon system*

After the discovery of CP violation, the forbidden $K_L \to \pi^+\pi^-$ decay was demonstrated to be caused mainly (and perhaps exclusively) by an unequal mixture of K^0 and $\overline{K}^0$ components in the K_L state:

$$|K_L\rangle \sim |K_{odd}\rangle + \varepsilon|K_{even}\rangle \tag{3·1}$$

$$= \frac{1+\varepsilon}{\sqrt{2}}|K^0\rangle + \frac{1-\varepsilon}{\sqrt{2}}|\overline{K}^0\rangle. \tag{3·2}$$

The K_{even} component can decay to the CP even $\pi^+\pi^-$ state without violating the CP symmetry in the decay process. Let us briefly describe a basic mechanism to generate such an imbalance between K^0 and $\overline{K}^0$ in the K_L state, referred to as indirect CP violation. In the following, we assume CPT symmetry.

A wave function of a particle with mass m and the decay width Γ is

$$\psi(t) = \exp\left(-imt - \frac{\Gamma}{2}t\right), \tag{3·3}$$

and its Schrödinger equation is

$$i\frac{\partial}{\partial t}\psi(t) = H\psi(t) \tag{3·4}$$

$$= \left(m - \frac{i}{2}\Gamma\right)\psi(t). \tag{3·5}$$

For a neutral kaon, the wave function is expanded to have two components,

$$\psi(t) = \begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix}, \tag{3·6}$$

where the top and bottom elements show the K^0 and $\overline{K}^0$ amplitudes, respectively. The Schrödinger equation is then written as

$$i\frac{\partial}{\partial t}\begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix} = H\begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix} \tag{3·7}$$

$$= \left[\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}\right]\begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix}. \tag{3·8}$$

The first and second matrices are called the mass matrix and decay matrix, respectively. CPT symmetry requires the same mass and decay width for K^0 and $\overline{K}^0$: $M_{11} = M_{22} \equiv M_0$, and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma_0$. Γ_{12} is the sum of amplitudes of $\overline{K}^0 \to f \to K^0$, where f represents the decay final states common to both K^0 and $\overline{K}^0$, such as $\pi\pi, \pi\pi\pi$, etc. M_{12} comes from a sum of amplitudes of $\overline{K}^0 \to i \to K^0$, where i represents intermediate virtual states. The Hermeticity of mass and decay matrices requires $M_{21} = M_{12}^*$ and $\Gamma_{21} = \Gamma_{12}^*$. Equation (3·8) is then written as

$$i\frac{\partial}{\partial t}\begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix} = \begin{bmatrix} M_0 - i\Gamma_0/2 & M_{12} - i\Gamma_{12}/2 \\ M_{12}^* - i\Gamma_{12}^*/2 & M_0 - i\Gamma_0/2 \end{bmatrix} \begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix} \tag{3·9}$$

$$\equiv \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix}. \tag{3·10}$$

The top-right component of the Hamilton matrix, $H_{12} = M_{12} - i\Gamma_{12}/2$, determines the probability of $\overline{K}^0 \to K^0$ transition, and the bottom-left component, $H_{21} = M_{12}^* - i\Gamma_{12}^*/2$, determines the probability of $K^0 \to \overline{K}^0$ transition.

If M_{12} and Γ_{12} are not parallel in the complex plane, as shown in Fig. 2, the magnitudes of H_{12} and H_{21} are different, resulting in different transition probabilities for $\overline{K}^0 \to K^0$ and $K^0 \to \overline{K}^0$. This is called *indirect CP violation*, or CP violation in K^0-$\overline{K}^0$ *mixing*. The pure K_L state is in an equilibrium state with a constant mixture of K^0 and $\overline{K}^0$ components, requiring the same flow of amplitudes in both directions, $|H_{12}\overline{K}^0(t)| = |H_{21}K^0(t)|$. As illustrated in Fig. 3, this means $|K^0(t)| \neq |\overline{K}^0(t)|$ in the pure K_L state.

The unequal mixture of K^0 and $\overline{K}^0$ in K_L was proven experimentally by observing a charge asymmetry in $K_L \to \pi^\pm e^\mp \nu$ decays. In K_L, the $K^0(\bar{s}d)$ component decays to $\pi^- e^+ \nu$, while the $\overline{K}^0(s\bar{d})$ component decays to $\pi^+ e^- \bar{\nu}$. In 1967, Bennett et al.[6] measured $(N^+ - N^-)/(N^+ + N^-) = (2.24 \pm 0.36) \times 10^{-3}$, where $N^\pm$ is the number of semileptonic decay events with $e^\pm$, and proved that the K^0 component is

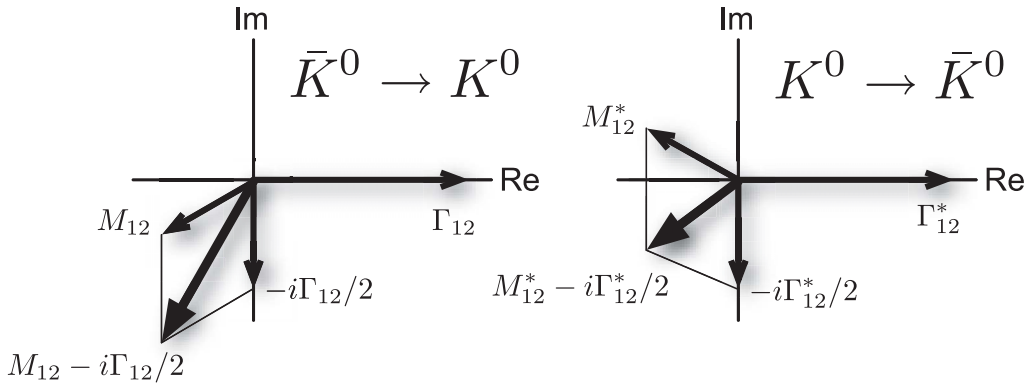

Fig. 2. If M_{12} and Γ_{12} are not parallel in the complex plane, the $\overline{K}^0 \to K^0$ and $K^0 \to \overline{K}^0$ transition amplitudes will be different.

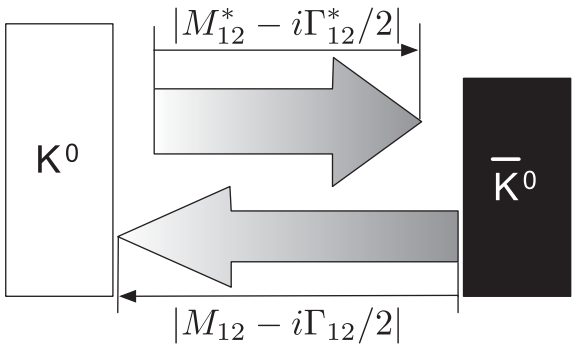

Fig. 3. If $\overline{K}^0 \to K^0$ transition probability is larger than $K^0 \to \overline{K}^0$, the K_L has larger fraction of K^0 component than $\overline{K}^0$.

larger than $\overline{K}^0$ in K_L. The difference in $\overline{K}^0 \to K^0$ and $K^0 \to \overline{K}^0$ transition rates was confirmed directly by CPLEAR experiment in 1998 by tagging K^0 and $\overline{K}^0$ in both initial and final states.[7)]

The question was then to explain the source of the relative phase between M_{12} and Γ_{12}.

3.2. *Superweak model*

In 1964, Wolfenstein proposed the Superweak Model.[8)] In this model, there is a very weak interaction that changes $K^0 \leftrightarrow \overline{K}^0$ and introduces a phase in M_{12} (relative to Γ_{12}), causing the indirect CP violation. Since such an interaction changes strangeness by 2, however, it cannot contribute to kaon decays in which the strangeness changes by 1. Thus, the Superweak Model cannot violate CP in the decay process itself.

3.3. *Direct CP violation*

Is CP violated in decay process? This question can be recast as "Can a CP odd K_{odd} state decay into a CP even $\pi\pi$ state?" Such CP violation is called *direct CP violation*. If a decay amplitude $\langle\pi\pi|H|K^0\rangle$ has an imaginary part, then $K_{odd} \to \pi\pi$ would be non-zero since $\langle\pi\pi|H|K_{odd}\rangle \propto \langle\pi\pi|H|K^0\rangle - \langle\pi\pi|H|\overline{K}^0\rangle \propto \mathrm{Im}\langle\pi\pi|H|K^0\rangle$.

The problem is that the directly accessible state is K_L instead of K_{odd}, and in the decay amplitude,

$$\langle\pi\pi|H|K_L\rangle \sim \langle\pi\pi|H|K_{odd}\rangle + \varepsilon\langle\pi\pi|H|K_{even}\rangle, \tag{3·11}$$

the amplitude of *direct CP* violation, $\langle\pi\pi|H|K_{odd}\rangle$, was predicted to be much smaller than the *indirect CP* violating $\varepsilon\langle\pi\pi|H|K_{even}\rangle$ term, making it hard to observe. The trick was then to use a small difference between $\langle\pi^+\pi^-|H|K_{odd}\rangle$ and $\langle\pi^0\pi^0|H|K_{odd}\rangle$ produced by their isospin (I) dependence. If direct CP violation does not exist, such a difference will not exist anyway. Let us define decay amplitudes:

$$\langle\pi\pi; I=0|H|K^0\rangle = A_0 e^{i\delta_0} \quad \text{and} \quad \langle\pi\pi; I=2|H|K^0\rangle = A_2 e^{i\delta_2} \tag{3·12}$$

for isospin 0 and 2, respectively, where δ_I is a phase shift due to a final state interaction for isospin I. For $\overline{K}^0$ decay,

$$\langle \pi\pi; I=0|H|\overline{K}^0\rangle = A_0^* e^{i\delta_0} \quad \text{and} \quad \langle \pi\pi; I=2|H|\overline{K}^0\rangle = A_2^* e^{i\delta_2}. \tag{3·13}$$

Note that the final state interaction phase does not flip its sign because it is independent of the initial state. The $\pi^+\pi^-$ and $\pi^0\pi^0$ systems have isospin $I_3 = 0$, and consist of $I = 0$ and $I = 2$ states:

$$|\pi^+\pi^-\rangle = \sqrt{\frac{2}{3}}|I=0\rangle + \sqrt{\frac{1}{3}}|I=2\rangle \quad \text{and} \quad |\pi^0\pi^0\rangle = -\sqrt{\frac{1}{3}}|I=0\rangle + \sqrt{\frac{2}{3}}|I=2\rangle. \tag{3·14}$$

The direct CP violating amplitudes are then

$$\langle \pi^+\pi^-|H|K_{odd}\rangle \propto \langle \pi^+\pi^-|H|K^0\rangle - \langle \pi^+\pi^-|H|\overline{K}^0\rangle \tag{3·15}$$

$$\propto \sqrt{2}\mathrm{Im}A_0 e^{i\delta_0} + \mathrm{Im}A_2 e^{i\delta_2}, \quad \text{and} \tag{3·16}$$

$$\langle \pi^0\pi^0|H|K_{odd}\rangle \propto -\mathrm{Im}(A_0)e^{i\delta_0} + \sqrt{2}\mathrm{Im}(A_2)e^{i\delta_2}. \tag{3·17}$$

In general, the relative contribution of direct CP violation, $\langle \pi\pi|H|K_{odd}\rangle / \langle \pi\pi|H|K_{even}\rangle$, is different for $\pi^+\pi^-$ and $\pi^0\pi^0$ decays.

In terms of the measurable K_L and K_S states, the ratios are

$$\eta_\pm \equiv \frac{\langle \pi^+\pi^-|H|K_L\rangle}{\langle \pi^+\pi^-|H|K_S\rangle} \tag{3·18}$$

$$\sim \frac{\langle \pi^+\pi^-|H|K_{odd}\rangle + \varepsilon\langle \pi^+\pi^-|H|K_{even}\rangle}{\langle \pi^+\pi^-|H|K_{even}\rangle} \tag{3·19}$$

$$\sim \varepsilon + i\frac{\sqrt{2}\mathrm{Im}A_0 e^{i\delta_0} + \mathrm{Im}A_2 e^{i\delta_2}}{\sqrt{2}\mathrm{Re}A_0 e^{i\delta_0} + \mathrm{Re}A_2 e^{i\delta_2}} \tag{3·20}$$

$$= \epsilon + \epsilon', \quad \text{and} \tag{3·21}$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0|H|K_L\rangle}{\langle \pi^0\pi^0|H|K_S\rangle} \tag{3·22}$$

$$\sim \epsilon - 2\epsilon', \tag{3·23}$$

by using $|A_2| \ll |A_0|$ where

$$\epsilon = \varepsilon + i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}, \quad \text{and} \tag{3·24}$$

$$\epsilon' = \frac{i}{\sqrt{2}}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)e^{i(\delta_2-\delta_0)}. \tag{3·25}$$

Thus, if $\arg A_0 \neq \arg A_2$ and $\delta_0 \neq \delta_2$, we can see a ratio difference between $\pi^+\pi^-$ and $\pi^0\pi^0$ decay modes. Experimentally, we measure the double ratio of decay widths:

$$R = \frac{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^0\pi^0)/\Gamma(K_S \to \pi^0\pi^0)} \tag{3·26}$$

$$\sim 1 + 6\mathrm{Re}(\epsilon'/\epsilon). \tag{3·27}$$

If $R \neq 1$, it demonstrates the existence of direct CP violation.

3.4. *Kobayashi-Maskawa model*

In 1973, Kobayashi-Maskawa showed that CP violation can be produced naturally if there are three generations of quarks.[9] Equation (3·28) shows the Cabibbo-Kobayashi-Maskawa matrix written in terms of the Wolfenstein parameters (λ, A, ρ, η):[10]

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (3{\cdot}28)$$

where λ is the Cabibbo angle and η parameterizes any CP violation. In this model, which became a part of the Standard Model, a box diagram (Fig. 4(a)) that changes $\overline{K}^0$ to K^0 with a t quark has a complex amplitude proportional to $-\lambda^6(1-\rho-i\eta)$. Since a box diagram with two intermediate c quarks has an amplitude proportional to λ^2, it can introduce an imaginary part of about $\lambda^4\eta \sim O(10^{-3})$ to M_{12}.

In addition, the Standard Model predicts that there can be *direct CP violation.* A second-order penguin diagram with a t quark, as shown in Fig. 4(c), can introduce an additional phase into the decay. While a $K \to \pi\pi$ tree diagram, as shown in Fig. 4(b), has an amplitude $\sim V_{us}V_{ud}^* \propto \lambda$, the amplitude of the penguin diagram is $\sim V_{ts}V_{td}^* \propto \lambda^5(1-\rho+i\eta)$. Thus, the penguin diagram can introduce an imaginary part of about $\lambda^4\eta \sim O(10^{-3})$ in Γ_{12}.

Therefore, a non-zero value of $\mathrm{Re}(\epsilon'/\epsilon)$ would support the Cabibbo-Kobayashi-Maskawa Model and reject the Superweak Model as the sole source of CP violation.

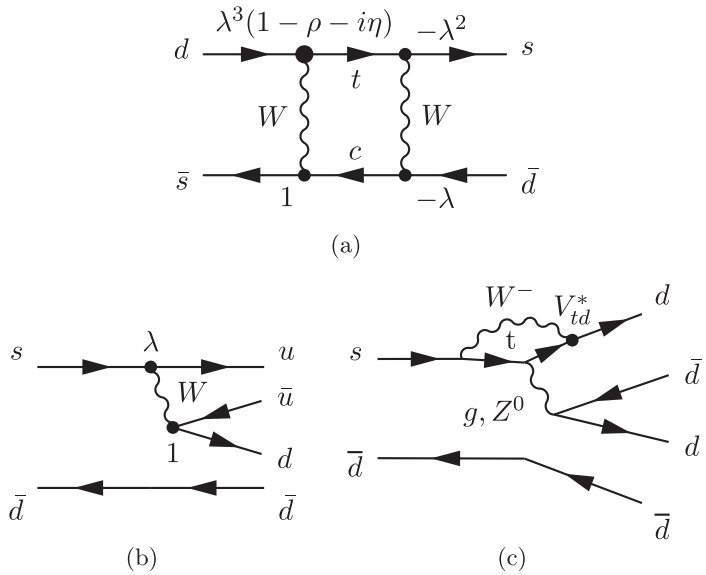

Fig. 4. (a) Box diagram that introduces indirect CP violation in K^0-$\overline{K}^0$ mixing. (b) Tree diagram for $K \to \pi\pi$. (c) Penguin diagram that can generate direct CP violation.

§4. Direct CP violation experiments

In this section, we will give a brief history of measurements of $\mathrm{Re}(\epsilon'/\epsilon)$.

4.1. *The early era: 1965 – 1980*

Attempts to look for a direct CP violation effect in 2π decays began immediately after the discovery of CP violation. The challenge was to detect the neutral pions while distinguishing them from copious backgrounds with photons. The two most precise experiments prior to the paper by Kobayashi and Maskawa were both published in 1972; one was a CERN experiment led by Carlo Rubbia[11] and the other a Princeton experiment led by Jim Cronin.[12] Each found a null result with a precision around 2% (in $\mathrm{Re}(\epsilon'/\epsilon)$).

The first calculation of the parameter ϵ' that we are aware of was published by Ellis, Gaillard, and Nanopoulos[13] in 1976. Their calculation was done in the framework of the Kobayashi-Maskawa model, which provided a connection between quark mixing and CP violation. By now, we know that nature does exploit such a connection, but at that time, this idea was not universally accepted. This Ellis, Gaillard, and Nanopoulos paper, together with subsequent ones by Gilman and Hagelin[14] and others, provided motivation for the next round of experiments. Although there were significant uncertainties, the predicted level of about $\mathrm{Re}(\epsilon'/\epsilon) = 2 \times 10^{-3}$ gave some guidance to the experimenters.

4.2. *The intermediate years: 1980 – 1995*

Fermilab E617[15] was proposed in early 1979. It deployed two side-by-side K_L beams. A regenerator placed in one of the beams provided a K_S component proportional to ρ (the regeneration amplitude). (Since $\sigma_T(\overline{K}^0 N) > \sigma_T(K^0 N)$, passing a K_L beam through material made of matter (N) changes the balance of K^0 and $\overline{K}^0$, "regenerating" a K_S component.) Then the ratio of the decay rates in the two beams is, to first order, proportional to either $\frac{\rho}{\eta_{+-}}$ or $\frac{\rho}{\eta_{00}}$, depending if one is detecting charged or neutral pions. The final states were detected then with large planar drift chambers and a lead-glass array. The experiment collected about 3000 $K_L \to \pi^0\pi^0$ decays. It was limited by a large neutron halo, which was about an order of magnitude greater than expected.

At around the same time, a group at BNL began their own experiment[16] with the same goal: finding another source of CP violation. This experiment also used lead glass and a regenerator, and collected about 1000 $K_L \to \pi^0\pi^0$ decays.

Measurement errors for both these experiments were dominated by statistics and while not making a positive detection, both significantly improved on the earlier generation of experiments, as can be seen in Fig. 5.

Later in this period, new efforts were born to further pursue direct CP violation. The E731 group at Fermilab built new wire chambers and reused the lead glass from E617; they also cleaned up the beam significantly.

The NA31 effort at CERN was launched with the novel idea of using no magnet, relying only on calorimetry to distinguish the decay modes from background. They ran at separate times to collect K_S or K_L decays. During the K_S run, a produc-

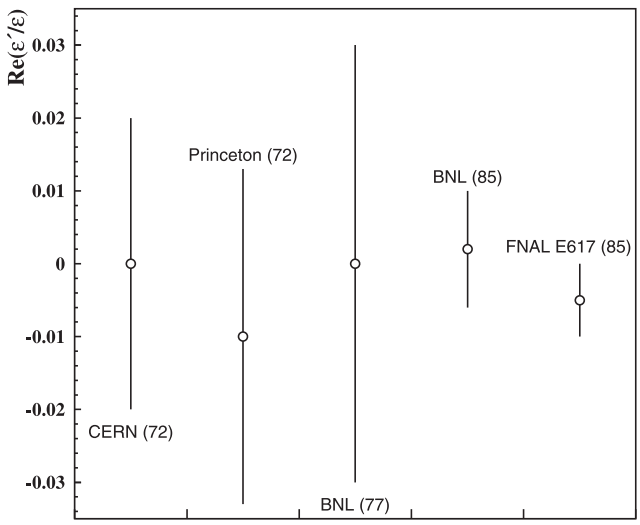

Fig. 5. Early measurements of $\mathrm{Re}(\epsilon'/\epsilon)$; the years of publication are indicated in parentheses.

tion target was moved inside the decay region to mimic the flat K_L decay vertex distribution. In this era, there were also two new "collider" experiments: CPLEAR at CERN producing K^0 via $p\bar{p} \to K^0 K^- \pi^+$, and KLOE at Frascati producing $e^+e^- \to \phi \to K^0\overline{K}^0/K_L K_S$. Clearly the question was of great interest!

Figure 6 shows the results on $\mathrm{Re}(\epsilon'/\epsilon)$ for E731, NA31, and all following experiments. After a first result from E731 in 1987,[17] NA31[18] presented "evidence" for direct CP violation, a result at exactly 3σ. The final result by E731[19] was consistent with 0 within 1.2σ, while that by NA31[20] was 3.5σ away from 0.

4.3. *The last generation, 1995 – 2009: KTeV and NA48*

The most recent Fermilab and CERN experiments used approaches that were similar in many respects but with important differences. The FNAL group in KTeV continued using basically the same technique as the earlier FNAL experiment, collecting decays from side-by-side K_L and K_S beams **simultaneously**. Especially at higher intensities, they did not want to separately normalize these two different decay modes, with such different conditions in the detectors. The CERN group also moved to a double-beam experiment with NA48. By this time, the theoretical predictions were at the level of a few times 10^{-4}, setting a scale for the experiments.

The experimental challenge in measuring $\mathrm{Re}(\epsilon'/\epsilon)$ with a sensitivity of 10^{-4} was to collect millions of each of the four decay modes in Eq. (3·27), and to understand relative acceptances between different modes at better than the 10^{-3} level. Both KTeV (Fig. 7) and NA48 (Fig. 8) collected all 4 decay modes simultaneously using two beams — one for K_L decays and one for K_S decays. Each detector included a long, evacuated decay region, followed by a charged spectrometer and a very precise electromagnetic calorimeter. The KTeV calorimeter used pure CsI crystals and NA48

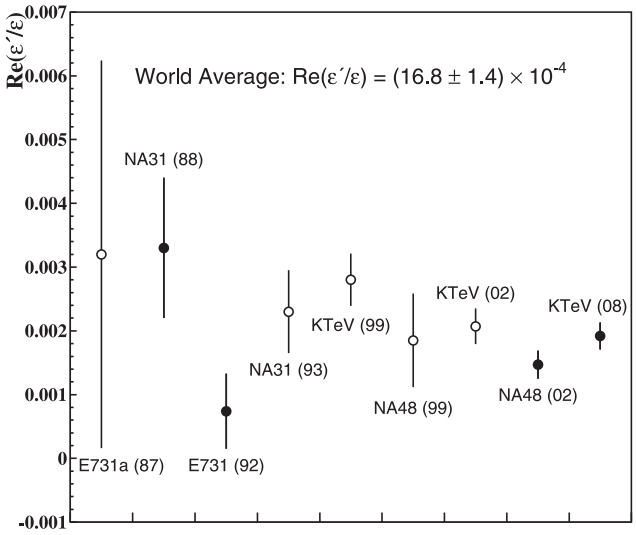

Fig. 6. Recent measurements of $\mathrm{Re}(\epsilon'/\epsilon)$; the years of publication are indicated in parentheses. The solid points, which supersede earlier measurements from the same groups, are used in the world average.

used liquid krypton. Both calorimeters had excellent energy and position resolution; the average energy resolution was better than 1% and the average position resolution was about 1 mm for both experiments. The performance of these calorimeters was crucial to the success of the experiments because the reconstructed position of decays along the beamline depends directly on the energy scale of the calorimeter. The excellent energy resolution also reduced background for both the $\pi^+\pi^-$ and $\pi^0\pi^0$ decay modes.

The principal difference between KTeV and NA48 was the method used to produce K_S decays. KTeV used a regenerator in one of the two beams to produce a K_S component through coherent regeneration. The KTeV regenerator was fully active to reduce the background from inelastic interactions. Figure 9 illustrates the interference of K_L and K_S downstream of the regenerator. NA48 used a bent crystal to transport a small fraction of protons that did not interact in the primary (K_L) target to a secondary (K_S) target close to the experiment. A time coincidence between the detector (e.g., the calorimeter for the $K \to 2\pi^0$ decay mode) and a counter placed in the proton beam upstream of the K_S target was used to identify K_S decays. Figure 10 shows the time difference between the tagging counter and the detector for charged and neutral decays.

The difference between the K_L and K_S lifetimes means that the distribution of decay positions along the beam (z) direction will be very different for the K_L and K_S decays which must be compared to extract $\mathrm{Re}(\epsilon'/\epsilon)$. Figure 11 shows z distributions from KTeV for the 4 decay modes. KTeV corrected for the variation in detector acceptance as a function of z with a Monte Carlo simulation. The quality

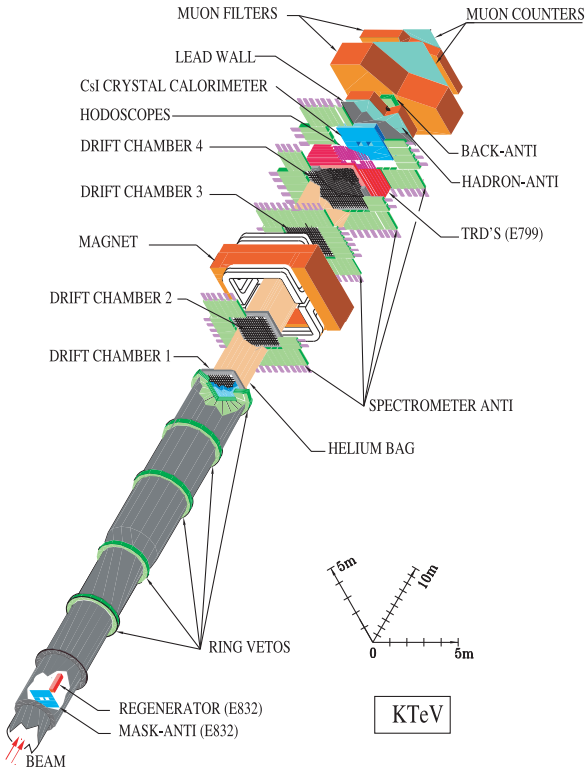

Fig. 7. Diagram of the KTeV detector.

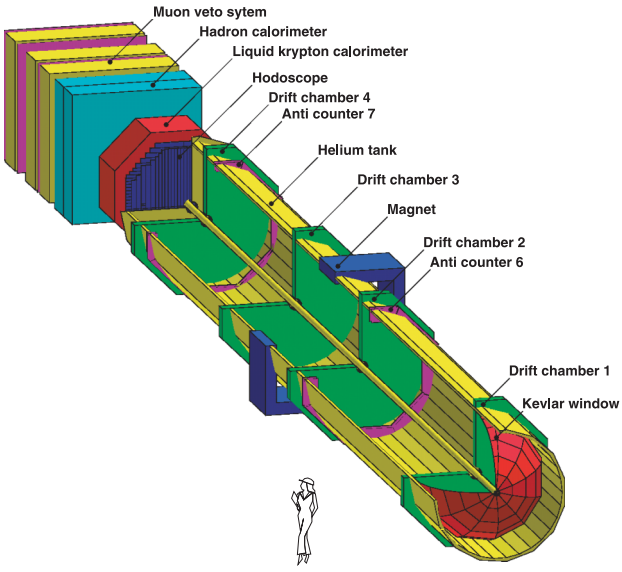

Fig. 8. Diagram of the NA48 detector.

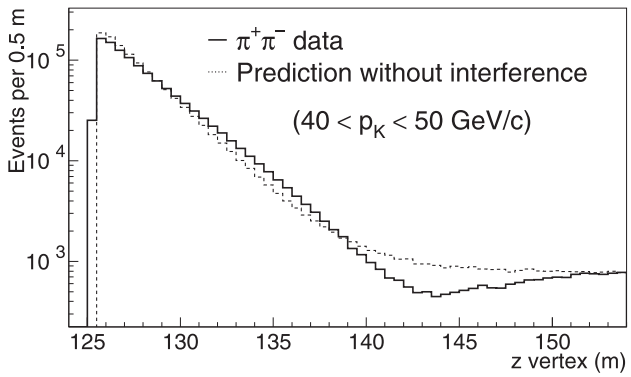

Fig. 9. z decay distribution of $K_L \to \pi^+\pi^-$ decays downstream of the KTeV regenerator for the restricted momentum range 40–50 GeV/c.

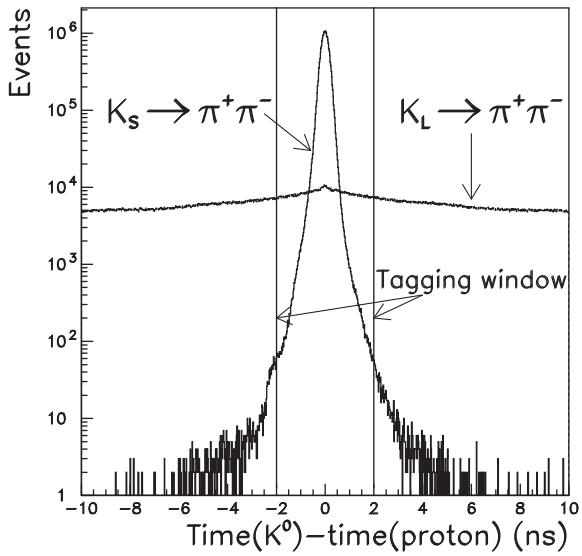

Fig. 10. Time difference between NA48 K_S tagging counter and detector for $K_S \to \pi^+\pi^-$ and $K_L \to \pi^+\pi^-$ events, identified by the reconstructed vertex.

of the simulation was studied using distributions from both the 2π decays, as well as higher statistics $K_L \to 3\pi^0$, and $K_L \to \pi e\nu$ decays.*)

NA48 greatly reduced the necessary acceptance correction by reweighting K_L decays to have the same z distribution as K_S decays (see Fig. 12). In exchange, it increased the statistical uncertainty in the result by about a factor of 2.

KTeV published its first result in 1999; it was 6.8 standard deviations from

*) The acceptance correction amounts to $\sim 5\%$, which comes mainly from a geometrical piece with negligible uncertainty. More subtle features of the detector response near edges, the effects of cuts in the analysis, etc, amount to a correction of only about 3×10^{-4} in $\mathrm{Re}(\epsilon'/\epsilon)$.

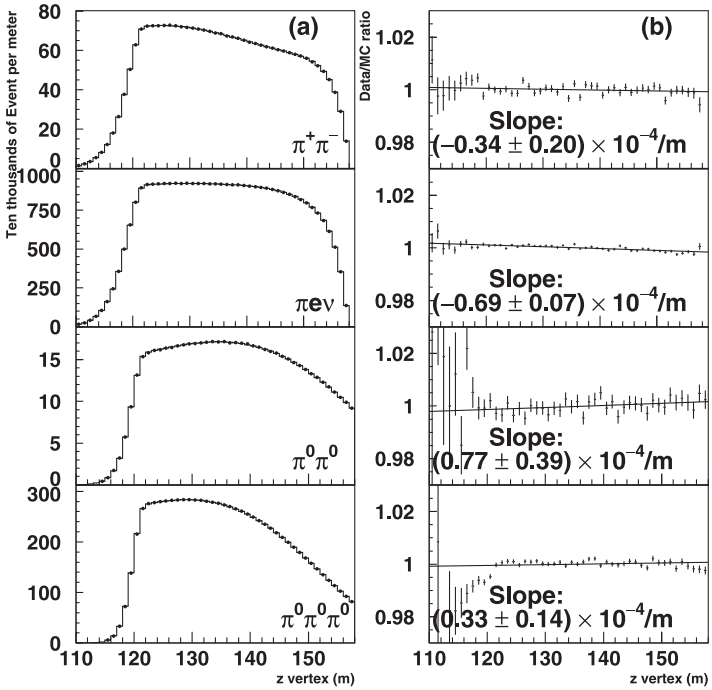

Fig. 11. KTeV comparison of the vacuum beam z distributions for data (dots) and MC (histogram). The data-to-MC ratios on the right are fit to a line, and the slopes are shown.

zero,[21] enough to claim that the effect was "established". This analysis was done "blind", with an unknown offset added which was only removed after all cuts and acceptance criteria and systematic error studies had been completed. In that same year, NA48 presented its first result,[22] between that from E731 and NA31, and agreeing well with both. It should be noted that this result appeared before CP violation was first observed in the B meson system.[23),24]

The final results of the experiments are[25),26]

$$\mathrm{Re}(\epsilon'/\epsilon) = (19.2 \pm 2.1) \times 10^{-4}, \quad \text{(KTeV)}$$
$$\mathrm{Re}(\epsilon'/\epsilon) = (14.7 \pm 2.2) \times 10^{-4}. \quad \text{(NA48)}$$

These measurements agree at the 1.5 sigma level, and clearly demonstrate the existence of direct CP violation. A weighted average of all measurements gives $\mathrm{Re}(\epsilon'/\epsilon) = (16.8 \pm 1.4) \times 10^{-4}$ with a confidence level of 13%.

Although these experiments were designed to measure $\mathrm{Re}(\epsilon'/\epsilon)$, they also have made greatly improved measurements of many parameters of the neutral kaon system. For example, both KTeV and NA48 made measurements of the K_S lifetime that are more precise than the average of all previous measurements. KTeV also measured the interference pattern downstream of the regenerator (see Fig. 9) to make precise measurements of the $K_L - K_S$ mass difference, and to measure the relative phases between the CP violating and CP conserving decay amplitudes for

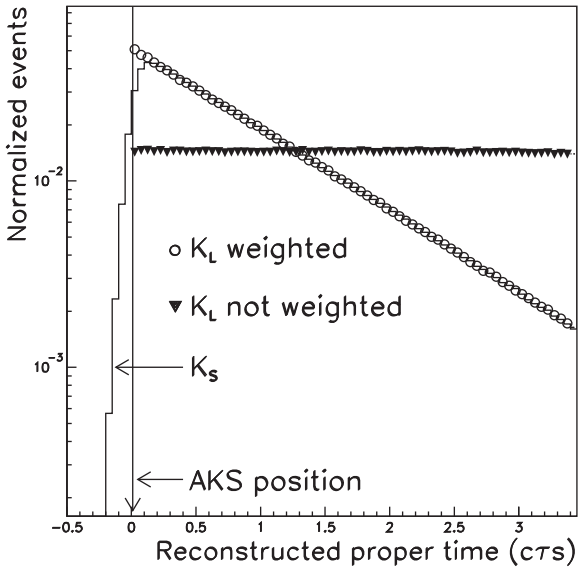

Fig. 12. Decay vertex distributions from NA48 for $K \to \pi\pi$ before and after lifetime reweighting.

$K_L \to \pi^+\pi^-$ (ϕ_{+-}) and for $K_L \to \pi^0\pi^0$ (ϕ_{00}).

We note that tremendous progress has been made: the early experiments collected about 100 K_L decays to $\pi^0\pi^0$; E731 had 30 times more: 3100 events; and the full KTeV sample has about 6 million $K_L \to \pi^0\pi^0$ events. The large increase in statistics was matched by a great improvement in control of systematic uncertainties (using event reweighting in NA48 and a Monte Carlo simulation of acceptance in KTeV). The investment in understanding detector performance for the measurement of $\mathrm{Re}(\epsilon'/\epsilon)$ also laid the groundwork for a host of additional measurements, such as the measurement of V_{us} described in the following section.

§5. Measurements of V_{us}: Testing CKM unitarity

Kaon experiments have also played a key role in testing the consistency of the CKM model. In particular, kaon decays are used to determine $V_{us} = \lambda$, which is a critical ingredient in determinations of other parameters and in CKM unitarity tests.

The first row of the Cabibbo-Kobayashi-Maskawa matrix provides the stringent test of the unitarity of the matrix. For about 20 years, measurements deviated from unitarity at the 2 sigma level. For example, the PDG 2002 review[27)] quoted $1-(|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2) = 0.0043\pm0.0019$. $|V_{us}|$, which contributes an uncertainty of 0.0010 to this unitarity test, is determined from charged and neutral semileptonic kaon ($K\ell3$) decay rates:

$$\Gamma_{K\ell3} \propto \frac{G_F^2 M_K^5}{192\pi^3}\,|V_{us}|^2\, f_+^2(0) I_K^\ell.$$

Here, ℓ refers to either e or μ, G_F is the Fermi constant, M_K is the kaon mass, $f_+(0)$ is the calculated form factor at zero momentum transfer for the $\ell\nu$ system, and I_K^ℓ is the phase-space integral, which depends on measured semileptonic form factors. There are additional factors for radiative corrections and to account for difference between charged and neutral kaons. Until recently, most determinations of $|V_{us}|$ were based only on $K \to \pi e\nu$ decays; $K \to \pi\mu\nu$ decays were not been used because of large uncertainties in I_K^μ. The experimental measurements are the semileptonic decay widths (based on the semileptonic branching fractions and lifetime) and form factors (allowing calculation of the phase space integrals). Theory is needed for $f_+(0)$ and radiative corrections.

Many new measurements during the last few years have resulted in a significant shift in V_{us}. Most importantly, recent measurements of the $K \to \pi e\nu$ branching fractions are significantly different than earlier PDG averages, probably as a result of inadequate treatment of radiation in older experiments. This effect was first observed by BNL E865[28] in the charged kaon system and then by KTeV[29),30)] in the neutral kaon system; subsequent measurements were made by KLOE,[31)–34)] NA48,[35)–37)] and ISTRA+.[38)] Current averages (e.g., by the PDG[39)] or Flavianet[40)]) of the semileptonic branching fractions are based only on recent, high-statistics experiments where the treatment of radiation is clear. In addition to measurements of branching fractions, new measurements of lifetimes[41)] and form factors,[42)–46)] have resulted in improved precision for all of the experimental inputs to V_{us}. Precise measurements of form factors for $K_{\mu 3}$ decay now make it possible to use both semileptonic decay modes to extract V_{us}.

Averaging recent kaon experiments and using the Leutwyler-Roos calculation of $f_+(0)$ gives[47)]

$$|V_{us}| = \lambda = 0.2255 \pm 0.0019.$$

Combining this result with the value of V_{ud} from superallowed nuclear beta decays,[47)] $V_{ud} = 0.97418 \pm 0.00027$, gives excellent agreement with unitarity, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9991(10)$, and provides strong confirmation of CKM model.

§6. Measurements of ρ and η: Searching for physics beyond the Standard Model

After the Kobayashi-Maskawa mechanism for CP violation was established by the observation of a non-zero $\mathrm{Re}(\epsilon'/\epsilon)$ in kaon decays and CP violating asymmetries in B meson decays, K and B experiments shifted their focus to the precise determination of the ρ and η parameters, and the search for new physics beyond the Standard Model. If the Kobayashi-Maskawa mechanism were the only source of the CP violation, then all the measurements should give consistent values of ρ and η. We know, however, that the CP violation mechanism in the Standard Model is not large enough to explain the matter dominance in the universe. Effects of new physics processes, which could provide additional sources of CP violation, depend on the decays, and therefore could make the "measured ρ and η parameters" inconsistent between different physics processes.

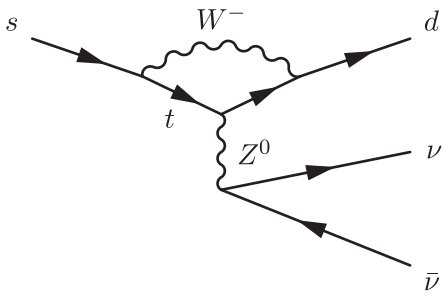

Fig. 13. Penguin diagram for $K \to \pi\nu\overline{\nu}$.

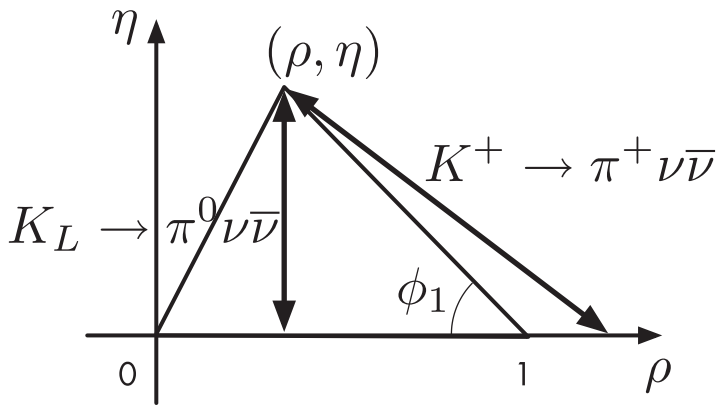

Fig. 14. Unitarity triangle.

New kaon experiments are focusing on $K \to \pi\nu\overline{\nu}$ decay modes because they have small theoretical uncertainties.[48] ($\mathrm{Re}(\epsilon'/\epsilon)$ is proportional to η, but hadronic uncertainties are too large to extract a meaningful value of η in spite of the precise measurements.) The $K \to \pi\nu\overline{\nu}$ decay modes proceed through a penguin diagram as shown in Fig. 13. In the Standard Model, the amplitude is dominated by t quark in the loop. Since quarks from three generations, d, s and t, are involved, the decay amplitude has an imaginary part of the CKM matrix, $V_{td} = A\lambda^3(1 - \rho - i\eta)$.

The $K_L \to \pi^0\nu\overline{\nu}$ decay amplitude is

$$\langle \pi^0\nu\overline{\nu}|H|K_L\rangle \simeq \langle \pi^0\nu\overline{\nu}|H|K_{odd}\rangle \tag{6·1}$$

$$\propto \langle \pi^0\nu\overline{\nu}|H|K^0\rangle - \langle \pi^0\nu\overline{\nu}|H|\overline{K}^0\rangle \tag{6·2}$$

$$\propto V_{td} - V_{td}^* \propto i\mathrm{Im}(V_{td}) = i\lambda^3\eta. \tag{6·3}$$

Therefore a measurement of $BR(K_L \to \pi^0\nu\overline{\nu})$ determines the height of the unitarity triangle, η, as shown in Fig. 14. The branching ratio is predicted to be $(2.76 \pm 0.40) \times 10^{-11}$, based on currently known Standard Model parameters.[49] The intrinsic theoretical uncertainty on the η measurement is 2.8%.

For the $K^+ \to \pi^+\nu\overline{\nu}$ decay mode, the branching ratio is proportional to $|V_{td}|^2$ with a correction for the c quark contribution. Thus, it effectively measures one of

the sides of the unitarity triangle. The estimated branching ratio is $(8.5\pm0.7)\times10^{-11}$ with a theoretical uncertainty of 5%.[50]

If there is another source of CP violation, particles related to the new physics can enter the penguin diagram for $K \to \pi\nu\overline{\nu}$ decays and change the branching ratios from the Standard Model predictions. For example, various SUSY models,[51] littlest Higgs model,[52),53] four-generation quark model,[54] etc., predict a wide range of branching ratios. The $K_L \to \pi^0\nu\overline{\nu}$ branching fraction is only constrained by the Grossman-Nir bound,[55] $BR(K_L \to \pi^0\nu\overline{\nu}) < 1.4 \times 10^{-9}$ (90% CL), set by applying an isospin rotation to the measured $K^+ \to \pi^+\nu\overline{\nu}$ branching ratio.

In order to see the deviation from the Standard Model prediction, we need precise measurements of the unitarity triangle parameters for comparison. For example, the $B \to J/\Psi K_S$ time-dependent CP asymmetry provides a precise measurement of the $\phi_1 = \beta$ in the Standard Model.

In the following sections, we will introduce experiments for the $K_L \to \pi^0\nu\overline{\nu}$ and $K^+ \to \pi^+\nu\overline{\nu}$ decay modes.

6.1. $K_L \to \pi^0\nu\overline{\nu}$

The signature of $K_L \to \pi^0\nu\overline{\nu}$ decay is two photons from a π^0 decay coming from a neutral beam with missing transverse momentum from the $\nu\overline{\nu}$. The major background is the $K_L \to \pi^0\pi^0$ decay, where two of the four photons are missed due to detector inefficiencies. Therefore, the decay region should be covered hermetically by photon detectors. A high intensity K_L beam is also needed to observe the $K_L \to \pi^0\nu\overline{\nu}$ decay because of the very small branching ratio.

6.1.1. KEK E391a experiment

The KEK E391a experiment was the first experiment dedicated for the $K_L \to \pi^0\nu\overline{\nu}$ decay mode.

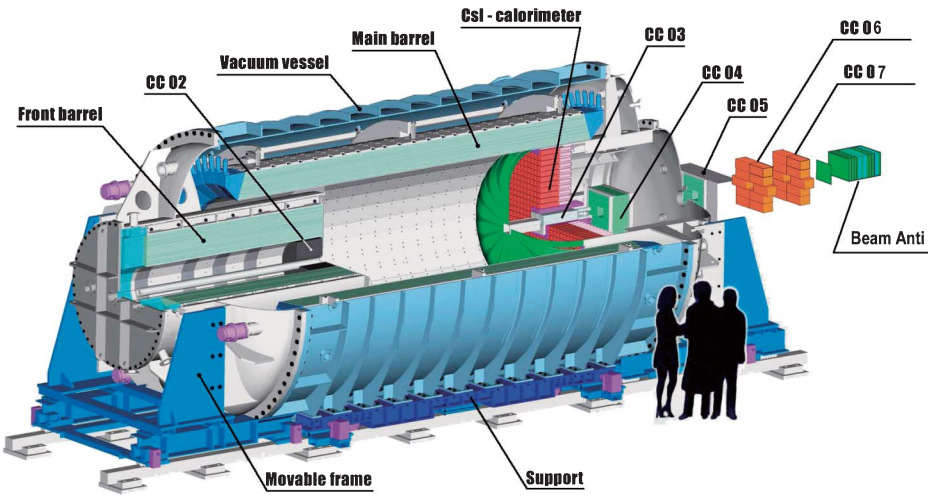

Fig. 15. KEK E391a detector.

Figure 15 shows the E391a detector. The energy and hit position of two photons from the decay were measured with the pure CsI calorimeter located downstream of the decay region. The decay region was covered with a hermetic photon veto detector made of Pb and scintillator. Another set of photon detectors were located downstream of the calorimeter to detect photons escaping through a beam hole in the calorimeter. In order to suppress π^0s produced by beam neutrons interacting with residual gas, the decay region was evacuated to 10^{-5} Pa. Most of the detector components were placed inside the vacuum tank, since otherwise the vacuum tank would absorb photons before hitting the detector. Based on no observed events inside the signal region, a limit of $BR(K_L \to \pi^0\nu\overline{\nu}) < 6.7 \times 10^{-8}$ (90% CL) was set.[56)]

6.1.2. J-PARC E14 K^{O}TO experiment

J-PARC E14 K^{O}TO experiment is being prepared to observe $K_L \to \pi^0\nu\overline{\nu}$ events. J-PARC is a new accelerator complex in Japan designed to deliver 3×10^{14} 30 GeV/c protons every 3.3 s.

The experiment uses a completely redesigned neutral beamline to suppress beam halo neutrons, and utilizes the existing E391a detector but with many modifications. To improve background rejection, the CsI crystals in the electromagnetic calorimeter will be replaced by much finer and longer CsI crystals used by the Fermilab KTeV experiment. A new photon veto with lead and aerogel modules will be used to veto photons escaping down the beam hole in the calorimeter, while being insensitive to neutrons in the beam. The waveform of all the detector elements will be recorded to understand events fully in a high rate environment.

With 2×10^{14} protons per spill and 3×10^7 s of running time, the experiment expects to observe 3.5 Standard Model signal events with an S/N ratio of 1.4.[57)]

After the E14 experiment, the plan is to make an optimized neutral beam line with a smaller targeting angle (to get a higher K_L yield) and larger detector to collect > 100 signal events.

6.2. $K^+ \to \pi^+\nu\overline{\nu}$

The signature of the $K^+ \to \pi^+\nu\overline{\nu}$ decay is a single π^+ coming from the K^+. Major backgrounds are $K^+ \to \pi^+\pi^0$ with two missing photons, $K^+ \to \mu^+\nu$ with misidentifying μ^+ as π^+, etc.

6.2.1. BNL E787/E949 experiments

The decay $K^+ \to \pi^+\nu\overline{\nu}$ was first observed by BNL E787, and by the succeeding E949 experiment. They stopped K^+ in a target, and looked for a single π^+ from the decay. Using stopped kaons made charged particles from two-body decay background events have a unique momentum, while the signal events had a wider spectrum. The target was surrounded by a spectrometer and range counters to identify π^+ using the momentum, range and energy. In addition, the $\pi^+ \to \mu^+ \to e^+$ decay chain was traced to help identify the π^+. In total, they observed 7 events, and measured $BR(K^+ \to \pi^+\nu\overline{\nu}) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$.[58)]

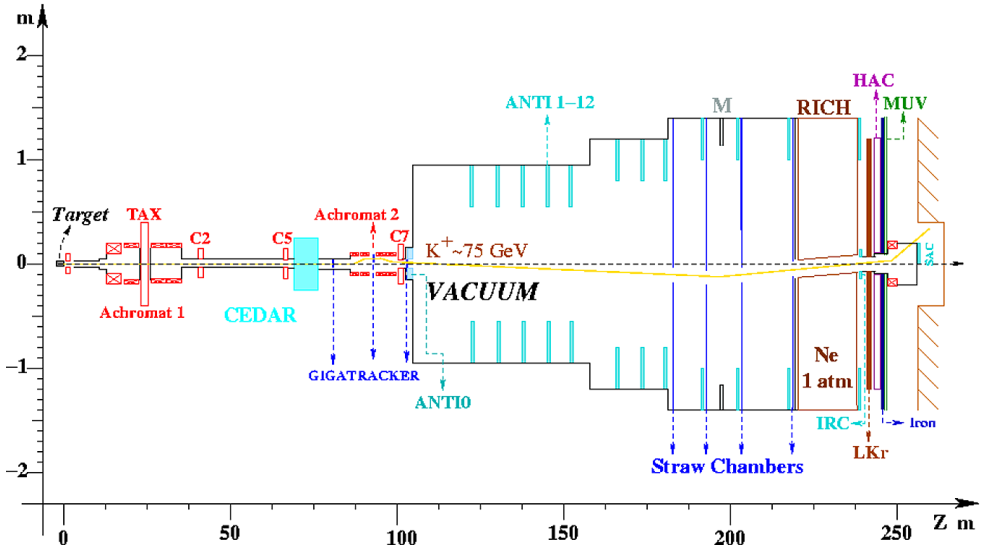

Fig. 16. Plan view of the beam line and detector for CERN NA62.

6.2.2. CERN NA62 experiment

The new CERN NA62 experiment[59)] takes a completely different approach to make a more accurate measurement of the branching ratio of $K^+ \to \pi^+\nu\overline{\nu}$. This experiment is designed to observe π^+s decaying from K^+s in flight, instead of from stopped kaons. By not having a kaon stopping target which produces extra particles, the experiment can run at higher intensity. Figure 16 shows the plan view of the experiment. Charged kaons in the 75 GeV/c 800 MHz beam are selected by a differential Cerenkov counter. The π^+s from the decay are identified by a fast ring imaging Cerenkov counter. Backgrounds with photons are suppressed by a series of annular photon veto counters and a liquid Kr calorimeter placed downstream. The momenta of the π^+s are measured with a magnetic spectrometer. The position and direction of incoming K^+s are measured with a special tracking device capable of running in a high intensity beam. This allows a high resolution measurement of the missing mass, $M^2_{miss} = P^2_K - P^2_\pi$, to suppress backgrounds kinematically. With one year of running, the experiment plans to collect 65 signal events with 9 background events.

§7. Conclusion

The neutral kaon has been studied for more than 60 years and continues to be a unique source of information about basic symmetries of nature. One fascinating aspect of the neutral kaon system is that their interactions with all four forces can be studied precisely (see Table I). Their mass comes from the strong interaction; the $K_L - K_S$ mass difference from the weak interaction; they have a charge radius

Table I. The (approximate) hierarchy in the neutral Kaon system. Phenomena span 20 orders-of-magnitude.

K Mass	M_K	500 MeV
K_S, K_L mass difference	Δm	3.5×10^{-6} eV
CP mixing parameter	ϵ	7×10^{-9} eV
$K^0, \overline{K}^0$ mass difference	$M_K^0 - M_{\overline{K}}^0$	$< 3.5 \times 10^{-10}$ eV
Direct CPV parameter	ϵ'	10^{-11} eV

due to their electromagnetic interaction; and a possible mass difference between the K^0 and $\overline{K}^0$ can be precisely limited from the effects the different gravitational pulls would have on the mixing.

After more than 30 years of effort, direct CP violation was established, giving strong support to the CKM model and ruling out the Superweak Model as the sole source of CP violation. In addition, all measurements in the kaon system are consistent with CPT invariance, meaning that CP violation is accompanied by T violation.

Now, kaons have changed their role, and have become a sensitive probe to search for CP violation in new physics beyond the Standard Model.

References

1) G. D. Rochester and C. C. Butler, Nature **160** (1947), 855.
2) P. Galison, *Image and Logic* (The University of Chicago Press, Chicago, 1997), p. 126.
3) M. Gell-Mann and A. Pais, Phys. Rev. **97** (1955), 1387.
4) K. Lande, E. T. Booth, J. Empeduglia, L. M. Lederman and W. Chinowsky, Phys. Rev. **103** (1956), 1901.
5) J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, Phys. Rev. Lett. **13** (1964), 138.
6) S. Bennett et al., Phys. Rev. Lett. **19** (1967), 993.
7) A. Angelopoulos et al. (CPLEAR Collab.), Phys. Lett. B **444** (1998), 43.
8) L. Wolfenstein, Phys. Rev. Lett. **13** (1964), 562.
9) M. Kobayashi and K. Maskawa, Prog. Theor. Phys. **49** (1973), 652.
10) L. Wolfenstein, Phys. Rev. Lett. **51** (1983), 1945.
11) M. Holder et al., Phys. Lett. B **40** (1972), 141.
12) M. Banner et al., Phys. Rev. Lett. **28** (1972), 1597.
13) J. Ellis, M. K. Galliard and D. V. Nanopoulos, Nucl. Phys. B **109** (1976), 213.
14) F. J. Gilman and J. S. Hagelin, Phys. Lett. B **133** (1983), 443.
15) R. H. Bernstein et al. (FNAL E617 Collab.), Phys. Rev. Lett. **54** (1986), 1631.
16) J. K. Black et al. (BNL E749 Collab.), Phys. Rev. Lett. **54** (1985), 1628.
17) M. Woods et al. (FNAL E731 Collab.), Phys. Rev. Lett. **60** (1988), 1695.
18) H. Burkhardt et al. (CERN NA31 Collab.), Phys. Lett. B **206** (1988), 169.
19) L. K. Gibbons et al. (FNAL E731 Collab.), Phys. Rev. Lett. **70** (1993), 1203.
20) D. G . Barr et al. (CERN NA31 Collab.), Phys. Lett. B **317** (1993), 233.
21) A. Alavi-Harati et al. (KTeV Collab.), Phys. Rev. Lett. **83** (1999), 22.
22) A. J. Bevan et al. (CERN NA48 Collab.), Phys. Lett. B **465** (1999), 335.
23) B. Aubert et al. (BaBar Collab.), Phys. Rev. Lett. **89** (2002), 201802.
24) K. Abe et al. (Belle Collab.), Phys. Rev. D **66** (2002), 071102.
25) E. Worcester, *Fermilab Wine and Cheese* (2008).
A. Alavi-Harati et al. (KTeV Collab.), Phys. Rev. D **67** (2003), 012005.
26) J. R. Batley et al. (CERN NA48 Collab.), Phys. Lett. B **544** (2002), 97.
27) Particle Data Group: K. Hagiwara et al., Phys. Rev. D **66** (2002), 010001.

28) A. Sher et al. (BNL865 Collab.), Phys. Rev. Lett. **91** (2003), 261802.
29) T. Alexopoulos (KTeV Collab.), Phys. Rev. Lett. **93** (2004), 181802.
30) T. Alexopoulos et al. (KTeV Collab.), Phys. Rev. D **70** (2004), 092006.
31) F. Ambrosino et al. (KLOE Collab.), Phys. Lett. B **632** (2006), 43.
32) F. Ambrosino et al. (KLOE Collab.), Phys. Lett. B **638** (2006), 140.
33) F. Ambrosino et al. (KLOE Collab.), Phys. Lett. B **636** (2006), 173.
34) F. Ambrosino et al. (KLOE Collab.), PoS(HEP2005)287, Frascati Phys. Ser. **41** (2006), 69, hep-ex/0510028.
35) A. Lai et al. (CERN NA48 Collab.), Phys. Lett. B **602** (2004), 41.
36) A. Lai et al. (CERN NA48 Collab.), Phys. Lett. B **645** (2007), 26.
37) J. R. Batley et al. (CERN NA48/2 Collab.), Eur. Phys. J. C **50** (2007), 329.
38) V. I. Romanovsky et al., arXiv:0704.2052.
39) Particle Data Group: C. Amsler et al., Phys. Lett. B **667** (2008), 1.
40) `http://www.lnf.infn.it/theory/flavianet/`
41) F. Ambrosino et al. (KLOE Collab.), Phys. Lett. B **626** (2005), 15.
42) T. Alexopoulos et al. (KTeV Collab.), Phys. Rev. D **70** (2004), 092007.
43) E. Abouzaid et al. (KTeV Collab.), Phys. Rev. D **74** (2006), 097101.
44) F. Ambrosino et al. (KLOE Collab.), Phys. Lett. B **636** (2006), 166.
45) A. Lai et al. (CERN NA48 Collab.), Phys. Lett. B **604** (2004), 1.
46) O. P. Yushchenko et al., Phys. Lett. B **589** (2004), 111.
47) E. Blucher and W. J. Marciano, Phys. Lett. B **667** (2008), 733.
48) L. S. Littenberg, Phys. Rev. D **39** (1989), 3322.
49) F. Mescia and C. Smith, Phys. Rev. D **76** (2007), 034017, and its update, `http://www.lnf.infn.it/wg/vus/content/Note_RareKdecays.pdf`
50) J. Brod and M. Gorbahn, Phys. Rev. D **78** (2008), 034006.
51) D. Bryman, A. J. Buras, G. Isidori and L. S. Littenberg, hep-ph/0505171, Int. J. Mod. Phys. A **21** (2006), 487, and references therein.
52) M. Blanke et al., J. High Energy Phys. **01** (2007), 066.
53) T. Goto, Y. Okada and Y. Yamamoto, Phys. Lett. B **670** (2009), 378.
54) W.-S. Hou et al., Phys. Rev. D **72** (2005), 115007.
55) Y. Grossman and Y. Nir, Phys. Lett. B **398** (1997), 163.
56) J. K. Ahn et al. (KEK PS E391a Collab.), Phys. Rev. Lett. **100** (2008), 201802.
57) J. Comfort et al. (J-PARC E14 Collab.), "Proposal for $K_L \to \pi^0 \nu\overline{\nu}$ Experiment at J-PARC" (2006).
58) A. V. Artamonov et al. (BNL E949 Collab.), Phys. Rev. Lett. **101** (2008), 191802; Phys. Rev. D **79** (2009), 092004.
59) G. Anelli et al. (CERN NA62 Collab.), "Proposal to Measure the Rare Decay $K^+ \to \pi^+ \nu\overline{\nu}$ at the CERNSPS", CERN-SPSC-2005-013 (2005).